A

WRITTEN ARITHMETIC,

FOR

COMMON AND HIGHER SCHOOLS;

TO WHICH IS ADAPTED

A COMPLETE SYSTEM OF REVIEWS,

IN THE FORM OF

DICTATION EXERCISES.

BY

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PREFACE.

THIS book is designed to prepare the pupil to meet the demands of actual life. It is itself copious in examples of a great variety of forms, and largely of a practical character; and the accompanying Key contains a set of DICTATION EXERCISES, adapted to every important topic treated in the book, to be used at the discretion of the teacher, by means of which the amount of practice may be increased almost indefinitely.

All that the book contains is written for the pupil; and if he will learn it understandingly, he may master the principles of arithmetic with but little aid from the teacher.

In the arrangement of subjects, that order has been adopted which experience has shown to be the best for all classes of learners. Some subjects, of little importance, have been briefly treated; others have been transferred to the Appendix. Should any subject, as Duodecimals, Circulating Decimals, or Average of Accounts, or any examples prove too difficult for the younger scholar, they can be omitted till the book is reviewed.

Answers are given to the examples, so far as is necessary to assure the pupil that he understands the principles; but every important principle is likewise tested by examples having no answers in the book. The answers not contained in the book may be found in the Key, from which they can easily be transferred to the black-board, if the teacher prefers to have them placed before his pupils.

To determine the adaptation of a text-book to school purposes, it

must be used in the school-room. This treatise has already been successfully tested by this standard, since its general character has been determined by the actual demands of a large grammar school, at present and for several years in the charge of the undersigned, and since it is largely illustrated by examples which have been repeatedly employed to familiarize students with the principles they here exemplify. Its practical character is fully certified by the testimony of many of its students, now business men, who practise its methods in the office and in the counting-room.

Though, at the request of the publishers, but one name appears upon the title-page as author, the book is the joint production of the person whose name it bears, and of E. N. L. WALTON, former teacher in one of the State Normal Schools of Massachusetts; and whatever merits or defects the book may be found to possess, may be attributed equally to each.

Our grateful acknowledgments are due to many teachers and business men for valuable suggestions, particularly to WM. J. ROLFE, A. M., of Cambridge, for important criticisms while the work was in preparation for the press; to FRANCIS COGSWELL, Esq., of Cambridge, for hints on methods of Reviews; and to the teachers of the Oliver Grammar School, Lawrence, for their kind assistance in solving and testing examples.

GEO. A. WALTON.

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ARITHMETIC.

ARTICLE **1.** ARITHMETIC is the science of numbers, and the art of computing by them.

2. A **Unit** is one.

3. A **Number** is a unit, or a combination of units.

4. A **Concrete,** or **Denominate Number,** is a number which is applied to some object or objects; as, *one boy, two apples, three slate pencils, four sounds.*

5. An **Abstract Number** is a number which is not applied to any object; as, *one, two, three.*

6. EXERCISE.

Name the *concrete numbers* in the following list: —

Four girls; seven swans; two; ten; nine chairs; five knives; eight; twelve horses; six mules; two oxen; four; eleven; seven pond lilies; one; ten; thirteen; nine days; fifteen lessons; two rabbits; six bushels.

Name the *abstract numbers* in the above.

7. The fundamental operations of written Arithmetic are based upon NOTATION, and consist of ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION.

NOTATION AND NUMERATION.

8. **Notation** is the art of *writing numbers.*

Numeration is the art of *reading numbers.*

9. Besides being expressed in *words,* numbers are repre-

sented by *letters* and *figures.* The method of representing them by *letters* is called the ROMAN method, because it was used by the ancient Romans. The method of representing them by *figures* is called the ARABIC method, because our first knowledge of it was obtained from the Arabs.

ROMAN METHOD.

10. The Roman Method is principally used in writing dates, and in numbering chapters and sections of books.

11. It employs seven capital letters; I representing one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand.

12. By combining these letters in various ways, all numbers may be expressed, the following principles being observed:—

(1.) When a letter is *repeated, its value is repeated.*

(2.) When a letter is placed *before* another of greater value, its value is to be *taken from* that of the greater; thus, IV denotes four.

(3.) When a letter is placed *after* another of greater value, its value is to be *added to* that of the greater; thus, VI denotes six.

(4.) When a letter is placed *between* two of greater value, its value is to be *taken from* their *united* value; thus, XIX denotes nineteen.

(5.) Any letter may be made to express thousands instead of units by placing a dash over it. Thus $\overline{\mathrm{X}}$ denotes ten thousand; $\overline{\mathrm{D}}$, five hundred thousand; $\overline{\mathrm{M}}$, one thousand thousand, or one million.

13. TABLE OF ROMAN NOTATION.

I	denotes	one.	V	denotes	five.
II		two.	VI		six.
III		three.	VII		seven.
IV *		four.	VIII		eight.

* IIII is sometimes used for four.

IX *	denotes	nine.	L	denotes	fifty.
X		ten.	LX		sixty.
XI		eleven.	LXX		seventy.
XII		twelve.	LXXX		eighty.
XIII		thirteen.	XC		ninety.
XIV		fourteen.	C		one hundred.
XV		fifteen.	CC		two hundred.
XVI		sixteen.	CCC		three hundred.
XVII		seventeen.	CD		four hundred.
XVIII		eighteen.	D		five hundred.
XIX		nineteen.	DC		six hundred.
XX		twenty.	DCC		seven hundred.
XXI		twenty-one.	DCCC		eight hundred.
XXX		thirty.	CM		nine hundred.
XXXI		thirty-one.	M		one thousand.
XL		forty.	$\overline{\text{M}}$		one million.
XLI		forty-one.	$\overline{\text{MM}}$		two million.

14. Exercises.

Read or write in words the following numbers:—

1. IV.
2. XI.
3. XIV.
4. XIX.
5. XXVI.
6. XXIX.
7. XXXVI.
8. XL.
9. XLV.
10. XLIX.
11. LVIII.
12. LXXI.
13. LXXXIX.
14. XCVIII.
15. CLV.
16. CXIX.
17. CCCXLVII.
18. CDLXXII.
19. DCCXLIV.
20. MDXCIV.
21. MDCCCLXIV.
22. $\overline{\text{MD}}$.

15. Write the following in Roman characters:—

1. All the numbers from one to twenty, inclusive.
2. All the numbers from thirty to forty, inclusive.
3. All the numbers from ninety to one hundred, inclusive.
4. One hundred thirty-eight.

* VIIII is sometimes used for nine.

5. Three hundred twenty-four.
6. Four hundred forty-nine.
7. Five hundred eighty-six.
8. Seven hundred sixty-seven.
9. Nine hundred fifty-three.
10. One thousand four hundred seven.
11. Five thousand eight hundred. *Ans.* $\overline{\text{V}}$DCCC.
12. Ten thousand ninety-nine.
13. One thousand eight hundred sixty-four.

Arabic Method.

16. The Arabic Method of representing numbers employs ten characters, or figures, as follows:—

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Zero.

17. The first nine are called *digits*, from the Latin word *digitus*, a *finger*, it being supposed that the ancients first counted by their fingers. They are also called *significant* figures, because they are *signs* for numbers. The character, 0, called *zero*, signifies nothing when it stands alone. It is called a *figure of place* because, in writing numbers, it is used to fill places not occupied by other figures.

Used singly, these characters can represent only the numbers from one to nine; but combined according to the following principles, they are used to represent all numbers.

18. The figures which represent *simple units* are placed at the left of a dot, called the *decimal point.* (See Art. 23 and note.) The *first* place, therefore, at the left of the decimal point is called the *units' place;* thus, 7. is read "seven units," or "seven."

Having no single figure to represent ten units, we consider the collection of ten units as *one ten*, or *a unit of the second order*, and represent it by the figure 1 put in the tens' place, which is the *second* place from the decimal point towards the left; thus, 10. represents ten, the zero being used to fill the units' place, which would otherwise be vacant. If we have any number of

tens and units to write together, we put the number of tens in the tens' place, and of units in the units' place; thus, thirty-six, or three tens and six units, is written 36.

A collection of *ten tens* is called *one hundred*, or *a unit of the third order*, and is represented by 1 in the *third* or *hundreds' place*, (100.); two hundreds are represented by 2 in the hundreds' place, (200.); three hundreds by 3 in the hundreds' place, (300.); etc.

A collection of *ten hundreds* is called *one thousand*, or, *a unit of the fourth order*, and is represented by 1 in the *fourth* or *thousands' place* (1000.); two thousands are represented by 2 in the thousands' place (2000.); etc.

Thousands.	Hundreds.	Tens.	Units.	Decimal point.
7	6	3	2	.

The above represents seven thousands, six hundreds, three tens, and two units, and is read, "Seven thousand six hundred thirty-two."

Exercises.

19. Read the following: —

1.	86.	6.	428.	11.	9000
2.	132.	7.	1302.	12.	9090
3.	6321.	8.	6006.	13.	9009
4.	7862.	9.	7801.	14.	8017.
5.	99.	10.	541.		

20. Write in figures, —

1. One thousand six hundred forty-four.
2. Two thousand eight hundred twenty-one.
3. Nine hundred nine.
4. Six thousand two hundred ten.
5. Eight thousand eight.
6. Five thousand fifty.
7. Seven thousand seven hundred seventy.
8. Twenty-nine.
9. Six hundred two.
10. Six thousand twenty.

QUESTIONS. — What is the first place at the left of the decimal point called? What is the second place at the left called? The third? How many units make one ten? How many tens make one hundred? How many hundreds make one thousand? How many units make one hundred? How many units make one thousand? How many tens make one thousand? What are units of the first order called? *Ans.* Simple units. What are units of the second order called? Of the fourth? Of the third?

In 7632 how many tens, and what number remains? How many hundreds, and what remains? How many thousands?

REMARK. — The number of units of any order is sometimes called a *term;* thus, the terms of 632 are 6 hundreds, 3 tens, and 2 units.

21. NUMERATION TABLE.

7th Period. Quintillions.	2	21st. Hundred-quintillions.
	8	20th. Ten-quintillions.
	4,	19th. Quintillions.
6th Period. Quadrillions.	9	18th. Hundred-quadrillions.
	6	17th. Ten-quadrillions.
	3,	16th. Quadrillions.
5th Period. Trillions.	3	15th. Hundred-trillions.
	4	14th. Ten-trillions.
	8,	13th. Trillions.
4th Period. Billions.	2	12th. Hundred-billions.
	9	11th. Ten-billions.
	1,	10th. Billions.
3d Period. Millions.	8	9th. Hundred-millions.
	7	8th. Ten-millions.
	6,	7th. Millions.
2d Period. Thousands.	3	6th. Hundred-thousands.
	2	5th. Ten-thousands.
	2,	4th. Thousands.
1st Period. Units.	1	3d. Hundreds.
	2	2d. Tens.
	4	1st. Units.
	.	Decimal point.

22. The *fifth* place from the decimal point towards the left is the *ten thousands'* place, each ten-thousand being equal to ten of the thousands; the *sixth* place is the *hundred thousands'* place, each hundred thousand being equal to ten ten-thousands; and so on, each unit of any order being equal to ten units of the order immediately preceding.

We now see that the *number* of units of any order is expressed by the *figure*, and the *order* of units by the *place* which the figure occupies; or, in other words, *the value represented by any*

figure depends upon the figure itself, and upon the place which that figure occupies. Thus, 2 in the first place means simply two (that is, two units); in the second place, it means two tens, or twenty; in the third place, two hundreds.

23. Since, by this method of writing numbers, the value represented by a significant figure increases as that figure is removed towards the left, and decreases as it is removed towards the right, *by a scale of tens*, the system is called the **Decimal System**, from the Latin word *decem*, which signifies *ten*.

NOTE. — The reason for calling the dot (Art. 18) a *decimal point* must now be obvious. This point is not always written, but, when not written, it is always understood.

24. By examining the table (Art. 21), we find it separated by commas into groups of three places each. These groups are called *periods*, the first period being that of units; the second that of thousands; the third, millions; the fourth, billions, etc. Thus we have simple units, tens of units, and hundreds of units; units, tens, and hundreds of thousands; units, tens, and hundreds of millions; etc.

25. EXERCISES ON THE TABLE.

1. Give the names of the first two periods from the decimal point, reading them towards the left; towards the right. Give the names of the first three periods in the same way; of the first four; five; six; seven. What is the second period called? third? sixth? seventh? fourth? fifth?

2. In which period are found thousands? millions? simple units? trillions? billions? quintillions? quadrillions?

3. In which place of what period are found tens of units? thousands? hundred-thousands? millions? hundreds of units? ten-thousands? billions? hundred-millions? ten-billions? ten-millions? quadrillions? ten-quintillions? hundred-billions? hundred-trillions? quintillions? ten-quadrillions? ten-trillions? hundred-quadrillions? trillions? hundred-quintillions?

4. Name the order of units of each number in paragraph 3. *Ans.* Tens are of the second order, thousands of the fourth order; etc.

5. What order of units is found in the first place of the second period? *Ans.* Fourth order, or thousands. In the third place of the

first period? In the second place of the third period? In the third place of the fourth period? In the first place of the fifth period? In the third place of the sixth period? In the second place of the seventh period? In the third place of the third period? In the first place of the seventh period? In the second place of the fourth period? In the first place of the sixth period?

6. In 6480921 how many tens, and what remains? *Ans.* 648092 tens, and 1 unit remaining. How many hundreds, and what remains? *Ans.* 64809 hundreds, and 21 remaining. How many millions, and what remains? thousands? ten-thousands? hundred-thousands?

26. The names of the periods employed to express numbers higher than Quintillions are, in their order from Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novendecillions, Vigintillions, etc.

27. To read numbers, observe the following

RULE. — *Beginning at the units' place, point off the expression into periods of three figures each; then begin at the left, and read each period in order from left to right, giving after each, excepting the last, the name of the period.*

EXERCISES.

28. Read or write in words the following: —

1.	361.	13.	987654321.
2.	786.	14.	89743208.
3.	3261.	15.	1122334455.
4.	96321.	16.	3670980347.
5.	9301.	17.	9008007006.
6.	80021.	18.	12400496623.
7.	654237.	19.	245607000000.
8.	9326429.	20.	94632748632.
9.	9000200.	21.	1781006390800.
10.	86320029.	22.	62876432019623
11.	324867.	23.	753248734762869.
12.	81402020.	24.	943300896402798.

☞ See Dictation Exercises, Key.

29. Name the terms in the first example above, commencing with units (Art. 20, Remark). *Ans.* One unit, six tens, three hundreds. Name the terms in the second example. In the third. In the other examples, in their order.

Read from the Table (Art. 21), the number represented by the first six figures from the decimal point; the first eight; the first ten; nine; twelve; fifteen; seventeen; twenty; fourteen; eighteen.

30. To write numbers, observe the following

RULE. — *Beginning with the highest period, write the figures of each period in their order from left to right, filling vacant places with zeros.*

31. EXERCISES.

Write the following: —

1. Three hundred sixty-four. *Ans.* 364.
2. Seven thousand eighty-nine. *Ans.* 7089.
3. Eighteen thousand eighteen.
4. Nine hundred thousand sixteen.
5. Four hundred twenty thousand, six hundred eighty-three.
6. Eight hundred ten thousand, two hundred four.
7. Two hundred fifty-nine thousand, seventy.
8. Forty-five million, seven hundred thousand, two hundred fifty-one.
9. Nine hundred one million, two hundred eighteen thousand, twenty-two.
10. Three billion, thirty-seven million, nine hundred six thousand, two hundred.
11. Two hundred thirty-four million, eight hundred sixty-three thousand, three hundred eighty-nine.
12. Seventeen billion, seven hundred fifty-nine million, ninety thousand, sixty-seven.
13. Three hundred thirty-three quadrillion, seven hundred seventy-nine billion, three hundred thousand, two.
14. Nine hundred ten quadrillion, four million, three thousand.
15. Fifty-four quintillion, eighty-three quadrillion, nine hundred million, seventeen thousand, one hundred eighty-two.
16. Eighteen billion, four.
17. Forty million, eight hundred thousand.

18. Eighty-nine million, four hundred five thousand, seven.
19. Thirty-seven trillion, ninety-three billion, eighty-one.
20. Seven hundred quintillion, one quadrillion, one.
21. Fifty quintillion, forty-nine thousand, thirty.

32. We have seen that the value represented by a figure increases by a scale of tens, as the figure is removed towards the left, and decreases in the same manner as it is removed towards the right.

Applying this principle, we can represent parts of units by placing figures at the *right* of the decimal point.

If we consider a unit to be composed of ten equal parts, we may represent one or more of these parts, which are called *tenths*, by a figure in the first place at the right of the point; again, if we consider one of these tenths to be composed of ten equal parts, we may represent one or more of these parts, which are called *hundredths*, by a figure in the second place, and so on.

The first place at the right of the point is the tenths' place, the second, the hundredths' place, the third, the thousandths' place.

Decimal point.
Tenths.
Hundredths.
Thousandths.

Thus: .7 8 5

Here the 7 at the right of the point represents seven *tenths* of a whole one, the 8 represents eight *hundredths*, and the 5 represents five *thousandths*. The entire number is read *seven hundred eighty-five thousandths*; .25 is read *twenty-five hundredths*; .3 is read *three tenths*.

EXERCISES.

Read the following:—

1.	.325;	.763;	.202;	.085;	.42;	.6.
2.	.87;	.03;	.504;	.004;	39;	.039.

Write the following:—

1. One hundred three thousandths. *Ans.* .103.
2. Eight hundred twenty-one thousandths.
ed forty-five thousandths.
Seven hundredths.

ADDITION.

33. Addition is the process of finding a number equal in value to two or more given numbers of the same kind. The number thus obtained is called the **sum**, or **amount.**

An upright cross, $+$, read *plus*, is the sign of *addition*, and, placed between two numbers, signifies that the one is to be added to the other. Two horizontal lines, $=$, read *equal to*, are the sign of *equality*, and signify that the quantities between which they are placed, are equal; thus, $2 + 5 = 7$, is read, *two plus five is equal to seven*, or, *two plus five equals seven.*

Illustrative Example.

34. Add the numbers 321, 285, and 937.

OPERATION.

$$\begin{array}{r} 321 \\ 285 \\ 937 \\ \hline \textit{Ans. } 1543 \end{array}$$

We first write these numbers, units under units, tens under tens, hundreds under hundreds, and draw a line beneath. Then, adding the units first, $7 + 5 + 1 = 13$ units $= 1$ ten and 3 units; we write the 3 in the units' place, under the column of units, and reserve the 1 ten to add with the column of tens. 1 ten $+$ 3 tens $+$ 8 tens $+$ 2 tens $=$ 14 tens $=$ 1 hundred and 4 tens; we write the 4 tens in the tens' place, and reserve the 1 hundred to add with the column of hundreds. 1 hundred $+$ 9 hundreds $+$ 2 hundreds $+$ 3 hundreds $=$ 15 hundreds $=$ 1 thousand and 5 hundreds; we write the 5 hundreds in the hundreds' place, and the 1 thousand in the thousands' place, and thus find the amount of the given numbers to be one thousand five hundred forty-three. Hence we derive the following

Rule for Addition. *Write the numbers, units under units, tens under tens, hundreds under hundreds, etc. Begin to add at the units' column. If the sum of the units is less than ten, write it under the column of units; if ten, or a number greater than ten, place the units' figure under the column of units, and reserve the tens to add with the tens. Proceed in the same way with the other columns, writing down the entire amount of the last column.*

PROOF I. — *Add each column in a reverse direction; if the same result be obtained as before, the work may be presumed to be correct.*

NOTE. — Greater readiness will be attained by mentioning only the results in adding columns. Thus, in the above example, instead of saying 7 and 5 are 12, and 1 are 13, say 7, 12, 13; and instead of saying, 1 ten and 3 tens are 4 tens, and 8 tens are 12 tens, and 2 tens are 14 tens, say 1, 4, 12, 14 tens.

35. EXAMPLES FOR PRACTICE.

1. What is the sum of twenty-one, sixty-seven, eighty-nine, thirty-two, forty-five, thirteen, ninety, and seventy-eight? *Ans.* 435.

2. What is the sum of six hundred four, nine hundred ninety-nine, seven hundred ten, six thousand nine hundred eighty-two, eleven thousand eight hundred seven? *Ans.* 21,102.

3. What is the sum of 326, 981, 362, 707, 889, and 864? *Ans.* 4129.

4. What is the sum of 246, 368, 909, 896, 763, and 892? *Ans.* 4074.

5. What is the sum of 32689, 86543, 94861, 18325, and 90026? *Ans.* 322,444.

6. What is the sum of all the numbers from one to thirty, inclusive? *Ans.* 465.

7. What is the sum of all the numbers from one hundred fifty to one hundred seventy-five, inclusive?

8. Add 99, 364, 77, 86, 912, 32678, 96542, and 32684.

9. Add 987, 5, 679, 369, 153, 888, 806, 17, 27, and 5654.

10. Add 915, 875, 617, 868, 575, 387, 694, 946, and 6377.

11. Find the sum of the last four answers. *Ans.* 189,506.

12. Add 987, 425, 672, 307, 216, 321, 111, 872, 564, 876, 818, 419, 187, 160, and 3453.

13. $875 + 466 + 327 + 942 + 286 + 424 + 309 + 429 + 482 + 317 + 406 + 466 + 111 + 171 + 1618 =$ what?

14. $324 + 868 + 522 + 297 + 789 + 524 + 286 + 361 + 472 + 884 + 472 + 287 + 649 + 592 + 1788 =$ what?

15. $876 + 205 + 918 + 468 + 207 + 948 + 572 + 618$ $1 + 594 + 872 + 206 + 48 + 500 + 918 + 1331 =$ what?

16. 36196 + 5384 + 2963 + 1200 + 100200 + 2560 + 74 + 36 + 5 + 4786 + 186 + 544 + 396486 = what?

17. Find the sum of the last five answers. *Ans.* 587,294.

Proof II. — *Separate the example into two or more parts by horizontal lines; add the parts separately, and then add their amounts; if the same result be obtained as before, the work may be presumed to be correct.* See Example 18.

	(18.)	Proof.	(19.)	(20.)
	4163314		7137500	7984172
	5949841		9345477	8194324
	4956811		1233198	4221001
	1726414	16796380	2122172	4754632
	9876431		8914619	3241320
	7325146		3141691	7987346
	9136719		4131261	7325789
	8677485	35015781	3286432	2941816
Ans.	51812161	51812161	9710100	2861423

(21.)	(22.)	(23.)	(24.)	(25.)
449	9250	81713	247742	3482
788	19	93957	303321	6327
435	8158	38	478984	8618
663	7901	4885	98517	9532
67	6850	3750	232326	2419
455	5102	15	879416	4671
399	4372	21901	123192	8384
517	3911	86462	10921	3476
31	2514	71557	800467	2123
205	1677	99108	93219	519
871	3501	8298	63496	9600
431	5528	33984	876201	4520
219	7332	57310	23407	5418
868	9415	83568	89467	7317
189	8267	97371	77111	2982
598	6408	76503	98121	8415
529	4641	36294	267137	3618
721	2286	45939	689642	3976
256	3719	36815	232864	6521
583	5931	81541	98518	9357
9266	86672	[illegible]	[illegible]	[illegible]

36. Table for Practice in the Fundamental Operations.

	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	
A	–9	8	7	4	4	9	0	2	5	9	8	7	9	1	5	3	1	7	1	8	4–	A
B	–9	0	5	7	8	8	8	8	4	4	2	5	8	7	5	9	9	5	3	3	7–	B
C	–6	7	9	4	3	5	5	6	8	6	7	2	6	1	7	8	7	2	6	9	2–	C
D	–3	6	9	6	6	3	5	3	6	3	0	7	8	6	8	3	2	7	4	7	6–	D
E	–1	5	3	9	6	7	8	5	6	2	1	6	5	7	5	4	3	6	2	0	8–	E
F	–8	8	8	4	5	5	5	5	0	3	2	1	3	8	7	2	1	6	3	5	6–	F
G	–8	0	6	3	9	9	5	1	6	1	1	1	6	9	4	5	0	3	5	7	5–	G
H	–9	1	7	5	1	7	2	4	4	8	7	2	9	4	6	8	4	2	9	9	3–	H
I	–9	2	7	9	3	1	3	3	0	5	6	4	6	4	9	4	7	1	8	8	8–	I
J	–9	5	3	2	0	5	6	4	9	8	7	6	8	0	7	1	7	4	4	4	7–	J
K	–6	6	7	8	7	1	6	6	8	3	1	8	2	0	6	8	6	2	2	0	6–	K
L	–7	2	8	4	3	1	6	6	9	4	1	9	5	2	6	3	5	4	4	1	7–	L
M	–8	4	5	2	1	9	2	4	8	1	8	7	8	4	9	5	3	4	6	2	1–	M
N	–1	4	4	8	6	8	1	5	2	1	6	0	2	1	6	1	1	1	5	8	4–	N
O	–2	2	5	1	8	9	3	9	3	8	7	5	4	5	4	1	9	7	2	9	0–	O
P	–1	9	9	5	9	8	9	8	2	2	8	4	5	7	5	4	9	0	9	4	0–	P
Q	–9	4	1	5	2	9	5	5	5	2	9	4	1	8	0	8	7	6	7	0	0–	Q
R	–7	9	5	7	2	1	8	2	3	8	9	6	5	9	4	9	0	2	2	9	8–	R
S	–7	3	4	2	5	6	3	2	1	5	1	7	8	0	6	3	9	6	4	7	5–	S
T	–3	2	3	5	8	3	5	3	0	8	9	4	5	1	6	4	8	4	6	2	7–	T
U	–8	2	4	1	7	4	7	7	4	2	8	7	4	9	9	2	3	5	5	8	6–	U
V	–7	6	9	4	1	6	2	6	8	4	0	6	3	6	4	3	8	7	2	1	9–	V
W	–1	0	8	8	2	4	8	2	4	7	1	6	7	6	2	8	9	7	9	8	4–	W
X	–8	7	2	8	9	2	9	8	2	8	7	2	8	9	8	6	2	4	7	6	8–	X
Y	–4	4	4	7	6	4	4	2	5	5	7	4	4	5	7	4	7	6	5	1	1–	Y

Add in the above Table (as units, tens, and hundreds) columns,

26. 1, 2, and 3. *Ans.* 13382.
27. 4, 5, and 6.
28. 7, 8, and 9.
29. 10, 11, and 12.
30. 13, 14, and 15.
31. 16, 17, and 18.
32. 19, 20, and 21.
33. 1 to 21 inclusive.

For further Exercises on the Table, see Key

34. Paid $2400 (dollars) for my farm, $155 for my horse and cart, $26 for garden utensils, $86 for a mowing-machine, $10 for a horse-rake, and $108 for a pair of oxen. Required the amount.

35. A body of troops were furnished with 3622 Springfield rifled muskets, 7690 smooth-bores, and 13185 Enfield rifles. Required the amount.

36. J. R. bought of the Seneca Knitting Mills, 39600 pairs of socks; of Whitten, Hopkins & Co., 9782 pairs; of Pierce Brothers & Co., 9353 pairs; of Allen, Lane & Washburn, 5664 pairs; of George C. Bosson, 4296 pairs; of Cushing, Pierce & Co., 1315 pairs; of Samuel Dennis, 276 pairs. Required the amount.

37. Required the average number of pupils attending the Grammar Schools of Boston during the year 1859–60, the average number attending the Adams School being 493; the Bigelow School, 469; Bowdoin School, 538; Boylston, 941; Brimmer, 575; Chapman, 626; Dwight, for boys, 622; Dwight, for girls, 489; Eliot, 708; Franklin, 559; Hancock, 719; Lawrence, 761; Lincoln, 466; Lyman, 370; Mayhew, 367; Phillips, 549; Quincy, 720; Wells, 494; Winthrop, 933.

38. In the year 1861, Massachusetts furnished for the U. S. army, from her several counties, as follows: From Barnstable, 3 commissioned officers and 108 enlisted men; Berkshire, 21 officers, 614 men; Bristol, 59 officers, 1681 men; Dukes, 0 officers, 1 man; Essex, 148 officers, 4134 men; Franklin, 12 officers, 482 men; Hampden, 35 officers, 845 men; Hampshire, 15 officers, 575 men; Middlesex, 141 officers, 4200 men; Nantucket, 1 officer, 7 men; Norfolk, 70 officers, 2031 men; Plymouth, 44 officers, 1363 men; Suffolk, 278 officers, 4111 men; Worcester, 110 officers, 3464 men. Besides these, there joined her regiments, 647 men whose residences were not given, and 20 officers and 955 men from other States. Required the whole number of enlisted men in her regiments; of commissioned officers; of both.

39. Massachusetts furnished army shoes, 16649 + 4480 +

7139 + 3228 + 2022 + 2336 + 2220 + 1000 + 1200 + 1236 + 1013 + 240 pairs; cavalry boots, 336 + 1008 + 336 + 192 + 160 + 168 + 150 pairs. Required the number of pairs of boots; of shoes; of both.

40. She furnished hats, 12000 + 4704; caps, 12130 + 2934 + 2069 + 450 + 251 + 98 + 160. Required the number of hats; of caps.

41. On commencing business a merchant had $7752 in cash, $7719 in real estate, goods valued at $9728, a lot of cattle valued at $6930, a ship valued at $16834; during the first year he was in trade he gained above all his expenses $3195. What was he worth at the end of the year?

42. What is the number of square miles in the British Isles, there being in Scotland 30000, in England 31200, in Wales 7200, and in Ireland 32500?

43. The United States contain 3284100 square miles more than the British Isles; required the area of the United States?

44. What is the length of the Grand Trunk Railway from Detroit to Portland, the distance from Detroit to Stratford being 143 miles; from Stratford to Georgetown, 59 miles; from Georgetown to Toronto, 30 miles; from Toronto to Coburg, 69 miles; from Coburg to Belleville, 44 miles; from Belleville to Kingston, 48 miles; from Kingston to Brockville, 47 miles; from Brockville to Prescott, 12 miles; from Prescott to Cornwall, 46 miles; from Cornwall to Montreal, 67 miles; from Montreal to Richmond, 73 miles; from Richmond to Island Pond, 71 miles; from Island Pond to Gorham, 58 miles; from Gorham to Bethel, 21 miles; from Bethel to Danville, 42 miles; from Danville to Portland, 28 miles?

45. How far is it from Detroit to Toronto?

46. How far from Toronto to Montreal?

47. How far from Kingston to Montreal?

48. How far from Montreal to Portland?

49. How far from Portland to Gorham?

50. From Boston to Portland is 111 miles; how far is it from Boston to Montreal?

☞ *For Dictation* Exercises, see Key.

SUBTRACTION.

37. SUBTRACTION is the process of taking one number from another of the same kind, to find the difference.

The number which is subtracted is called the **subtrahend,** from the Latin *subtrahendus, to be taken from under*, as that is the number taken away. The number from which the subtrahend is taken is called the **minuend,** from the Latin *minuendus, to be made smaller*, as that is the number to be diminished. The result is called the **difference,** or **remainder.**

A short horizontal line, —, read *minus* or *less*, is the sign of subtraction, and, placed between two numbers, signifies that the number after it is to be taken from that before it; thus, $7 - 3 = 4$, read, *seven minus three equals four*, shows that, if 3 be taken from 7, the remainder is 4.

ILLUSTRATIVE EXAMPLE, I.

38. From 267 take 135.

OPERATION.

Minuend,	267	
Subtrahend,	135	
Remainder,	132	*Ans.*

For convenience, we write the subtrahend under the minuend, placing units under units, tens under tens, hundreds under hundreds, and draw a line beneath; 5 units from 7 units $= 2$ units, which we write in the units' place, under the units; 3 tens from 6 tens $= 3$ tens, which we write in the tens' place; 1 hundred from 2 hundreds $= 1$ hundred, which we write in the hundreds' place; and the result is 132, which is the difference between 267 and 135.

39. PROOF. If 132 is the difference between 267 and 135, it is evident that, if we add 132 to 135, the sum will equal 267. Hence, to prove subtraction, *add the difference to the subtrahend. If the sum thus obtained is equal to the minuend, the work may be presumed to be correct.*

NOTE. The pupil should prove each example, till he is sure that he makes no mistakes.

40. EXAMPLES.

1. $368 - 334 = ?$ *Ans.* 34.	4. $868879 - 42135 = ?$
2. $2769 - 2631 = ?$ *Ans.* 138.	5. $974968 - 721265 = ?$
3. $862785 - 250122 = ?$	6. $37879868 - 1244045 = ?$

Sum of the last four answers $= 37828933$.

41. ILLUSTRATIVE EXAMPLE, II.

$9861 - 8674 =$ what?

OPERATION.
9861
8674
Ans. 6187

Here a difficulty presents itself. We cannot take 4 units from 1 unit. In order to perform the operation, we must reduce one of the tens in the minuend to units, which with the 1 unit we already have, equals 11 units; 4 units from 11 units $= 7$ units, which we put in the units' place. Having reduced one of the tens to units, we have but 5 tens left, and as 7 tens cannot be taken from 5 tens, we must reduce one of the hundreds to tens, which $= 10$ tens; 10 tens $+ 5$ tens $= 15$ tens; 7 tens from 15 tens $= 8$ tens, which we write in the tens' place; 6 hundreds from 7 hundreds $= 1$ hundred; 3 thousands from 9 thousands $= 6$ thousands, and the answer is 6187. Hence the

RULE FOR SUBTRACTION. *Write the subtrahend beneath the minuend, units under units, tens under tens, etc. Begin to subtract at the units' place, taking each term * in the subtrahend from the one above it, and placing the remainder beneath. If the upper term is less than the lower, increase it, by adding to it one of the next higher denomination reduced to its own denomination, and then subtract, bearing in mind, in the next operation, that the upper term has been diminished by the one reduced.*

EXAMPLES.

What are the remainders in the following examples?

	(1.)	(2.)	(3.)	(4.)
Minuend,	849	321	8642	3084
Subtrahend,	278	219	730	2427
Remainder,	571	102	7912	657
Proof,	849	321		

* See Art. 20. Remark.

	(5.)	(6.)	(7.)	(8.)
From	3228	3256	7862	98731
Take	409	2948	7589	19829

Sum of the last four remainders, 82,302.

9. A man had 375 oranges in a box; if he should sell 259 of them, how many would he have left? *Ans.* 116 oranges.

10. A man, having 451 acres of land, gave 349 acres to his son; what remained? *Ans.* 102 acres.

11. If a teacher is now 57 years old, and has taught 38 years, at what age did he begin to teach? *Ans.* 19 years.

12. How old was a person in 1865 who was born in 1789? *Ans.* 76 years.

13. If I had $625 in a bank, and withdrew $249, what remained? *Ans.* $376

42. Illustrative Example, III.

From 20000 take 9.

OPERATION.

```
(1)(9)(9)(9)(10)
 2  0  0  0  0
             9
 -------------
 1  9  9  9  1
```

Here we have no tens to reduce to units, no hundreds, and no thousands. We must then take one of the 2 ten-thousands (leaving 1 ten-thousand), and reduce it to thousands, making 10 thousands. Reducing one of the thousands to hundreds, one of the hundreds to tens, and one of the tens to units, we leave 9 thousands, 9 hundreds, 9 tens, and have 10 units, from which, if we take 9 units, 1 unit will remain. Having no tens to take from 9 tens, no hundreds to take from 9 hundreds, no thousands to take from 9 thousands, and no ten-thousands to take from 1 ten-thousand, we write these figures in their respective places below the line, and have for a remainder 19991.

43. Examples.

1. From 2017 years take 1028 years. *Ans.* 989 years.

2. A man, who had 1205 yards of cloth, sold 429 yards. How many yards were left? *Ans.* 776 yards.

3. There are 205 sheep in a flock; if 109 of them should be driven to market, how many would remain? *Ans.* 96 sheep.

4. A merchant bought goods for $1084, and sold them for $177 less than he gave; how much did he receive for them? *Ans.* $907.

5. 30070 men went into battle; 4564 were slain, and 1300 were taken prisoners; how many were left? *Ans.* 24,206 men.

6. Take 229 oxen from 2006 oxen. *Ans.* 1777 oxen.

7. Subtract 25 hundred from 81 thousand. *Ans.* 78,500.

8. How many more in 47000 than in 702? *Ans.* 46,298.

9. 47000 less 46298 equals how many? *Ans.* 702.

10. 9832147 less 3472108 equals how many?

11. What number added to 9213628 will give 23475310?

12. What number subtracted from 7654321 will leave 369?

13. 86293210 minus 329876 equals how many?

14. 987621085 — 329875232 = how many?

15. Find the sum of the last five answers. *Ans.* 771,984,860.

16. 360080 + 7002 — 72824 = what?

17. 3478921 + 368754 — 2878796 = what?

18. From 7654321 — 1234567 take 53899.

19. From 4673214 + 2792 take 98264.

20. 98432231 — 32636841 — 808994 = what?

21. 8087670 — 7549094 — 89699 = what?

22. Find the sum of the last six answers. *Ans.* 77,642,007.

23. What is the difference between 19360742 and 9643278?

24. How many times can I take 7642 gallons from 21002 gallons, and what will remain?

25. If the minuend is 36 quadrillion and the subtrahend 95 million 86, what is the remainder. *Ans.* 35,999,999,904,999,914.

26. If the minuend be 69 trillion and the difference 85 billion, what is the subtrahend?

27. Philadelphia was founded in 1682. In what year was New York city settled, it having been settled 68 years before?

28. Victoria ascended the throne of England in 1837. How many years has she reigned?

29. Napoleon commenced his brilliant career in 1795. How many years before his final defeat in 1815?

30. The Israelites left Egypt in 1491 B. C., and 40 years after entered the land of Canaan. In what year did that event happen?

31. In the year 1851, London had 2362000 inhabitants; Pekin was estimated to have 1500000. How many more inhab- 'ad London than Pekin?

32. The equatorial diameter of the earth is 41843330 feet, and the polar diameter 41704788 feet; required the difference.

33. The population of St. Louis in 1850 was 77860, and in 1860, 160773; required the increase in 10 years.

34. James Nye has in his possession $172; he owes $28 to A, $36 to B, and $19 to C. After paying his debts, what will remain?

35. I have saved from my income $362, and have $2180 in government bonds; how much more must I save that I may purchase a house worth $3500?

44. General Review, No. 1.

1. Two persons, who are 200 miles apart, travel towards each other, one 46 miles, the other 51 miles a day; how far apart will they be at the end of one day?

2. If the above persons travel away from each other, how far apart will they be at the end of one day?

3. A man gave to his eldest son $3575, to his youngest son $4680, and to his daughter $2495 less than to the youngest son; his whole property was worth $20000; what sum remained?

4. A ship, which was valued at $15590, was sold at a loss of $4975; what did she bring?

5. If the subtrahend be 369 quadrillion, and the remainder 99 quadrillion 13 billion, what is the minuend?

6. The difference between two numbers is 95478. The larger number is 148769; what is the smaller?

7. How many times can 18640 be subtracted from 46806, and what will remain?

8. Which of the two numbers 15672 or 10560 is nearer to 13465, and how much?

9. From what number must 846 be taken twice to leave 15684?

10. To what number must 962 be added three times to make 8472?

11. Which is nearer to 348628, 63248 + 93264, or 600063 − 59321?

☞ **For Dictation Exercises, see Key.**

MULTIPLICATION.

45. Multiplication is the process of finding a number equal in value to one number taken as many times as there are units in another number. The number which is multiplied is called the **Multiplicand,** the number by which we multiply is called the **Multiplier**, and the result obtained is called the **Product.**

The multiplicand and multiplier are often called **factors** of the product, from the Latin *facio*, I make, because, being multiplied together, they make up the product. The product is also said to be the **multiple** of the factors. Thus, 7 times 6 = 42. Here, 7 is the multiplier, 6 the multiplicand, and 42 the product; or 7 and 6 are the factors of 42, which is their multiple.

The sign of multiplication is a small, oblique cross, ×, read, *times*, or, *multiplied by.* Thus, 7 × 6 may be read either 7 *times* 6, or 7 *multiplied by* 6. In the former case 7 is the multiplier and 6 the multiplicand, while in the latter 6 is the multiplier and 7 the multiplicand. The product is the same, whichever is the multiplier.

NOTE. — In the process of multiplication, the multiplier must be an *abstract* number. We cannot multiply pencils by pencils, or pencils by apples, but either may be multiplied by an abstract number, and give a product of the same denomination as the concrete factor. (Art. 4.)

46. ILLUSTRATIVE EXAMPLE, I.

Multiply 2364 by 7.

OPERATION.

Factors	2364	Multiplicand.
	7	Multiplier.
Multiple,	16548	Product.

Seven times 4 units = 28 units = 2 tens and 8 units. We write the 8 in the units' place, and reserve the 2 tens for the tens' place. 7 times 6 tens = 42 tens, which, with the 2 reserved tens, = 44 tens = 4 hundreds and 4 tens; we write the 4 tens in the tens' place, and reserve ...nds for the hundreds' place. 7 times 3 hundreds = 21

hundreds, which, with the 4 reserved hundreds, = 25 hundreds = 2 thousands + 5 hundreds; we write the 5 hundreds in the hundreds' place, and reserve the 2 thousands for the thousands' place. 7 times 2 thousands = 14 thousands, which, with the 2 thousands reserved, = 16 thousands = 1 ten-thousand + 6 thousands; we write the 6 thousands in the thousands' place, and the 1 ten-thousand in the ten-thousands' place, and thus obtain for our product 16548.

NOTE. — This result might be obtained by finding the sum of the number 2364 taken seven times; that is, by adding 2364 to itself six times.

Hence, Multiplication may be regarded as a short way of performing Addition.

47. EXAMPLES.

Multiply

1. 267 by 2; by 3; by 4; and add the products. *Ans.* 2,403.
2. 628 by 5; by 6; by 7; " " " *Ans.* 11,304.
3. 3401 by 8; by 9; " " " *Ans.* 57,817.
4. 90021 by 10; by 11; " " " *Ans.* 1,890,441.
5. 66285 by 12; by 8; " " " *Ans.* 1,325,700.

6. 4364 × 8 = what?	10. 9832 × 7 = what?
7. 7762 × 9 = what?	11. 8349 × 6 = what?
8. 5391 × 4 = what?	12. 22078 × 11 = what?
9. 3409 × 5 = what?	13. 19869 × 12 = what?

14. Add the last eight products, and multiply by 7. *Ans.* 5,205,081.
15. 123456 × 6 = ? 16. 987654321 × 7 = ?
17. Add the last two products. *Ans.* 6,914,320,983

48. ILLUSTRATIVE EXAMPLE, II.

Multiply 3648 by 294.

OPERATION.

```
   3648
    294
  -----
  14592
 32832
7296
-------
1072512 Ans.
```

Here we are to multiply, not only by units, but by tens and hundreds. We write the numbers units under units, tens under tens, &c., and multiply first by the units, as before, and then by the tens. It is evident that the product of any number multiplied by tens will be ten times as great as if multiplied by the same number of units multiplied by hundreds, one hundred times as great as if multiplied by units; multiplied by

thousands, one thousand times as great, etc. Hence, when a number is multiplied by tens, hundreds, or thousands, the products thus obtained are written one, two, or three places farther to the left than when multiplied by units; or, in other words, we multiply by the other terms as we multiply by the units, placing the first figure of each product under the term by which we multiply. The sum of these partial products is the entire product. Hence the

RULE FOR MULTIPLICATION.

Write the multiplier under the multiplicand. Beginning at the right, multiply each term of the multiplicand by each term of the multiplier, successively, placing the right hand figure of each partial product under the term by which you multiply, carrying as in addition. Add all the partial products, and the result will be the entire product.

49. PROOF I. *Take the multiplicand for the multiplier, and the multiplier for the multiplicand. If the result thus obtained be like the first result, the work is probably correct.*

50. PROOF II. *By casting out the 9's.* This method is much the easier, though not always sure.

NOTE. — To cast out the 9's from any number, *commence at the left, and add the digits towards the right. When their sum equals 9 or more, reject 9 and add the remainder to the next digit, and so on.* The last remainder is called the excess of 9's.

TO PROVE MULTIPLICATION BY CASTING OUT THE 9'S.

Cast out the 9's from each of the factors. Then multiply the remainders, should there be any, cast out the 9's from the product, and note the last remainder. Cast out the 9's from the answer, and if the remainder equals the one obtained above, the work may be presumed to be right; thus,

```
   36184      3 + 6 = 9.  1 + 8 = 9.          4, 1st remainder.
    2681  2 + 6 + 8 = 16 = 9 + 7.  7 + 1 = 8, 2d remainder.
   -----                                      --
   36184                                      32
  289472                              3 + 2 = 5, last remainder.
 217104
 72368
 --------
 97009304 Ans.   7 + 3 = 10 = 9 + 1.  1 + 4 = 5, which equal-
```

ling the remainder above, the work is right.

NOTE. — *For demonstration of rule, see Appendix.*

Examples.

51. Perform and prove the following examples:—

1. $3684 \times 36 = ?$ *Ans.* 132,624.
2. $2842 \times 28 = ?$ *Ans.* 79,576.
3. $18762 \times 236 = ?$
4. $128124 \times 402 = ?$
5. $189003 \times 836 = ?$
6. $12053 \times 972 = ?$

7. Add the answers to the last four examples, and multiply the sum by 3798. *Ans.* 857,040,362,792.

8. Multiply 123456789 by 98765.

52. Any number may be multiplied by 10, 100, 1000, or a unit of any order, *by annexing as many zeros to the multiplicand as there are zeros in the multiplier, and placing the decimal point at the right.*

Examples.

9. Multiply 68432 by 10, by 100, 10000, 1000, 1000000, and add the products. *Ans.* 69,192,279,520.

10. Multiply 3682 by 10000, 10, 1000, 100, 100000, and add the products.

53. Illustrative Example, III.

Multiply 68432 by 86000.

Operation.

```
  68432
  86000
 ------
 410592
547456
----------
5885152000  Ans.
```

Here, by multiplying first by 86, and then annexing three zeros, which multiplies the first product by one thousand, the true result is obtained, and labor saved.

Illustrative Example, IV.

Multiply 832000 by 210.

Operation.

```
 832000
   210
 ------
   832
 1664
---------
174720000  Ans.
```

Here the zeros in both the multiplicand and multiplier are disregarded until after muliplying the other terms together.

54. Examples.

11. $6320 \times 80 = ?$
12. $4682 \times 360 = ?$
13. $92373 \times 86300 = ?$
14. $76000 \times 8020 = ?$
15. $82680 \times 900100 = ?$
16. $9876002 \times 10001 = ?$
17. $32001 \times 20206 = ?$
18. $987987 \times 654653 = ?$
19. $368043 \times 77665 = ?$
20. $23698 \times 84293 = ?$

21. Add the last ten answers, and multiply the sum by 100.
Ans. 81,482,871,584,800.

22. How many hills of corn have I in my cornfield, which contains 97 rows and 45 hills in a row?

23. If each hill produces 18 ears, how many ears does the field produce?

24. I have four corn bins, containing severally 63 bushels, 54 bushels, 37 bushels, and 29 bushels. There are four pecks in a bushel. How many pecks do they all hold?

25. Allowing 23 ears of corn to a peck, how many ears are there in the bins?

26. If a barrel of flour costs 9 dollars, what will 368 barrels cost?

27. If a person by working 12 hours a day can do a piece of work in 37 days, in how many days can he do it working 1 hour a day?

28. I have 5 bins, which contain 69 bushels each. What will be the capacity of a bin which will contain as much as all of them?

29. If 6 yards of cloth will make one pair of shirts, how many yards will make one dozen or 12 shirts? How many will make 8 dozen?

30. What will 8 dozen cost at 15 cents per yard for the cloth, 80 cents apiece for bosoms, wristbands, and buttons, and 50 cents apiece for making?

31. It takes 7 yards of ticking for a single bed-sack; what must I pay for cloth for 18 single bed-sacks, at 16 cents per yard?

32. If sheeting can be bought for 17 cents a yard, what must I pay for cloth for 21 sheets, allowing 10 yards for a pair?

33. What will be the cost of 9 dressing gowns at 5 dollars apiece, 3 pairs slippers at 1 dollar a pair, 2 pairs boots at 4 dollars a pair, and 3 dozen stockings at 2 dollars a dozen?

34. Suppose in 1 yard of cloth there are 580 fibres of warp and 432 of filling, and that each fibre of warp contains 32 strands, and each of filling 48, how many strands in the yard?

35. The Lawrence Pacific Mills turn out material for about 65000 dresses in a week; how many will they make in a year, or 52 weeks?

36. Allowing 12 yards to a dress, how many yards do they make in a year?

☞ For Contractions in Multiplication, see Appendix.

☞ For Dictation Exercises, see Key.

DIVISION.

55. **Division** is the process of ascertaining *how many times* one number is contained in another, or of finding *one of the equal parts* of a number.

NOTE. — In the example, "John has 10 apples, which he wishes to give to as many boys as he can, giving them 2 apples apiece, to how many can he give them?" — it is evident he can give them to as many boys as 2 *is contained times* in 10. In the example, "If 16 pears are divided equally among 4 boys, how many pears does 1 boy receive?" it is evident that 1 boy must receive *one fourth* of what the 4 boys receive, or one fourth of 16 pears; that is, *one of the four equal parts* of the number, 16 pears.

The number which is divided is called the **Dividend,** the number by which we divide is called the **Divisor,** and the result the **Quotient,** from the Latin *quoties,* how many times.

The sign of Division is a short horizontal line between two dots, $\div$; thus, $9 \div 3$ shows that 9 is to be divided by 3. Sometimes the dividend and divisor take the place of the dots; thus, $\frac{9}{3}$. This expression may be read, 9 divided by 3, nine thirds, or one third of nine, and is the fractional * form of division.

* See Art. 82.

Short Division.

Note. — This method is to be preferred where the divisor is not greater than 12.

56. Illustrative Example, I.

Divide 936 by 6.

Operation.

Divisor 6) 936 Dividend.

Quotient 156

We place the divisor at the left of the dividend, from which we separate it by a curved line, and, drawing a straight line beneath the dividend, proceed thus: 6 is contained in 9 hundreds 1 hundred times, with 3 hundreds remaining. We write the 1 hundred beneath the hundreds in the dividend, and reduce the 3 hundreds remaining to tens. 3 hundreds equal 30 tens, which, with the 3 tens of the dividend, equal 33 tens. 6 in 33 tens, 5 tens times, with a remainder of 3 tens; writing the 3 tens in the tens' place, and reducing the remainder as before, we have 36 units. 6 in 36, 6 times; writing the 6 in the units' place, we have 156 as the quotient of 936 divided by 6.

Illustrative Example, II.

Divide 17869 by 7.

Operation.

7) 17869

Ans. 2552 – 5 Remainder.

In this example, as 7 is not contained in 1 (ten thousand) any number of (ten thousand) times, we shall have no ten thousands in the quotient, and therefore take 17 (thousands) for our first partial dividend. We find also that the dividend does not contain the divisor an exact number of times, but that there is a remainder of 5. As this does not contain 7 any whole number of times, we can indicate the division by placing the 5 in the quotient above the divisor, and have for the answer $2552\frac{5}{7}$, which is read, two thousand five hundred fifty-two and five sevenths.

From the above examples we derive the

Rule for Short Division. *Beginning at the left, divide the first term or terms of the dividend by the divisor, make the result the first term of the quotient.*

Prefix the remainder, should there be any, to the next term of the dividend, divide as before, and thus continue till all the terms of the dividend are divided.

Should there be a remainder after the last division, place the divisor beneath it, and annex the result to the quotient.

57. Proof I. Division is the converse of Multiplication, the divisor and quotient being factors of the dividend: hence, to prove an example in division, *multiply the quotient by the divisor, and to the product add the remainder. The sum thus obtained should equal the dividend.*

58. Examples.

Divide

1. 36945 by 3. *Ans.* 12,315.
2. 987654 by 4. *Ans.* 246,913$\frac{2}{4}$.
3. 864024 by 6.
4. 369801 by 9.
5. 120087 by 11.
6. 906102 by 3.
7. Find the sum of the last four answers. *Ans.* 498,044.
8. Divide 10101019 by 7.
9. Divide 16444006488 by 4.
10. $23456983241 \div 9 = ?$
11. $30089043921 \div 7 = ?$
12. $\frac{786491}{8} =$ what?
13. $\frac{369472}{5} =$ what?

14. How many barrels of flour, at 7 dollars a barrel, can I buy for 259 dollars?

15. At 11 cents a yard, how many yards of cloth can I buy for 368972 cents?

16. If 12 pieces of cloth contain 408 yards, how many yards in a piece?

17. How many weeks are there in 4781 days?

18. How many hours will it take me to walk 1378 miles, at 5 miles an hour?

19. 9 times a certain number equals 324783; what is that number? *Ans.* 36,087.

20. $8 \times$ what $= 36924$?   21. $12 \times$ what $= 46817$?

Long Division.

59. Long Division is the process of dividing where the divisor is large, and the work written down.

60. Illustrative Example, I.

Divide 85232 by 23.

64. When the divisor is 10, 100, or 1000, &c., we can divide by simply *removing the decimal point in the dividend as many places towards the left as there are zeros in the divisor; the number at the right of the point will be the remainder:* thus, $368. \div 100 = 3.68$ (or $3\frac{68}{100}$). Hence, if the divisor consists of any number of significant figures with zeros annexed, *first cut off the zeros from the divisor and an equal number of figures from the right of the dividend, then divide what remains of the dividend by what remains of the divisor. To the remainder, if any, annex the figures that were cut off in the dividend;* thus,

```
38|00 ) 968|42 ( 25
        76
        ---
        208
        190
        ----
         1842
```

Disregarding the tens and units, we find how many times 38 (hund.) is contained in 968 (hund.), which is 25, with a remainder of 18 (hund.); this, with the 4 tens and 2 units left in the dividend, makes the entire remainder 1842, $\therefore$ 3800 is contained in 96842, $25\frac{1842}{3800}$ times.

65. Examples.

15. Divide 42179 by 1000; by 18000. *Ans.* $42\frac{179}{1000}$; $2\frac{6179}{18000}$

16. Divide 76532102 by 4800; by 91000.

17. Divide 98000269 by 32600000; by 980000.

18. Bought a farm for \$18715, at \$95 an acre; how many acres were there in the farm? *Ans.* 197.

19. Paid \$4505 for 27 acres of woodland; what was the price per acre?

Solution. If 27 acres cost \$4505, one acre will cost one twenty-seventh of \$4505, which equals, &c.

20. Paid \$35328 for 368 acres of land; find the price per acre.

21. The distance from Boston to Albany is 198 miles, from Albany to Buffalo, 298 miles. How long will it take a train to pass over the road at the rate of 28 miles an hour, allowing 2 hours for detentions between Boston and Albany, 1 hour at Albany, and 3 between Albany and Buffalo? *Ans.* $23\frac{20}{28}$ hours.

22. The Ohio Canal descends 1832 feet in 152 locks; what is the average descent in each lock?

23. If 8 presses can coin 19000 pieces of money in an hour, how many pieces can one press coin in a minute, 60 minutes making an hour?

24. In how many days, of 12 hours each, can the president of a bank sign 9000000 bank notes, if he signs 8 in a minute?

☞ For Contractions in Division, see Appendix.

☞ For Dictation Exercises, see Key.

66. Questions for Review.

1. What is ARITHMETIC? What are numbers? What is an abstract number? a concrete? What is a unit? Define Notation and Numeration. How are numbers represented? Describe the Roman method; — the Arabic. Which is more used? Why is this sometimes called the Decimal System? What is the decimal point? By what is the *number* of units of any order expressed? By what is the *order* of units expressed?

2. How do you write numbers? How do you read numbers? Name the first seven periods. Name others as far as you can. How are these periods separated? What are the names of the *places* of each period?

3. What is the least number of figures that will express units? — thousands? — billions? — trillions? — millions? — quadrillions.

4. In 189654238761, what is the largest number of thousands? — of millions? — of ten-millions? — of hundred-billions? — of trillions? — of tens? — of hundreds — of ten-thousands?

5. How will zeros at the right of a number affect it? — at the left?

6. What does a figure in the first place at the right of the decimal point represent? — in the second place? — in the third?

7. What is ADDITION? What is the sign for Addition? — for Equality? How do you arrange numbers to be added? Is this absolutely necessary? Perform and explain an example containing four numbers of at least seven figures each. Give the rule.

8. What is SUBTRACTION? Name and define the terms used. What is the sign for Subtraction? Take 3684 from 7000068, and explain. Give the rule; — the proof. When the minuend and difference are given, how can you find the subtrahend? When the subtrahend and difference are given, how can you find the minuend?

9. What is MULTIPLICATION? Name and define the terms used. What is the sign for Multiplication? Perform and explain an example in which the multiplier has, at least, two figures. Give the rule — first method of proof; — second method. How do you multiply by

10, 100, 1000, &c.? How do you proceed if there are zeros at the right of the multiplicand or multiplier? Tens × units = what? Thousands × tens? Hundreds × tens? Ten-thousands × hundreds? Ten-thousands × ten-thousands?

10. What is DIVISION? Name and define the terms used. What is the sign for Division? Perform and explain an example in short division. Give the rule. Give the proof by multiplication. Perform and explain an example in long division. Give the rule. Give the proof by casting out the 9's.

11. How do you divide by 10, 100, 1000 &c.? How do you divide when the divisor contains zeros at the right of significant figures? When the dividend and quotient are given, how can you find the divisor? When the divisor and quotient are given, how can you find the dividend? When the multiplier and product are given, how can you find the multiplicand. When the multiplicand and product are given, how can you find the multiplier?

67. MISCELLANEOUS EXAMPLES.

1. Add nine billion, six hundred ninety-two million, eighty-one thousand sixty-four; eighty-nine trillion, six hundred thirty-two million, ninety-one thousand eighteen; eighty-seven thousand thirty-four; and two hundred sixty-eight quadrillion, nine hundred eighty-four trillion, ninety-eight million one thousand ninety-four.

2. From (900362840218 — 986234681) take (7682 + 9619875.)*

3. Multiply (3684291 + 3642) by (8643264 — 8321628.)

4. Divide (3687291 — 86) by (3684 + 232.)

5. If 892 is one factor, and 28544 the product, what is the other factor?

6. 365 times what number = 298570?

7. If the dividend is 38493, and the divisor 4277, what is the quotient?

8. If the dividend = 42777, and the quotient 9, what is the divisor?

9. There were 52 schools in Antigua in 1858, with 4467 scholars; required the average number in each.

* In examples 2, 3, and 4, first perform the operations indicated within *the parentheses.*

10. David was born 1085 years B. C., and Washington 1732 A. D.; what time elapsed between these events?

11. What do I save a year, my income being $1600 a year, and my expenses $24 a week, 52 weeks making the year?

12. Illinois produced in 1860, 1515594 pounds of maple sugar; what was its value at 8 cents per pound?

13. Mississippi produced 1195699 bales of cotton; what was its value at 13 cents per pound, 400 pounds to the bale?

14. Missouri produced 4164 tons of lead, worth $356660, what was the value per ton?

15. The population of Chicago in 1850 was 29963; in 1860, 109260; what was the average increase a year?

16. If 8 men can do a piece of work in 24 days, how long will it take one man to do it?

17. If 768 be one factor, and 861 — 237 the other factor, what is the product?

18. Smith & Co. consume 74 tons of coal in a year; how much more must they pay for their coal in 1864, when coal is $14 a ton, than in 1860, when it was $8 a ton?

19. From the invention of parchment to the invention of paper was 782 years; to the use of quills in writing 741 years more; to the invention of printing, 804 years more; to the invention of stereotyping, 345 years more; how many years from the invention of parchment to that of stereotyping?

20. Parchment was invented 887 years B. C.; when was paper invented? *Ans.* 105 B. C.

21. When were quills first used in writing? *Ans.* A. D. 636.

22. When was printing invented?

23. When was stereotyping invented?

24. 76854 divided by what number, gives a quotient of 56 and a remainder of 22?

25. What number divided by 87, gives a quotient of 3842 and a remainder of 76?

26. In 1853, Wheeler & Wilson made 799 sewing machines; in 1854, 956; in 1855, 1171; in 1856, 2210; in 1857, 4591; in 1858, 7978; in 1859, 21306; in 1860, 19265; in 1861, 19725. *Required the amount.*

27. If a sewing machine can set 640 stitches in a minute, how many can it set in an hour? — in a day of 12 hours? — in 6 working days, or a week? — in 52 weeks, or a year?

28.* There was sent to the U. S. mint, from 1823 to 1836, $4377984 worth of gold; what was the average value sent a year? If gold was worth 16 dollars an ounce, how many pounds were sent, allowing 12 ounces to a pound?

29.* In the Pacific Mills, 200000000 gallons of water are used in a day. How many weeks would it take a man to pump it if he could pump a gallon in six strokes of the pump, 20 strokes a minute for 16 hours a day, allowing 6 working days per week?

30.* If the earth is 95000000 of miles from the sun, and the moon at its full is 224000 miles farther on, and light travels at the rate of 191500 miles a second, how many seconds is it in passing from the sun to the moon and back to the earth?

Ans. $498\frac{81000}{191500}$ seconds.

31. If 3871 be divided by 79, and the quotient be multiplied by 133, to this product 6523 be added, the amount divided by 40, and that quotient multiplied by 970, what will be the product? *Ans.* 316,220.

32. $(17-2)\div 3=?$† 33. $(7+3)\times 2=?$†

34. $\dfrac{(1863+7982)\times 3}{7}$† $=?$ 35. $\dfrac{19360\div\overline{9+2}+43}{368}$† $=?$

36.* $(\overline{2+1}\times 7+4)\div 5+(8+6)\times 2=?$†

37.* $(81+9)\div 10+67+(\overline{2+3}\times 7+7)\div 6$† $=?$

☞ For Dictation Exercises, see Key.

† A *vinculum*, ——, or *parenthesis* ( ), signifies that the same operation is to be performed upon all the quantities thus connected. In solving examples, it is generally better first to reduce all quantities connected by a vinculum, or parenthesis, to their simplest forms. Thus, in Ex. 32. $(17-2)\div 3=15\div 3=5$. Ex. 33. $(7+3)\times 2=10\times 2=20$.

Ex. 36. $(\overline{2+1}\times 7+4)\div 5+(8+6)\times 2=$
$(3\times 7+4)\div 5+,\quad 14\times 2=5+28=33.$

NOTE. — Examples with stars are "optional examples." They may be omitted by younger pupils until a review, or altogether, if the teacher desire.

FEDERAL MONEY.

68. **Federal Money** is the medium of exchange in the United States. Federal is derived from the Latin *fœdus,* a league; the money being used by states leagued or united under one government. Federal money consists of eagles, represented by E.; dollars, represented by $; dimes, by d.; cents, by cts., and mills by m.

Table of United States Currency, or Federal Money.

10 m. = 1 ct.
10 cts. = 1 d.
10 d. = $1
$10 = 1 E.

69. As these denominate numbers increase and decrease like simple numbers, *by a scale of tens,* they are written as simple numbers are written, and operations are performed upon them as upon simple numbers, the dollar being regarded as the unit. The sign for dollar, $, is placed before any number which we wish to designate as representing United States currency.

Eagles.	Dollars.	Dimes.	Cents.	Mills.
$8	9.	4	4	5

In business operations the denominations eagles and dimes are commonly disregarded, eagles being considered tens of dollars, and dimes, tens of cents; thus, the above illustration is read 89 dollars, 44 cents, 5 mills.

Examples.

70. Write the following:—

1. Seven hundred sixty-four dollars eighteen cents four mills. *Ans.* $764.184
2. 972 dollars 17 cents 2 mills.
3. 5768 dollars 9 cents 2 mills.
4. 10 thousand dollars sixty cents.
5. 9 million dollars 9 mills.

71. Read the following:—

6. $2789.842.	11. $2009147.00.
7. $9872.406.	12. $98705481.052.
8. $9084.007.	13. $4897.007.
9. $864201.90.	14. $987801.94.
10. $329871.045.	15. $81746.807.

16. What is the largest number of cents contained in example 6?—7?—8?—9?—10? *1st Ans.* 278,984 cts.

17. What is the largest number of dimes?—of mills?—of eagles? *1st Ans.* 27,898 dimes.

18. Read examples 11 to 15, making cents the unit of numeration.

19. Reduce $86452. to cents; to mills.

Ans. 8,645,200 cents; 86,452,000 mills.

20. Reduce $9841.72 to mills.

21. Reduce 8712647 cents to dollars.

22. Reduce 3687514 mills to dollars.

How do you reduce dollars to cents? to mills?

How do you reduce mills to dollars? to cents?

How do you reduce cents to mills?

23. $9843.621 + $4687.32 + $84.321 + $.07 + $.64 + $973.241 = ? *Ans.* $15,589.213.

Note.—In addition and subtraction of Federal Money, dollars should be written under dollars, cents under cents, etc.

24. $3684.271 + $765.42 + $1763.417 + $8645.217 — 3.68 = ? *Ans.* $14,854.645.

25. From $8643.271 + $98367.489 take ($37.862 + $33695.41).

26. From $3471.009 — $.71 take ($987.541 + $862.73).

27. From $4645. + $8178. take ($9827. — $6712.86).

28. $34865.002 × 46 = ? *Ans.* $1,603,790.092.

Note.—In the example above, as mills are multiplied, the answer must be mills.

29. 11 × $3687.40 = ?	31. $98417.83 × 791 = ?
30. $946.918 × 478 = ?	32. 984 × $7654216.69 = ?

NOTE.—It will be obvious that in the four following examples, the quotient must be of the same denomination as the dividend.

33. $13428. ÷ 9 = ? 35. $241364.48 ÷ 56 = ?

34. $7352.88 ÷ 12 = ? 36. $3712471.712 ÷ 488 = ?

72. ILLUSTRATIVE EXAMPLE.

$1725. ÷ 18 = ?

OPERATION.

```
18)1725(95.833 6/18. Ans.
   162
   ---
   105
    90
   ---
   150
   144
   ---
     60
     54
     --
     60
     54
     --
      6
```

In this example, after dividing the dollars, we have a remainder of 15 dollars; this we reduce to dimes by annexing a zero, and dividing, obtain 8 dimes for the quotient figure, and have a remainder of 6 dimes, which we reduce to cents and divide, and have a remainder of 6 cents, which we reduce to mills and divide, and have a remainder of 6 mills, and for the entire quotient, \$95.833$\frac{6}{18}$. *Ans.*

NOTE.—In the four following examples continue the division to mills.

37. Divide $9867. by 37; by 91; by 416.

38. Divide $89000. by 17; by 42; by 368.

39. Divide $36421.90 by 18; by 48.

40. Divide $6003489. by 96; by 543.

41. How many times are $.34 contained in $36.72?

42. How many times are $.25 contained in $645.?

NOTE.—In dividing Federal Money by Federal Money, when the denominations are unlike, it is necessary first to reduce the dividend and divisor to the same denomination. The answer will be an abstract number; thus, $645. ÷ $.25 = 64500 ÷ 25 = 2580.

43. Divide $186432.18 by $0.032.

44. Divide $382971.21 by $93.

45. Bought 1 pair of boots for $1.37; 1 pair for $1.65.; slippers for $.95; shoes for $.65; and shoes for $.82. Required the entire cost.

46. Bought a horse for $95.00; a wagon for $63.00, and harness for $ 15.00; kept them a week, paying $ 2.50 for board

for the horse, then sold them all for $ 175.00. Did I gain or lose, and how much?

47. What cost 8 pairs geese at $1.28 per pair?

48. Bought 2 dozen pigeons at $.85 per dozen, 2 dozen at $1.10 per dozen, and 1 dozen for $.90. What should I pay?

49. 8874 sheep were sold at $4.13 per head; what did they bring?

50. There were shipped to Great Britain in one year from New York, 20602243 pounds of butter. What would it bring at 15 cents per pound?

51. 39479897 pounds of cheese were shipped the same year. Required the receipts at 7 cents per pound?

52. 4778 beeves were sold in New York market in one week, averaging 874 lbs. apiece, at 7 cents per pound; what was received for them?

53. Bought 2 pieces of flannel, each containing 62 yards, for $39.68, and sold them for 40 cents per yard. What did I gain?

54. Paid a man $16.25 for 13 days' work; what was that a day?

55. Paid $5.10 for 17 boxes strawberries; what was that a box?

56. Among how many boys may $10 be distributed, that each may receive $0.625?

57. Sold 35 barrels Greenings at $1.75 per barrel, 17 barrels Baldwins at $1.80 per barrel, 12 barrels fall Harveys at $1.25 per barrel, and 25 of Russets at $2.25 per barrel. Paid 17 cents a barrel for picking, and $12.00 for transportation. What remained after all my bills were paid?

58. Paid $3.00 for 1 dozen apple trees, $3.36 for 1 dozen peach trees, $3.30 for one half dozen pear trees; what did I pay for the whole, and how much a piece for each kind?

59. Paid a carpenter for stock and work for a house, $450.75; for mason's work, $38.25; for digging and stoning cellar, $47.18; for painting, $40.00; to the plumber, $8.125. I then sold it, and lost, in so doing, $14.305; what did I sell it for? *Ans.* $570.

60. Bought a farm, containing 40 acres meadow and 17 woodland, for \$2850.00. Sold to one man 10 acres woodland for \$85.00 per acre; to another a house lot of one acre for \$90.00; and the remainder to a third for \$2025.00. What did I gain by the operation; and for how much per acre did I sell the remainder? *Ans.* \$115; \$44.02$\frac{8}{46}$.

BILLS.

73. When, in a business transaction, one person receives money, property, or services from another, he becomes *indebted* or is *debtor* for the amount he receives.

The person who parts with the money, property, or services, is *credited* for the amount he has given, and hence is called the *creditor*.

A written statement of the amount of the debt, with the items included, is called a *bill*, and is usually written in forms like those on the following pages.

When the creditor is paid the amount due, he acknowledges the receipt by his signature at the foot of the bill, after the words "Received payment." A bill thus signed is said to be *receipted*.

74. Find the cost of each article in the following bills, and their several amounts.

(1.) BUFFALO, *November* 10, 1862.

MR. JAMES CROCKER,

Bought of HENRY SHEDD,

10 bbls. St. Louis Flour, extra,				at	\$ 9.50	
12 "	Western	"	medium,	"	7.75	
8 "	Canada	"	extra,	"	6.72	
14 "	Canada	"	choice extra,	"	7.87	
3 "	Corn Meal,			"	4.25	
20 bu.	Northern Oats,			"	.61	
						\$376.89.

Received payment,

HENRY SHEDD,
By GEORGE BAKER.

(2.) LAWRENCE, *November* 18, 1862.

MR. D. DANFORTH,

Bought of J. SMITH,

65 bu.	Potatoes,	at	$0.55	
300 lbs.	Squashes,	"	.01	
450 "	Pork,	"	.11	
35 bu.	Beans,	"	2.50	
85 "	Rye flour,	"	2.25	

Received payment, $367.00 *Ans.*

J. SMITH.

(3.) BOSTON, *April* 17, 1863.

MR. JAMES BLAKE,

Bought of BRECK & Co.,

8 bu.	Herds Grass,	at	$2.25	
75 lbs.	Clover Seed,	"	.11	
25 bu.	Canary Seed,	"	3.62	
18 lbs.	Mustard Seed,	"	.13	
25 "	Hops,	"	.17	
22 "	Hops,	"	.16	

Received payment, $115.61 *Ans.*

JOSEPH BRECK, for BRECK & Co.

(4.) NEW BEDFORD, *October* 9, 1862.

MR. J. L. RICE,

To HENRY BROWN, *Dr.**

To	2	bbls. Pork, prime,	at	$ 15.50	
"	250	lbs. Hams,	"	.09	
"	475	" Butter,	"	.24	
"	432	" "	"	.18	

Received payment, $

HENRY BROWN.

* This means that Mr. Rice is debtor to Mr. Brown. *Dr.* is read "debtor."

(5.) NEW YORK, *March 1*, 1863.

A. M. PHIPPS, ESQ.,

To SAMUEL SLOANE, *Dr.*

Jan. 7.	To	12 lbs.	Tartaric Acid,	at	$ .85	
" "	"	7 "	Blue Vitriol,	"	.25	
" 12.	"	3 oz.	Morphine,	"	7.00	
" 13.	"	5 "	Quinine,	"	4.00	
Feb. 2.	"	2 lbs.	Cardamoms,	"	3.50	
" "	"	10 "	Cream Tartar,	"	.51	
" "	"	8 "	Cubebs,	"	.53	
" "	"	5 "	Gum Copal,	"	.68	
" "	"	8 galls.	Cod Liver Oil,	"	1.75	
		Received payment,				$

SAMUEL SLOANE.

(6.)* BRISTOL, *January* 1, 1863.

OTIS BUTLER, ESQ.,

1862. To RALPH BURNSIDE, *Dr.*

Apr. 3.	To	96 lbs.	Rice,	at	$ .07	
" "	"	3 "	Saleratus,	"	.08	
" "	"	28 "	Castile Soap,	"	.16	
May 9.	"	25 "	Pearl Starch,	"	.09	
" 15.	"	196 "	Crushed Sugar,	"	.13	
" "	"	196 "	Brown Havana,	"	.12	
June 5.	"	46 "	Hyson Tea,	"	1.12	
July 10.	"	37 "	Gunpowder Tea,	"	.95	
						$

Cr.†

May 10.	By	1 Wagon,	$42.00	
" 16.	"	2 Cows, at $35.00,		
" "	"	Cash,	10.00	
			$	
		Balance due R. B.		$

Received payment,

RALPH BURNSIDE.

† This means that Mr. Butler is credited for goods or cash delivered. *Cr.* is read "creditor."

75. Find the amounts due in the following examples, and make out the bills.

7. Charles Fuller purchased of James Munroe, Jan. 4, 1863, 1 horse for $95.00, 2 cows at $50 apiece, 1 wagon for $62.00, 2 shovels at $1.12 apiece, 30 bushels corn at $.65 per bushel, and 17 bushels wheat at $1.62 per bushel.

8. Samuel Banks sold to Abraham Seward, March 10, 1863, 2 pieces flannel, of 62 yards each, at $.49 per yard; 5 pieces cotton, bleached, at $.24 per yard, 2 of the pieces containing 36 yards each, and 3 containing 35 yards each; 38 yards ticking, at $.29; 86 yards brown sheeting, at $.27; 42 yards broadcloth, at $3.65 per yard. 290.78

9. Dr. Cardamom bought of James Mortar 3 gallons castor-oil at $2.50; 9 pounds oil peppermint at $2.50; 4 pounds oil cassia at $3.62; 4 pounds oil orange at $3; 6 pounds oil lemon at $4.25; 5 pounds oxalic acid at $.33; and 5 pounds Seneca root at $.95.

10. Baldwin & Lewis, of Cincinnati, bought of Balch & Rayner, Boston, 24 sack coats at $15.75; 36 vests at $3.50; 95 pairs pants at $4.38; 4 dozen pairs suspenders at 42 cts. per pair; 23 dozen pairs gloves at 68 cts. per pair; and 15 dozen collars at 13 cts. apiece.

11. Hiram Teachwell bought of Mark Thrifty, Nov. 8, 1862, 2 Dictionaries, at $.87 apiece; 9 Vocal Cultures, at $.70; 12 Walton's First Steps, at $.13, and 24 Worcester's Spellers, at $.20. Dec. 2, he bought 2 reams paper at $2.12, 3 dozen pencils at $.50, and 12 slates at $.17. Dec. 10, he paid Mr. Thrifty $20.00, and Jan. 1, 1863, Mr. Thrifty made out his bill. Required the balance due.

12.* Solomon Katchall bought of Hiram Southack, Aug. 11, 1862, 12 pairs congress gaiters, at $2.75; 12 pairs misses' gaiters, at $1.12; 8 pairs kip boots, at $2.75; 12 pairs, at $.95; 9 pairs boys' metallic-toed shoes, at $.72; 12 pairs gents.' boots, at $6.75; 12 pairs, at $4.25; 5 pairs, at $3.15. He sold Mr. Southack 18 yards black silk, at $1.17; 48 yards brown sheeting, at $.19; 18 yards crash, at $.13, and 20 yards flannel, at $.45.

☞ For Dictation Exercises, see Key.

ANALYSIS.

76. Analysis in arithmetic consists in determining the solution of an example from the relations of the numbers given in that example.

The given number which is of the same denomination as the required answer forms the basis of all the reasoning, and should be the first written in performing an example.

The value of any number of things may be obtained by first finding the value of a single thing or *unit* of the same denomination. This unit is sometimes called the *unit of computation.*

Illustrative Example.

If 25 barrels of flour cost \$175, what cost 17 barrels?

Operation.

$\frac{175}{25} \times 17 = \$119$. *Ans.*

\$175 is the term of the same denomination as the required answer. Before finding the value of 17 barrels, we must know the value of 1 barrel.* If 25 barrels cost \$175, 1 barrel will cost 1 twenty-fifth of \$175, and 17 barrels will cost 17 $\times$ 1 twenty-fifth of \$175, = \$119.

Examples.

1. If 13 acres of land produce 780 bushels of corn, how many bushels will 5 acres produce? *Ans.* 300.

2. If 5 boxes of oranges cost \$21.80, what cost 21 boxes? *Ans.* \$91.56.

3. If a car runs 207 miles in 9 hours, how far will it run in 25 hours?

4. If 18 rows of potatoes yield 54 bushels, how many bushels will 405 similar rows yield?

5. If \$19.74 were paid for 14 bushels of rye, what must be paid for 25 bushels?

6. If 19 tons of coal run an engine 266 miles, how far will 14 tons run it?

7. If 5 oxen consume 85 pounds of hay in 1 day, how much will be required for 1 yoke of oxen of the same size, and for the same time?

* 1 barrel is the unit of computation.

8. How many pounds of coffee can be bought for $15, if 40 lbs. cost $8?

NOTE. — If $8 pay for 40 pounds, $1 will pay for 1 eighth of 40 pounds, and $15 will pay for 15 × 1 eighth of 40 pounds = 75 pounds.

9. If 150 barrels of apples were bought for $200 and sold for $350, what would be gained by selling 45 barrels at the same rate?

10. If a quantity of hay lasts 22 oxen 105 days, how many days will it last 5 yoke?

NOTE. — If it lasts 22 oxen 105 days, it will last 1 yoke 11 × 105, and it will last 5 yoke 1 fifth of 11 × 105 days = 231 days.

11. A field of wheat was reaped by 10 men in 6 days; what length of time would be required for 15 men to reap the same amount?

12. A cistern can be emptied in 35 minutes by 7 pipes; in what time can it be emptied, if 5 only of the pipes are open?

13. If 1423 operatives can do a piece of work in 12 days, in what time will 2400 operatives perform the same work?

14. If a certain piece of work can be performed by 250 men in 14 weeks, how many more must be employed to perform it in a week?

15. A garrison of 10000 men have provision to last them 6 weeks; if 2000 men be killed in a sally, how long will the provisions last the remainder?

77. QUESTIONS FOR REVIEW.

1. FEDERAL MONEY. What are the denominations of federal money? Give the table. How do you write numbers in federal currency? What is considered the unit? Give the sign for dollars. How do you reduce eagles to dollars? dollars to cents? dollars to mills? cents to mills? mills to dollars?

2. How do you add numbers in this currency? How do you subtract? When you multiply, of what denomination is the product? When you divide by an abstract number, of what denomination is the quotient? Divide $185 by 7, continue the division to mills, and ...in. What is necessary in order to divide mills by dollars? by lividing cents by dollars, is the quotient abstract or con-

crete? In dividing dollars by an abstract number, is the quotient abstract or concrete?

3. Bills. What is a bill? *Ans.* It is a writing given by the creditor to the debtor, showing the amount of the debt. Who is the creditor? the debtor? What is the receipt of a bill?

4. Analysis. What is analysis? Which number forms the basis of the reasoning.

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78. General Review, No. 2.

1. 287 + 5 million + 36 thousand + 59481 = ?

2. Add 567 to the sum of the following numbers: 121; 232; 343; 454; 565; 676; 787; 898.

3. Take 987 from each of the following numbers, and add the remainders: 9876; 5678; 3644; 7573; 2432; 4001.

4. What number must be added to the difference between 58 and 7003 to equal 938425?

5. What number, taken from the quotient of 1833000 ÷ 47 leaves 25?

6. What number equals the product of 1785, 394, and (624 — 48)?

7. If 5872 is the multiplicand, and half that number the multiplier, what is the product?

8. If 4832796 is the product, and 1208199 the multiplicand, what is the multiplier?

9. If 894869 is the minuend, and the sum of all the numbers in the third example is the subtrahend, what is the remainder?

10. If 700150 is the dividend, and 3685 the quotient, what is the divisor?

11. If 28936 is the divisor, and 86 is the quotient, what is the dividend?

12. Divide 87 million by 15 thousand.

13. $3.75 + $9.32 + $.75 + $10. + $2.185 + 4 cents = ?

14. $19. — $.75 — $8.25 + $3.54 = ?

15. From 18 × $5.873, take $3.68 ÷ 4.

16. If $183.30 is the dividend, and $3.90 the divisor, what is the quotient?

17. If $98 60 is the dividend, and 17 the divisor, what is the quotient?

☞ For changes, see Key.

PROPERTIES OF NUMBERS.

79. SIGNS. — RECAPITULATION.

$+$ signifies plus, or more.	$=$ signifies equal to.
$-$ signifies minus, or less.	$\times$ signifies multiplied by.
$>$ signifies greater than.	$\div$ signifies divided by.
$<$ signifies less than.	$\therefore$ signifies therefore.

( ) parenthesis, and $\overline{}$, vinculum, signify that the same operation is to be performed upon all the quantities thus connected.

DEFINITIONS.

80. Numbers are either **integral** or **fractional.**

81. **Integral** numbers, or **Integers,** are *whole* numbers.

82. **Fractional** numbers are *parts* of whole numbers.

83. A **Factor** or **Divisor** of a number is any number which is contained in it without a remainder; thus, 2 is a factor of 6.

84. A **Prime Number** is a number which contains no integral factor but itself and 1; as, 1, 2, 3, 11.

85. A **Composite Number** is a number which contains other integral factors besides itself and 1; as, 4, 6, 8, 25.

86. A **Prime Factor** is a factor which is a prime number.

87. A composite number equals the product of all its prime factors; thus, $12 = 2 \times 2 \times 3$.

88. Two numbers are said to be *prime to each other* when they contain no common factor except 1; thus, 8 and 15 are prime to each other.

89. The **Power** of a number is the number itself, or the product obtained by taking that number a number of times as a factor.

The number itself is the *first power;* if it is taken *twice* as a factor, the product is called the *second power*, or *square;* if *three* times, it is called the *third power*, or *cube;* if *four* times, the *fourth* power, &c. Thus, the second power of 3 is 3×3
e third power of 3 is $3 \times 3 \times 3 = 27$; the fifth
3 is $3 \times 3 \times 3 \times 3 \times 3 = 243$.

90. The **Index** or **Exponent** of a power is a figure which shows how many times the number is taken as a factor. It is written at the right of the number, and above the line. Thus, in 5^3, 7^2, 2^4, the exponent ^{3} shows that 5 is taken three times as a factor, ^{2} that 7 is taken twice, and ^{4} that 2 is taken four times as a factor.

91. The **Root** of a number is one of the equal factors which produce that number. If it is one of the two equal factors, it is the *second*, or *square* root; if one of the three, the *third*, or *cube* root; if one of the four, the *fourth* root, &c. Thus the square root of 9 is 3, the cube root of 125 is 5.

92. $\sqrt{\ }$ is the **Radical Sign**, and, by itself, denotes the square root; with a figure placed above, it denotes the root of that degree indicated by the figure; thus, $\sqrt[3]{\ }$ signifies the third root, $\sqrt[6]{\ }$ the sixth root.

Divisibility of Numbers.

93. (1.) Any number whose unit figure is 0, 2, 4, 6, or 8, is even.

(2.) Any number whose unit figure is 1, 3, 5, 7, or 9, is odd.

(3.) Any even number is divisible by 2.

(4.) Any number is divisible by 3 when the sum of its digits is divisible by 3; thus, 2814 is divisible by 3, for $2+8+1+4 = 15$, is divisible by 3.

(5.) Any number is divisible by 4, when its tens and units are divisible by 4; for, as 1 hundred, and consequently any number of hundreds, is divisible by 4, the divisibility of the given number by 4 must depend upon the tens and units; thus, 86324 is divisible by 4, while 6831 is not.

(6.) Any number is divisible by 5 if the units' figure is either 5 or 0; for, as 1 ten, and consequently any number of tens, is divisible by 5, the divisibility of the given number by 5 must depend upon the units.

(7.) Any number is divisible by 6, if divisible by 3 and by 2.

(8.) Any number is divisible by 8, if its hundreds, tens, and units are divisible by 8; for, as 1 thousand, and consequently any

number of thousands is divisible by 8, the divisibility of the given number by 8 must depend on the hundreds, tens, and units.

(9.) Any number is divisible by 9 if the sum of its digits is divisible by 9 *; thus, 368451 is divisible by 9, and 23476 is not.

(10.) Any number is divisible by 10, 100, or 1000, if it contain at the right 1, 2, or 3 zeros; and so on.

(11.) Any number is divisible by 11, if the difference between the sums of the alternate digits is 0, or a number divisible by 11; thus, in 126896, as $(1+6+9)-(2+8+6)=0$, the number is divisible by 11; and in 9053, as $(9+5)-(0+3)=11$, the number is divisible by 11.

(12.) A number is divisible by any composite number, if it is divisible by all the factors of that number.

93. There are no rules of sufficient practical importance for determining when numbers are divisible by other numbers than those spoken of above. Their divisibility must be ascertained by trial. To do this,—

Divide the number successively by higher and higher primes, until one is found which divides it, or until the quotient is smaller than the divisor. If no divisor is then found, the number is prime; for, if a number contain any prime factor greater than its square root, its corresponding factor must be less.

94. If the odd numbers are written in order, and every third one from 3, every fifth one from 5, every seventh one from 7, and so on, be marked, and the figures 3, 5, 7, &c., be written under the figures as they are marked, the remaining numbers will be primes, and those marked will have for their factors the numbers written beneath; † thus,—

$$1,\ 3,\ 5,\ 7,\ \underset{3}{\cancel{9}},\ 11,\ 13,\ \underset{3,5}{\cancel{15}},\ 17,\ 19,\ \underset{3,7}{\cancel{21}},\ 23,\ \underset{5}{\cancel{25}},\ \underset{3,9}{\cancel{27}},\ 29,\ 31,$$

$$\underset{3,11}{\cancel{33}},\ \underset{5,7}{\cancel{35}},\ 37,\ \underset{3,13}{\cancel{39}},\ 41,\ 43,\ \underset{\substack{3,5\\9,15}}{\cancel{45}},\ 47,\ \underset{7}{\cancel{49}},\ \underset{3,17}{\cancel{51}},\ \&c.$$

* See Appendix.

† Eratosthenes, in the third century B. C., discovered this method of finding primes and factors of numbers, and as he made his table of parchment, cutting out the composite numbers as he found them, this parchment called *Eratosthenes' Sieve.*

By applying this principle, a table can easily be made of the primes and of the composites, with their factors.

Table of Prime Numbers to 1201.

1	61	151	251	359	463	593	701	827	953	1069
2	67	157	257	367	467	599	709	829	967	1087
3	71	163	263	373	479	601	719	839	971	1091
5	73	167	269	379	487	607	727	853	977	1093
7	79	173	271	383	491	613	733	857	983	1097
11	83	179	277	389	499	617	739	859	991	1103
13	89	181	281	397	503	619	743	863	997	1109
17	97	191	283	401	509	631	751	877	1009	1117
19	101	193	293	409	521	641	757	881	1013	1123
23	103	197	307	419	523	643	761	883	1019	1129
29	107	199	311	421	541	647	769	887	1021	1151
31	109	211	313	431	547	653	773	907	1031	1153
37	113	223	317	433	557	659	787	911	1033	1163
41	127	227	331	439	563	661	797	919	1039	1171
43	131	229	337	443	569	673	809	929	1049	1181
47	137	233	347	449	571	677	811	937	1051	1187
53	139	239	349	457	577	683	821	941	1061	1193
59	149	241	353	461	587	691	823	947	1063	1201

Table of the Composite Numbers to 917,

Which contain no prime factor less than 7 (*excepting* 1*).

Nos.	Factors.	Nos.	Factors.	Nos.	Factors.	Nos.	Factors.	Nos.	Factors.
49	7^2	289	17^2	469	7, 67	623	7, 89	779	19, 41
77	7, 11	299	13, 23	473	11, 43	629	17, 37	781	11, 71
91	7, 13	301	7, 43	481	13, 37	637	7^2, 13	791	7, 113
119	7, 17	319	11, 29	493	17, 29	649	11, 59	793	13, 61
121	11^2	323	17, 19	497	7, 71	667	23, 29	799	17, 47
133	7, 19	329	7, 47	511	7, 73	671	11, 61	803	11, 73
143	11, 13	341	11, 31	517	11, 47	679	7, 97	817	19, 43
161	7, 23	343	7^3	527	17, 31	689	13, 53	833	7^2, 17
169	13^2	361	19^2	529	23^2	697	17, 41	841	29^2
187	11, 17	371	7, 53	533	13, 41	703	19, 37	847	7, 11^2
203	7, 29	377	13, 29	539	7^2, 11	707	7, 101	851	23, 37
209	11, 19	391	17, 23	551	19, 29	713	23, 31	869	11, 79
217	7, 31	403	13, 31	553	7, 79	721	7, 103	889	7, 127
221	13, 17	407	11, 37	559	13, 43	731	17, 43	893	19, 47
247	13, 19	413	7, 59	581	7, 83	737	11, 67	899	29, 31
253	11, 23	427	7, 61	583	11, 53	749	7, 107	901	17, 53
259	7, 37	437	19, 23	589	19, 31	763	7, 109	913	11, 83
287	7, 41	451	11, 41	611	13, 47	767	13, 59	917	7, 131

* 1 is a factor of all numbers.

FACTORING OF NUMBERS.

95. Illustrative Example, I.

Resolve 48 into its prime factors.

OPERATION.

$48 = 6 \times 8$; $6 = 2 \times 3$; $8 = 2 \times 2 \times 2$; $\therefore 48 = 2 \times 2 \times 2 \times 2 \times 3$, or $2^4 \times 3$. Hence,

Rule I. To resolve a number into its prime factors.— *First separate it into any two factors; separate these factors, if they are composite, into others, and so on, till all are prime.*

Proof. *Multiply the factors thus obtained together, and the product, if the work is correct, will equal the given number.*

96. Examples.

Resolve the following numbers into their prime factors.—

1. 32. *Ans.* 2^5.	4. 56.	7. 100.	10. 81.	13. 64.	16. 130
2. 84. *Ans.* $2^2 \times 3 \times 7$.	5. 49.	8. 150.	11. 99.	14. 77.	17. 125
3. 88. *Ans.* $2^3 \times 11$.	6. 72.	9 69.	12. 144.	15. 108.	18. 2500

97. Illustrative Example, II.

Resolve 42075 into its prime factors.

OPERATION.

$$\begin{array}{r} 3\,)\,\underline{42075} \\ 3\,)\,\underline{14025} \\ 5\,)\,\underline{4675} \\ 5\,)\,\underline{935} \\ 11\,)\,\underline{187} \\ 17 \end{array}$$

Ans. $3^2, 5^2, 11, 17$.

Here we divide, successively, by such prime numbers as will leave no remainder, till we obtain a prime number for a quotient; since the product of these prime numbers, 3, 3, 5, 5, 11, and 17 equals the given number, they must be the prime factors of that number. Hence,

Rule II. *Divide the number by any prime number which is contained in it without a remainder. Divide the quotient in the same manner, and thus continue till a quotient is obtained which is a prime number. This quotient and the several divisors are the prime factors.*

NOTE. — The work may sometimes be shortened by dividing by a composite number, remembering afterwards to substitute the factors of that number for the number itself. Thus, in the above we may divide by 9 instead of dividing by 3 twice.

98. EXAMPLES.

Resolve the following numbers into their prime factors.

19.	176.	*Ans.* $2^4 \times 11$.	23.	260.	27.	357.
20.	180.	*Ans.* $2^2 \times 3^2 \times 5$.	24.	285.	28.	644.
21.	192.	*Ans.* $2^6 \times 3$.	25.	329.	29.	684.
22.	208.	*Ans.* $2^4 \times 13$.	26.	338.	30.	2310.

99. Select the prime numbers in the columns below, and find the factors of the composite numbers.

1.	341.	6.	450.	11.	704.	16.	947.
2.	344.	7.	590.	12.	719.	17.	971.
3.	362.	8.	560.	13.	769.	18.	2681.
4.	367.	9.	596.	14.	808.	19.	1163.
5.	409.	10.	689.	15.	839.	20.	3248.

☞ For Dictation Exercises, see Key.

GREATEST COMMON DIVISOR.

100. A **Common Divisor** of two or more numbers is any number that will exactly divide each of them; thus, 2 is a common divisor of 12 and 18.

101. The **Greatest Common Divisor** is the *greatest* number that will exactly divide each of them; thus, 6 is the greatest common divisor of 12 and 18.

102. ILLUSTRATIVE EXAMPLE.

Find the greatest common divisor of 12, 30, and 42.

OPERATION.

$12 = 2 \times 2 \times 3.$
$30 = 2 \times 3 \times 5.$
$42 = 2 \times 3 \times 7.$
G. C. D. $= 2 \times 3 = 6$ *Ans.*

As 2 and 3 are the only common factors of 12, 30, and 42, it follows that 2×3, or 6, is the greatest common divisor. Hence,

RULE I. To find the greatest common divisor of two or

more numbers: *Separate the numbers into their prime factors, and find the product of such as are common.*

103. EXAMPLES.

Find the G. C. D.* of

1. 48, 56, and 60. *Ans.* 4.	3. 108, 45, 18, and 63.
2. 24, 42, and 54. *Ans.* 6.	4. 18, 36, 12, 48, and 42.

NOTE.—In Example 4, 18 is a factor of 36, and 12 of 48. The G.C.D. of 18 and 12 must be the G. C. D. of 18, 12, and their *multiples*, 36 and 48; ∴ we need only find the G. C. D. of 18, 12, and 42.

Find the G. C. D. of

5. 42, 28, and 84.	7. 32, 18, 108, and 25.
6. 26, 52, and 65.	8. 114, 102, 78, and 66.

104. When numbers cannot readily be separated into their factors, the following method may be adopted:—

ILLUSTRATIVE EXAMPLE. Find the G. C. D. of 91 and 325.

OPERATION.

```
91 ) 325 ( 3
     273
      52 ) 91 ( 1
           52
           39 ) 52 ( 1
                39
                13 ) 39 ( 3
                     39
                     00
```

We divide 325 by 91, to see if it is a divisor of 325, for 91 is the greatest divisor of itself; if it is a divisor of 325, it is the G. C. D. of 91 and 325. It is not a divisor of 325, for there is a remainder of 52. 52 is the greatest divisor of itself; if it is a divisor of 91, it is the G. C. D. of 52 and 91. It is not a divisor of 91, for there is a remainder of 39; 39 is the greatest divisor of itself; if it is a divisor of 52, it is the G. C. D. of 39 and 52. It is not a divisor of 52, for there is a remainder of 13; 13 is the G. C. D. of itself and 39. It must therefore be of 39 and 52, for $52 = 1 \times 39 + 13$. If it is the G. C. D. of 39 and 52, it must be of 52 and 91, for $91 = 1 \times 52 + 39$. If it is the G. C. D. of 52 and 91, it must be of 91 and 325, for $325 = 3 \times 91 + 52$. Hence the following:

RULE II. To find the G. C. D. of two numbers: *Divide the greater number by the less, and the less number by the remainder, if there is any, and thus proceed, dividing the last*

* Greatest Common Divisor.

divisor by the last remainder, until nothing remains. The last divisor is the G. C. D. sought.

To find the G. C. D. of more than two numbers, *find the G. C. D. of any two of them, and then of that divisor and a third number, and so on till all the numbers are taken.*

105. EXAMPLES.

Find the G. C. D. of

9. 198 and 297. *Ans.* 99.
10. 222 and 564. *Ans.* 6.
11. 529, 782, and 1127. *Ans.* 23.
12. 229 and 954.
13. 392, 1008, and 224.
14. 6581, 6611, and 249.

15. What is the width of the widest carpeting that will exactly fit either of two halls, 45 feet and 33 feet wide respectively? *Ans.* 3 ft.

16. A has a piece of ground 90 feet long and 42 feet wide, what is the length of the longest rails that will exactly suit its length and its width? *Ans.* 6 ft.

17. A lady has one flower bed measuring 10 feet around, and another measuring 18 feet. If she borders the beds with pinks, what is the greatest distance she can set her pink roots apart, and have them equally distant in the two beds? *Ans.* 2 ft.

18. A man has 90 bushels Kidney potatoes, 60 bushels Jackson Whites, and 105 bushels Red Rileys. If he puts them all into the largest bins of equal size that will exactly measure either lot, how many bushels will each of his bins contain?

19. What is the length of the longest stepping-stones that will exactly fit 3 streets, 72, 51, and 87 feet wide, respectively?

20. What is the length of the longest curb-stones that will exactly fit 4 strips of sidewalk, the first being 273 feet long, the second 294, the third 567, and the fourth 651?

☞ For Dictation Exercises, see Key.

FRACTIONS.

106. A **Fraction** is one or more of the equal parts of a unit; thus, $\frac{3}{4}$, read *three fourths*, shows that a unit has been divided into four equal parts, and that three of those parts are taken.

107. The number which shows into how many equal parts a unit is divided, is called the **Denominator** of the fraction, because it *denominates* or *names* the parts; thus, 4 is the denominator of $\frac{3}{4}$.

108. The number which shows how many parts are taken, is called the **Numerator**; thus, 3 is the numerator of $\frac{3}{4}$.

109. The numerator and denominator are called the **Terms** of a fraction.

110. A **Common** or **Vulgar Fraction** is a fraction whose denominator and numerator are both expressed, the numerator being written above, and the denominator below, a dividing line; as, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{9}{25}$.

111. A **Decimal Fraction** is one whose denominator is 10, or some integral power of 10. The denominator is not generally expressed. $\frac{2}{10}$ written .2, and $\frac{36}{100}$ written .36, are decimal fractions.

112. A **Mixed Number** is a whole number and a fraction expressed together, as $7\frac{5}{8}$, $21\frac{3}{5}$.

113. Common fractions may be either **Proper, Improper, Compound,** or **Complex.**

114. A **Proper Fraction** is one whose numerator is *less* than its denominator, as $\frac{2}{3}$.

115. An **Improper Fraction** is one whose numerator *equals* or *exceeds* its denominator, as $\frac{3}{3}\frac{5}{5}$, $\frac{5}{4}$.

116. A **Compound Fraction** is a fraction of a fraction, as $\frac{3}{4}$ of $\frac{5}{6}$.

117. A **Complex Fraction** is one which contains a fraction in either or both of its terms, as $\frac{2\frac{5}{8}}{7}$, $\frac{\frac{8}{9}}{7\frac{2}{3}}$.

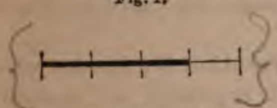
QUESTIONS. What does the denominator of a fraction show? What does the numerator show?

What is meant by the expression $\frac{5}{8}$? *Ans.* 5 of the 8 equal parts into which a unit is divided.

What is meant by the expression $\frac{7}{8}$? $\frac{9}{10}$? $\frac{18}{20}$? $\frac{13}{25}$?

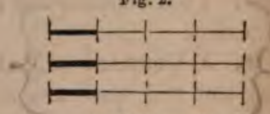
118. If we compare common fractions with the last expression for division in Art. 55, we shall see that their forms are alike. A fraction implies division, the numerator being the dividend, and the denominator the divisor. Thus, $\frac{3}{4}$ may be considered either three fourths of 1 or one fourth of 3. The following diagram will show that these are equivalent expressions, $\frac{3}{4}$ of the one line in figure 1 being equal to $\frac{1}{4}$ of the three lines in figure 2.

Fig. 1.



$\frac{3}{4}$ of $1 = \frac{3}{4}$.

Fig. 2.



$\frac{1}{4}$ of $3 = \frac{3}{4}$.

What does $\frac{5}{7}$ denote? *Ans.* It denotes either 5 of the seven equal parts into which 1 is divided, or one seventh of 5.

What does $\frac{9}{11}$ show? $\frac{25}{27}$? $\frac{15}{16}$?

119. GENERAL PRINCIPLES.

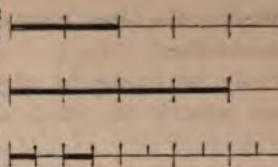
NOTE.—The following propositions should be copiously illustrated by the teacher, and frequently referred to, until they are fully comprehended by the pupil.

PROPOSITION I. As the denominator of a fraction shows the number of parts into which a unit is divided, and the numerator shows how many parts are taken, it follows that if we *multiply the numerator* of a fraction by a whole number, we *multiply the number of parts*, and thus *increase* the value of the fraction; but if we *multiply the denominator* of a fraction, we *multiply the number of parts into which a unit is divided*, and thus *diminish* the *size of the parts*, and consequently decrease the value of the fraction.

$\frac{2}{5}$

ILLUSTRATION. $\frac{2 \times 2}{5} = \frac{4}{5}$ Fraction increased.

$\frac{2}{5 \times 2} = \frac{2}{10}$ Fraction diminished.

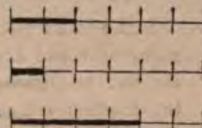


PROPOSITION II. If we *divide the numerator* of a fraction by a whole number, we *divide the number of parts* and thus *diminish* the value of the fraction; but if we *divide the denominator* of a fraction, we *divide the number which shows into how many parts the unit is divided*, and thus *increase the size of the parts*, and consequently increase the value of the fraction.

$\frac{2}{6}$

ILLUSTRATION. $\frac{2 \div 2}{6} = \frac{1}{6}$ Fraction diminished.

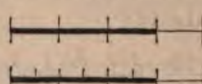
$\frac{2}{6 \div 2} = \frac{2}{3}$ Fraction increased.



PROPOSITION III. If we multiply the numerator and denominator, each by the same number, we *increase the number of parts of the fraction*, but *diminish their size in the same proportion;* consequently the value of the fraction is not altered.

ILLUSTRATION. $\frac{3}{4}$

$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

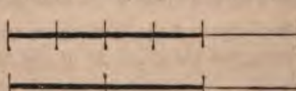


Fraction not altered in value.

PROPOSITION IV. If we divide the numerator and denominator, each by the same number, we *diminish the number of parts* in the same proportion as we *increase their size*, consequently the value of the fraction is not altered.

ILLUSTRATION. $\frac{4}{6}$

$\frac{4 \div 2}{6 \div 2} = \frac{2}{3}$



Fraction not altered in value.

QUESTIONS. How does *multiplying the numerator* of a fraction affect the value of the fraction? Why? How does *multiplying the denominator* affect the value of the fraction? Why?

How does *dividing the numerator* of a fraction affect the value

of the fraction? Why? How does *dividing the denominator* affect the value of the fraction? Why?

If, then, you multiply the numerator and denominator each by the same number, what is the effect upon the fraction? Why?

If you divide the numerator and denominator each by the same number, what is the effect upon the fraction? Why?

REDUCTION OF FRACTIONS TO LOWEST TERMS.

120. A Fraction is expressed in its lowest terms when the numerator and denominator are prime to each other.

121. ILL. EX. Reduce $\frac{8}{10}$ to its lowest terms.

OPERATION.

$$\frac{8}{10}=\frac{4\times\cancel{2}}{5\times\cancel{2}}=\frac{4}{5}, \textit{ Ans.}$$

$8=4\times 2$; $10=5\times 2$. Dividing the numerator and denominator each by striking out the common factor 2, the value of the fraction will not be altered (Art. 119, Prop. IV.), and will be expressed in its lowest terms. Hence the

RULE. To reduce a fraction to its lowest terms: *Remove from the numerator and denominator all their common factors.*

122. EXAMPLES.

Reduce to their lowest terms,

1. $\frac{10}{15}$. *Ans.* $\frac{2}{3}$.	4. $\frac{64}{88}$. *Ans.* $\frac{8}{11}$.	7. $\frac{200}{350}$.	10. $\frac{39}{271}$
2. $\frac{20}{32}$. *Ans.* $\frac{5}{8}$.	5. $\frac{99}{108}$.	8. $\frac{75}{100}$.	11. $\frac{750}{800}$.
3. $\frac{49}{56}$. *Ans.* $\frac{7}{8}$.	6. $\frac{18}{81}$.	9. $\frac{450}{675}$.	12. $\frac{375}{525}$.

13. Reduce $\frac{15}{45}$ to its lowest terms. *Ans.* $\frac{1}{3}$.

NOTE I. — 1, being a factor of all numbers, will remain when all other factors are struck out, as in the numerator of example 13.

NOTE II. — In case the factors of the numerator and denominator cannot readily be ascertained, *find the G. C. D. of the two terms, and divide each of them by it.*

Reduce to their lowest terms,

14. $\frac{75}{300}$.	16. $\frac{342}{1710}$.	18. $\frac{2391}{10361}$.	20. $\frac{1891}{7471}$
15. $\frac{810}{7290}$.	17. $\frac{562}{2528}$.	19. $\frac{737}{6589}$.	21. $\frac{1135}{9761}$

☞ For Dictation Exercises, see Key.

CANCELLATION.

123. Cancellation consists in rejecting equal factors from dividend and divisor.

124. All arithmetical operations in division may be expressed in the form of a fraction, the dividend being the numerator, and the divisor the denominator; since dividing both terms of a fraction by the same number does not alter its value, it follows, that we may strike out, or *cancel*, any factors common to the dividend and divisor without changing their relative value.

N. B. *All operations upon arithmetical quantities should first be expressed, as far as possible, by signs, that the processes may be clearly indicated to the teacher, and that the work to be done may be reduced, if possible, by cancellation.*

ILL. EX. Divide 3 times 4 times 6 times 5 times 7, by 2 times 8 times 6 times 9 times 10.

$$\frac{\cancel{3}\times\cancel{4}\times\cancel{6}\times\cancel{5}\times 7}{2\times\underset{2}{\cancel{8}}\times\cancel{6}\times\underset{3}{\cancel{9}}\times\underset{2}{\cancel{10}}}=\frac{7}{2\times 2\times 3\times 2}=\frac{7}{24},\ \textit{Ans.}$$

125. EXAMPLES.

Express, cancel, and perform the following: —

1. Divide $8 \times 6 \times 3 \times 9 \times 7 \times 4$, by $2 \times 5 \times 7 \times 10 \times 8$. *Ans.* $6\frac{12}{25}$.

2. Divide 81×42, by 99×7.

3. Multiply 75×10, by 3×6, and divide that product by $15 \times 25 \times 12$.

4. Divide $7 \times 8 \times 48$, by $63 \times 4 \times 5 \times 17$, and multiply the quotient by 51.

5. Divide $99 \times 28 \times 6$, by $5 \times 8 \times 18$, multiply the quotient by 4×4, and divide by 22×27.

6. Spent $\frac{1}{3}$ of \$75, which I received for work, for flour at \$5 a barrel; how many barrels did I buy?

7. If 25 pounds of lead costs \$4.60, what do 8 pounds cost?

8. What will be received for 27 pieces of broadcloth, if 6 pieces bring \$864?

9. If it requires 13 bushels of wheat to make 3 barrels of flour, how many bushels will be required to make 78 barrels of flour? *Ans.* 338 bushels.

10. If a tree 69 feet high casts a shadow of 90 feet, what length of shadow will be cast by a tree 92 feet high?

Ans. 120 feet.

11. A merchant exchanged 561 pounds of sugar, at 9 cents per pound, for eggs at 11 cents per dozen; how many dozen were received?

12. If 12 pieces of cloth, each piece containing 62 yards, cost $372, what cost 24 yards?

13. If a bar of iron 8 feet long weighs 36 pounds, what will a bar of the same size 100 feet long weigh?

14. How many boxes of oranges can be bought for $420, if $28 be paid for 7 boxes?

15. If the work of 7 men is equal to the work of 9 boys, how many men's work will equal the work of 63 boys?

16. If 15 men consume a barrel of flour in 6 weeks, how long would it last 9 men? *Ans.* 10 weeks.

17.* If the interest of $650 for 12 months is $52, what is the interest of three times that sum for eight months? *Ans.* $104.

18. If 12 men can build a wall in 42 days, how many days will be required for 21 men to build it?

19. If $15 purchase 12 yards of cloth, how many yards will $48 purchase? *Ans.* $38\frac{2}{5}$ yards.

20. A ship has provision for 15 men 12 months; how long will it last 45 men?

21. How many overcoats, each containing 4 yards, can be made from 10 bales of cloth, 12 pieces each, 42 yards in each piece?

22.* If 375 barrels of pork, each 200 pounds, cost $6000, what is the cost of 5 barrels, each 195 pounds?

23.* Sold 20 barrels of apples at $2.50 per barrel, and spent the money thus obtained for cloth at $.50 a yard, which I sold at $.60 a yard, and bought a horse with the proceeds. What did I pay for the horse?

126. Reduction of Whole or Mixed Numbers to Improper Fractions.

Ill. Ex. Change $2\frac{7}{8}$ to an improper fraction.

Operation.

$2\frac{7}{8} = \frac{16+7}{8} = \frac{23}{8}$. *Ans.*

In 1 unit there are $\frac{8}{8}$, ∴ in 2 units there are $2 \times \frac{8}{8}$ or $\frac{16}{8}$. $\frac{16}{8} + \frac{7}{8} = \frac{23}{8}$, *Ans.*

Hence the

Rule. To reduce a mixed number to an improper fraction;— *Multiply the whole number by the denominator of the fraction; to that product add the numerator; and write the result over the denominator.*

Examples.

Reduce to improper fractions,

1. $2\frac{3}{4}$. *Ans.* $\frac{11}{4}$.	3. $7\frac{3}{5}$.	5. $11\frac{18}{19}$.	7. $321\frac{2}{5}$.
2. $4\frac{1}{2}$. *Ans.* $\frac{9}{2}$.	4. $8\frac{2}{7}$.	6. $14\frac{7}{15}$.	8. $8\frac{564}{931}$.

9. Reduce 36 to fifths. $\frac{36 \times 5}{5} = \frac{180}{5}$, *Ans.*
10. Reduce $584\frac{2}{9}$, 368, $87\frac{5}{9}$, to ninths.
11. Add 784 to 916, and express the answer in sevenths.
12. Reduce 7×98 to eighths.
13. Reduce $(15 - 8) \times 16$ to fifths.
14. Reduce 8692 to a fraction whose denominator is 25.
15. Reduce $367\frac{29}{77}$ to an improper fraction.
16. Change $4567\frac{8}{9}$ to ninths, 43862 to elevenths.
17. Change $36\frac{564}{902}$ to an improper fraction.

☞ For Dictation Exercises, see Key.

127. Reduction of Improper Fractions to Whole or Mixed Numbers.

Ill. Ex. Change $\frac{29}{6}$ to a mixed number.

Operation.

$\frac{29}{6} = 4\frac{5}{6}$, *Ans.*

There are $\frac{6}{6}$ in 1 unit, ∴ in $\frac{29}{6}$ there are as many units as $\frac{6}{6}$ is contained times in $\frac{29}{6}$, which is $4\frac{5}{6}$ times. Hence the

Rule. To reduce an improper fraction to a mixed number;— *Divide the numerator by the denominator.*

Examples.

Change to whole or mixed numbers,

1. $\frac{34}{7}$. *Ans.* $4\frac{6}{7}$.	6. $\frac{931}{15}$.	11. $\frac{196}{98}$.
2. $\frac{85}{4}$. *Ans.* $21\frac{1}{4}$.	7. $\frac{3001}{19}$.	12. $\frac{71}{71}$.
3. $\frac{98}{5}$. *Ans.* $19\frac{3}{5}$.	8. $\frac{150}{24}$.	13. $\frac{23784}{25}$.
4. $\frac{105}{6}$.	9. $\frac{865}{321}$.	14. $\frac{368471}{13}$.
5. $\frac{684}{13}$.	10. $\frac{37048}{11}$.	15. $\frac{854321}{328}$.

☞ For Dictation Exercises, see Key.

128. Multiplication of Fractions by Whole Numbers.

As multiplying the numerator of a fraction multiplies the *number* of parts, their size remaining the same, and dividing the denominator multiplies the *size* of the parts, their number remaining the same (Art. 119), it follows that, —

To multiply a fraction by a whole number, we may either *multiply the numerator by the whole number, or divide the denominator.*

The latter method is preferable when the denominator can be divided without a remainder, as it gives the answer in lower terms.

Ill. Ex. Multiply $\frac{7}{8}$ by 4

1st Operation.

$$\frac{7\times 4}{8}=\frac{28}{8}=3\tfrac{4}{8},\ \textit{Ans.}$$

2d Operation.

$$\frac{7}{8\div 4}=\frac{7}{2}=3\tfrac{1}{2},\ \textit{Ans.}$$

We might have cancelled in the first operation, and thus have obtained the same result as in the second; thus, $\frac{7\times\cancel{4}}{\underset{2}{\cancel{8}}}=\frac{7}{2}=3\frac{1}{2}$

Examples.

Multiply

1. $\frac{4}{9}$ by	5.	*Ans.* $2\frac{2}{9}$.	9.	$\frac{8}{67}$ by	19.
2. $\frac{3}{29}$ by	6.	*Ans.* $\frac{18}{29}$.	10.	$\frac{38}{105}$ by	21.
3. $\frac{4}{85}$ by	504.	*Ans.* $23\frac{61}{85}$.	11.	$\frac{81}{100}$ by	95.
4. $\frac{7}{15}$ by	15.		12.	$\frac{82}{162}$ by	54.
5. $\frac{3}{8}$ by	4.		13.	$\frac{387}{548}$ by	274.
6. $\frac{3}{11}$ by	110.		14.	$\frac{15}{640}$ by	328.
7. $\frac{85}{108}$ by	9.		15.	$\frac{64}{2286}$ by	762.
8. $\frac{87}{99}$ by	11.		16.	$\frac{36874}{55}$ by	55.

17. If one yard of cloth costs $\frac{2}{3}$ of a dollar, what will 17 yards cost?

18. If a ton of coal costs $\frac{3}{4}$ of an Eagle, how much will 15 tons cost?

19. Required the cost of 28 pounds of candles, at $\frac{3}{8}$ of a dollar a pound.

20. Multiply $256\frac{7}{8}$ by 18. *Ans.* $4623\frac{3}{4}$.

21. Multiply $376\frac{23}{63}$ by 21.

☞ For Dictation Exercises, see Key.

129. Multiplication of Whole Numbers by Fractions.

Ill. Ex. Multiply 8 by $\frac{5}{6}$.

Operation.

$$\frac{\overset{4}{\cancel{8}} \times 5}{\underset{3}{\cancel{6}}} = \frac{4 \times 5}{3} = 6\tfrac{2}{3}, \textit{ Ans.}$$

8 multiplied by 5 is 5×8; if it is multiplied by $\frac{5}{6}$, a number one sixth as large as 5, the product must be one sixth as large as if 5 had been the multiplier, or $\frac{1}{6}$ of 5×8. The expression then becomes 8 times 5, divided by 6; after cancelling, $\frac{4 \times 5}{3} = \frac{20}{3} = 6\frac{2}{3}$, *Ans.* Hence the

Rule. To multiply a whole number by a fraction; — *Multiply the whole number by the numerator of the fraction, and divide that product by the denominator.*

Examples.

Multiply

1. 36 by $\frac{2}{3}$. *Ans.* 24.
2. 568 by $\frac{5}{6}$. *Ans.* $473\frac{1}{3}$.
3. 385 by $\frac{7}{25}$.
4. 3681 by $\frac{5}{18}$.
5. 5432 by $\frac{17}{24}$.
6. 87036 by $\frac{8}{27}$.
7. What cost $\frac{5}{8}$ of 1 ton of hay, at \$12 a ton?

Operation.

$$\frac{\$\overset{1.50}{\cancel{12.00}} \times 5}{\cancel{8}} = \$\,7.50, \textit{ Ans.}$$

If 1 ton of hay costs \$12, $\frac{1}{8}$ of a ton will cost $\frac{1}{8}$ of \$12, and $\frac{5}{8}$ of a ton will cost 5 times $\frac{1}{8}$ of \$12. Cancelling, we have $\$1.50 \times 5 = \$7.50$, *Ans.*

8. What cost $\frac{8}{9}$ of an acre of land, at \$100 an acre?
9. What cost $\frac{5}{12}$ of an acre of land, at \$150 an acre?
10. What cost $\frac{3}{4}$ of a gross of pens, at \$.96 a gross?
11. What cost $5\frac{7}{8}$ cords of wood, at \$7.56 a cord?
12. What cost $3\frac{5}{6}$ hogsheads of molasses, at \$18.80 a hogshead?

13. What cost $2\frac{6}{7}$ firkins of butter, at \$12.60 a firkin?

14. What cost $63\frac{2}{3}$ yards of flannel, at \$.54 a yard?

☞ For Dictation Exercises, see Key.

130. Multiplication of Fractions by Fractions.

Ill. Ex. Multiply $\frac{5}{6}$ by $\frac{8}{9}$.

Operation.

$$\frac{5 \times \overset{4}{\cancel{8}}}{\underset{3}{\cancel{6}} \times 9} = \frac{20}{27}, \textit{ Ans.}$$

$\frac{5}{6}$ multiplied by 8, is 8 times $\frac{5}{6}$; if it be multiplied by $\frac{8}{9}$, a number *one ninth* as large as 8, the product must be one ninth as large as if 8 had been the multiplier, or one ninth of 8 times $\frac{5}{6}$. The expression then becomes $\frac{5 \times 8}{6 \times 9}$; after cancelling, $\frac{5 \times 4}{3 \times 9} = \frac{20}{27}$, *Ans.* Hence the

Rule. To multiply a fraction by a fraction; — *Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

Examples.

Multiply

1. $\frac{2}{3}$ by $\frac{3}{4}$. *Ans.* $\frac{1}{2}$.	4. $\frac{39}{49}$ by $\frac{14}{15}$.
2. $\frac{5}{7}$ by $\frac{3}{10}$. *Ans.* $\frac{3}{14}$.	5. $\frac{86}{99}$ by $\frac{198}{172}$.
3. $\frac{10}{13}$ by $\frac{3}{5}$. *Ans.* $\frac{6}{13}$.	6. $\frac{714}{883}$ by $\frac{1766}{988}$.

7. $\frac{8}{11} \times 2\frac{1}{2} =$ what?

Note. — Reduce mixed numbers to improper fractions before multiplying by fractions.

8. $18\frac{6}{7} \times \frac{14}{33} = ?$	10. $15\frac{5}{6} \times \frac{8}{19} = ?$
9. $5\frac{12}{18} \times \frac{6}{257} = ?$	11. $3\frac{5}{6} \times 31\frac{2}{3} = ?$

Note. — In the ninth example, first reduce $\frac{12}{18}$ to lower terms.

12. $38\frac{27}{36} \times \frac{12}{31} = ?$	13. $2\frac{11}{55} \times \frac{405}{459} = ?$

14. What cost $2\frac{9}{10}$ boxes of raisins, at $1\frac{2}{3}$ dollars a box?

15. What cost $10\frac{3}{5}$ tons of coal, at \$$7\frac{5}{6}$ a ton?

16. What cost $5\frac{1}{2}$ pounds of coffee, at $\frac{3}{10}$ of a dollar a pound?

17. What cost $17\frac{8}{10}$ pounds of pork, at $\frac{3}{20}$ of a dollar a pound?

☞ For Dictation Exercises, see Key.

131. Reduction of Compound Fractions to Simple Fractions.

Ill. Ex., I. If 1 dollar buys $\frac{3}{5}$ of a yard of cloth, how much will $\frac{3}{4}$ of a dollar buy?

OPERATION.

$\frac{3 \times 3}{5 \times 4} = \frac{9}{20}$ yd., *Ans.*

EXPLANATION 1ST. If one dollar buys $\frac{3}{5}$ of a yard of cloth, $\frac{1}{4}$ of a dollar will buy $\frac{1}{4}$ of $\frac{3}{5}$ of a yard, and $\frac{3}{4}$ of a dollar will buy 3 times $\frac{1}{4}$ of $\frac{3}{5}$ of a yard, $= \frac{9}{20}$ of a yard.

EXPLANATION 2D. If one dollar buys $\frac{3}{5}$ of a yard of cloth, $\frac{3}{4}$ of a dollar will buy $\frac{3}{4}$ of $\frac{3}{5}$ of a yard. $\frac{1}{4}$ of $\frac{1}{5}$ of a yard is $\frac{1}{20}$ of a yard, which may be shown by dividing $\frac{1}{5}$ of a yard into 4 equal parts. The whole yard can thus be divided into 4×5, or 20 equal parts; therefore, 1 part will be $\frac{1}{20}$ of the whole yard. If $\frac{1}{4}$ of $\frac{1}{5} = \frac{1}{20}$, $\frac{1}{4}$ of $\frac{3}{5}$ must be $\frac{3}{20}$, and $\frac{3}{4}$ of $\frac{3}{5}$ must be 3 times $\frac{3}{20}$, or $\frac{9}{20}$. *Ans.*

ILL. EX., II. $\frac{3}{4}$ of $\frac{3}{5}$ = what? $\frac{3 \times 3}{4 \times 5} = \frac{9}{20}$, *Ans.*

$\frac{3}{4}$ of $\frac{3}{5}$ is a compound fraction (Art. 116), and is reduced to a simple fraction *by multiplication.* Let the explanation be similar to the second explanation of the illustrative example.

EXAMPLES.

1. $\frac{8}{9}$ of $\frac{9}{10}$ = what? *Ans.* $\frac{4}{5}$.
2. $\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{14}{21}$ = what? *Ans.* $\frac{2}{7}$.
3. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$ = what?
4. $\frac{10}{33}$ of $\frac{16}{15}$ of $\frac{25}{48}$ of $\frac{11}{32}$ = what?
5. $\frac{7}{8}$ of $2\frac{1}{5}$ of $\frac{8}{55}$ of $3\frac{1}{3}$ = what?
6. $\frac{11}{12}$ of $3\frac{3}{7}$ of $1\frac{13}{22}$ of 60 = what?

☞ For Dictation Exercises, see Key.

132. DIVISION OF FRACTIONS BY WHOLE NUMBERS.

As dividing the numerator of a fraction diminishes the *number* of parts, their size remaining the same, and multiplying the denominator diminishes the *size* of the parts, their number remaining the same (Art. 119), it follows, that

To divide a fraction by a whole number, we may either *divide the numerator by the whole number, or multiply the denominator.*

EXAMPLES.

Divide

1. $\frac{8}{13}$ by 4.	*Ans.* $\frac{2}{13}$.	4. $\frac{11}{12}$ by 4.	*Ans.* $\frac{11}{48}$.
2. $\frac{6}{7}$ by 3.	*Ans.* $\frac{2}{7}$.	5. $\frac{21}{25}$ by 3.	
3. $\frac{10}{13}$ by 7.	*Ans.* $\frac{10}{91}$.	6. $\frac{17}{32}$ by 4.	

7.	$\frac{3681}{3684} \div 9 =$ what?	10.	$\frac{684}{1084} \div 36 =$ what?
8.	$\frac{96492}{583217} \div 11 =$ what?	11.	$15\frac{177}{209} \div 9 =$ what?
9.	$\frac{4320}{9362} \div 864 =$ what?		*Ans.* $1\frac{159}{209}$.

NOTE.—In example 11, divide 15 by 9, then reduce the remainder to an improper fraction, and divide the fraction by 9.

12. Divide $28\frac{86}{97}$ by 18. | 13. Divide $4\frac{412}{541}$ by 368.

☞ For Dictation Exercises, see Key.

133. DIVISION OF WHOLE NUMBERS AND FRACTIONS BY FRACTIONS.

ILL. EX., I. How many times is $\frac{1}{2}$ contained in 3?

SOLUTION. $\frac{1}{2}$ is contained in any number twice as many times as 1 is contained in it. 1 is contained in 3, 3 times; $\therefore \frac{1}{2}$ is contained in 3, $2 \times 3 = 6$ times.

EXAMPLES.

1. How many times is $\frac{1}{3}$ contained in 6? *Ans.* 18.
2. How many times is $\frac{1}{7}$ contained in 18? *Ans.* 126.
3. How many times is $\frac{1}{8}$ contained in 8?

How do you divide by a fraction having 1 for its numerator?

Divide

4. 20 by $\frac{1}{9}$.	7. 56 by $\frac{1}{12}$.	10. 181 by $\frac{1}{81}$.
5. 27 by $\frac{1}{14}$.	8. 100 by $\frac{1}{10}$.	11. 96 by $\frac{1}{12}$.
6. 31 by $\frac{1}{28}$.	9. 702 by $\frac{1}{5}$.	12. 108 by $\frac{1}{90}$.

13. If it takes $\frac{1}{2}$ of a yard of cloth to make a vest, how many vests can be made from 8 yards?

14. If a man walks 1 mile in $\frac{1}{3}$ of an hour, how many miles will he walk in 8 hours?

15. If the cars can run a mile in $\frac{1}{36}$ of an hour, how many miles can they run in 10 hours?

16. At $\frac{1}{6}$ of a dollar a dozen, how many dozen eggs can be bought for 10 dollars?

ILL. EX., II. Divide 4 by $\frac{3}{5}$; that is, see how many times $\frac{3}{5}$ is contained in 4.

OPERATION.

$$\frac{4 \times 5}{3} = \frac{20}{3} = 6\tfrac{2}{3},\ \textit{Ans.}$$

1 is contained in 4, 4 times; $\frac{1}{5}$ is contained in 4, 5 times as many times as 1 is contained in it, or 5×4 times, and $\frac{3}{5}$,

which is 3 times $\frac{1}{5}$, can be contained in it only $\frac{1}{3}$ as many times as $\frac{1}{5}$ is contained in it, or $\frac{1}{3}$ of 5×4 times.

ILL. EX., III. Divide $\frac{2}{3}$ by $\frac{5}{6}$.

OPERATION.

$$\frac{2 \times \overset{2}{\cancel{6}}}{\cancel{3} \times 5} = \frac{4}{5}, \textit{ Ans.}$$

1 is contained in $\frac{2}{3}$, $\frac{2}{3}$ of a time; $\frac{1}{6}$ is contained in $\frac{2}{3}$, $6 \times \frac{2}{3}$ times; $\frac{5}{6}$ (which is 5 times $\frac{1}{6}$) is contained in it $\frac{1}{5}$ of $6 \times \frac{2}{3}$ times. The expression then becomes $\frac{2 \times 6}{3 \times 5}$; after cancelling, $\frac{2 \times 2}{5} = \frac{4}{5}$, *Ans.* Hence the

RULE. To divide a whole number or a fraction by a fraction; *Multiply the dividend by the denominator of the divisor, and divide by the numerator;* or,

Invert the divisor, and proceed as in Multiplication.

EXAMPLES.

17.	$18 \div \frac{5}{6} = ?$	*Ans.* $21\frac{3}{5}$.	24.	$\frac{5}{7} \div \frac{15}{28} = ?$
18.	$21 \div \frac{3}{10} = ?$	*Ans.* 70.	25.	$\frac{21}{25} \div \frac{18}{19} = ?$
19.	$98 \div \frac{7}{10} = ?$	*Ans.* 140.	26.	$\frac{36}{39} \div \frac{4}{13} = ?$
20.	$54 \div \frac{9}{16} = ?$	*Ans.* 96.	27.	$\frac{19}{45} \div \frac{114}{315} = ?$
21.	$108 \div \frac{12}{17} = ?$	*Ans.* 153.	28.	$\frac{17}{79} \div \frac{4}{79} = ?$
22.	$\frac{8}{9} \div \frac{3}{4} = ?$	*Ans.* $1\frac{5}{27}$.	29.	$\frac{106}{351} \div \frac{7}{351} = ?$
23.	$\frac{3}{5} \div \frac{12}{18} = ?$		30.	$\frac{185}{731} \div \frac{18}{731} = ?$

31. $2\frac{1}{2} \div 1\frac{3}{7} = ?$

OPERATION. $2\frac{1}{2} \div 1\frac{3}{7} = \frac{5}{2} \div \frac{10}{7} = \dfrac{5 \times 7}{2 \times \underset{2}{\cancel{10}}} = \frac{7}{4} = 1\frac{3}{4}$, *Ans.*

NOTE. — Reduce mixed numbers to improper fractions before dividing.

32. $3\frac{6}{9} \div 4\frac{2}{15} = ?$

33. $5\frac{5}{12} \div 6\frac{1}{2} = ?$

34. $26\frac{1}{2} \div 3\frac{7}{11} = ?$

35. $1 \div 541\frac{3}{4} = ?$

36. $\frac{3}{5}$ of $\frac{8}{9} \div \frac{2}{3}$ of $\frac{7}{15} = ?$

OPERATION.

$$\frac{3}{5} \text{ of } \frac{8}{9} \div \frac{2}{3} \text{ of } \frac{7}{15} = \frac{3 \times 8}{5 \times 9} \div \frac{2 \times 7}{3 \times 15} = \frac{\cancel{3} \times \overset{4}{\cancel{8}} \times \cancel{3} \times \overset{3}{\cancel{15}}}{\cancel{5} \times \underset{3}{\cancel{9}} \times \cancel{2} \times 7} = 1\frac{5}{7}, \textit{ Ans.}$$

37.* $\frac{5}{7}$ of $\frac{10}{12} \div \frac{5}{19}$ of $\frac{2}{3}$ of $\frac{2}{9}$? *Ans.* $15\frac{15}{56}$.

38.* $\frac{8}{11}$ of $\frac{9}{11}$ of $2\frac{1}{3} \div \frac{3}{22}$ of $\frac{12}{15}$ of $\frac{7}{25}$? *Ans.* $45\frac{5}{11}$.

39. $71\frac{2}{3} \times \frac{9}{13} \div \frac{5}{9}$ of $7\frac{1}{6}$? *Ans.* $12\frac{6}{13}$.

40. How many sevenths are there in $\frac{4}{5}$ of $7\frac{9}{16} \div 6\frac{2}{3}$?

41.* How many times $\frac{1}{3}$ of 14 in $2\frac{3}{16} \times \frac{5}{6}$ of 81?

☞ For Dictation Exercises, see Key.

134. Reduction of Complex Fractions to Simple Fractions.

Ill. Ex. $\dfrac{9\frac{6}{7}}{2\frac{7}{8}} =$ what? This is a complex fraction (Art. 117), and is reduced *by performing the division indicated;* thus,

$$9\tfrac{6}{7} \div 2\tfrac{7}{8} = \frac{\overset{3}{\cancel{69}} \times 8}{7 \times \cancel{23}} = \tfrac{24}{7} = 3\tfrac{3}{7},\ \textit{Ans.}$$

Examples.

1. $\dfrac{3}{\frac{4}{9}} =$ what? *Ans.* $6\frac{3}{4}$.

2. $\dfrac{\frac{3}{4}}{9} =$ what? *Ans.* $\frac{1}{12}$.

3. $\dfrac{6}{2\frac{8}{9}} =$ what? *Ans.* $2\frac{1}{13}$.

4. $\dfrac{86\frac{4}{13}}{5\frac{1}{13}} =$ what?

5. $\dfrac{\frac{3}{4}\frac{5}{5}}{\frac{7}{11}} =$ what?

6. $\dfrac{538\frac{1}{5}}{763\frac{1}{3}} =$ what?

7.* $\dfrac{9\frac{1}{3} \times 34\frac{4}{5}}{\frac{1}{3} \text{ of } 21\frac{1}{3}} =$ what?

8.* $\dfrac{7\frac{3}{5} \times 6 \times \frac{5}{17}}{\frac{1}{19} \text{ of } 3\frac{4}{7}} =$ what?

9.* $13 \times \dfrac{8\frac{9}{15}}{9} \times \dfrac{6\frac{3}{7}}{4\frac{1}{12}} \times 707 =$ what?

10.* If $\dfrac{7 \times 11 \times 8}{18\frac{2}{3}}$ is the dividend, and $\dfrac{1}{28 \times \frac{5}{11} \times 12}$ the divisor, what is the quotient?

11.* If $\dfrac{49\frac{1}{3} \times \frac{2}{5} \times \frac{5}{9}}{5\frac{2}{7} \times 9\frac{4}{9} \times \frac{5}{11}}$ is the divisor, and $\frac{15}{28}$ the quotient, what is the dividend?

12.* If $\dfrac{25\frac{1}{5} \times 5\frac{1}{2} \times 4}{2\frac{4}{7} \times \frac{3}{10} \times 126}$ is the dividend, and $18\frac{2}{3}$ the quotient, what is the divisor?

☞ For Dictation Exercises, see Key.

135. To find the Whole Number when a Fractional Part of it is given.

Ill. Ex. $38\frac{2}{3}$ is $\frac{4}{5}$ of what number?

Operation.

$$38\tfrac{2}{3}=\frac{116}{3}.\quad \frac{\overset{29}{\cancel{116}}}{3}\frac{\times 5}{\times \cancel{4}}=\frac{145}{3}=48\tfrac{1}{3},\ \textit{Ans.}$$

$38\frac{2}{3}=\frac{116}{3}$; if $\frac{116}{3}$ is $\frac{4}{5}$ of some number, $\frac{1}{5}$ of that number is $\frac{1}{4}$ of $\frac{116}{3}$, and $\frac{5}{5}$, or the entire number, is 5 times $\frac{1}{4}$ of $\frac{116}{3}$; cancelling, the expression becomes $\frac{29\times 5}{3}=48\frac{1}{3}$, *Ans.*

Examples.

1. $16\frac{1}{5}$ is $\frac{9}{10}$ of what number? *Ans.* 18.
2. $25\frac{3}{7}$ is $\frac{2}{3}$ of what number? *Ans.* $38\frac{1}{7}$.
3. $\frac{5}{8}$ of $\frac{18}{25}$ is $\frac{3}{7}$ of what number? *Ans.* $1\frac{1}{20}$.
4. $\frac{8}{9}$ of $6\frac{2}{3}$ is $\frac{4}{9}$ of what number? *Ans.* $13\frac{1}{3}$.
5. $\frac{5}{11}$ of $\frac{6}{13}$ is $\frac{8}{9}$ of what number?
6. $2\frac{1}{2}\times 7\frac{7}{8}$ is $3\frac{1}{2}$ times what number (or $\frac{7}{2}$ of what number)?
7. $182 \div \overline{12\times 2\frac{1}{2}}$ is $3\frac{1}{4}$ times what number?
8. From New York to Troy is 150 miles, which is $\frac{3}{7}$ of the distance from New York to Rouse's Point; what is the distance from New York to Rouse's Point? *Ans.* 350 miles.
9. Mr. Aborn owns $\frac{20}{21}$ of an acre of land; his neighbor Jones owns $\frac{2}{3}$ as much, which is $\frac{4}{7}$ of what Mr. Green owns; what does Mr. Green own?
10. If $\frac{2}{3}$ of a piece of work be performed in 25 days, what number of days will be required to do the remainder?
11. Paid \$6 a week for board in Boston, which was $\frac{3}{4}$ of what I paid in New York; this was $\frac{4}{5}$ of what I paid in Philadelphia; and this was $\frac{5}{7}$ of what I paid in Washington. What did I pay in Washington?
12. A vessel having lost $\frac{1}{5}$ of her cable, has 200 feet remaining; how many feet had she at first?

Solution.—If $\frac{1}{5}$ be lost, $\frac{4}{5}$ will remain; if $\frac{4}{5}=200$ feet, $\frac{1}{5}=\frac{1}{4}$ of 200 feet, and $\frac{5}{5}$, or the whole cable, $=5\times\frac{1}{4}$ ot 200 feet, $=250$ feet.

13. A ship's crew having lost $\frac{1}{8}$ of their bread, are obliged to ist on 14 ounces a day; what were they allowed at first?

14. Having lost $\frac{1}{11}$ of my money in trade, I now have \$2476.50 ; what had I at first?

15. A mother and her son together have \$45 in a purse; the son's part is $\frac{2}{3}$ as great as the mother's. Required the part of each?

SOLUTION. — The purse contained once the mother's money and $\frac{2}{3}$ as much more, (the son's). If $1\frac{2}{3}$ ($\frac{5}{3}$) times the mother's part $=$ \$45, $\frac{1}{3} = \frac{1}{5}$ of \$45, and $\frac{3}{3} = 3 \times \frac{1}{5}$ of \$45, $=$ \$27, the mother's part; $\frac{2}{3} = 2 \times \frac{1}{5}$ of \$45, $=$ \$18, the son's part.

16. The sum of the ages of a father and son is 155 years, the son's age being $\frac{2}{3}$ the age of the father; what is the age of each?

17. A body of 4800 troops has $\frac{1}{5}$ as many cavalry as infantry; what is the number of each?

18. A lot of land yielded 4140 bushels of grain in two years, yielding $\frac{4}{5}$ as much the second year as the first; what was the yield each year?

19. What number is that to which if $\frac{2}{3}$ of itself be added the sum will equal 275?

20. A carpenter, who has a number of floors to lay, estimates that it will cost $\frac{2}{5}$ more to lay them with hard pine, worth \$28 a thousand feet, than with white pine; what is the price of white pine per thousand?

21. In counting his fowls, a farmer finds that he has 396 in all, which is $\frac{1}{5}$ more than he had the previous year; how many had he then?

22. He has sold his eggs at an average of $13\frac{1}{7}$ cents per dozen, which is $\frac{1}{3}$ higher than the previous year; what did they average then?

23. He is paid for grain $\$1\frac{4}{5}$ per bag, which is $\frac{1}{6}$ less than he was paid last year; what was he paid last year?

24.* Mr. Ober owns $50\frac{1}{2}$ acres of wood land, $\frac{2}{3}$ of which he exchanges with Mr. Fay for $40\frac{1}{4}$ acres of meadow land, which is $\frac{7}{8}$ of what Mr. Fay owned; how much meadow land did Mr. Fay have at first? How much wood land after the exchange was made?

☞ For Dictation Exercises, see Key.

136. To find what Part one Number is of another.

Note. — Younger pupils may omit this article.

What part of 2 is 1, or 1 is what part of 2? *Ans.* 1 is $\frac{1}{2}$ of 2, because it is 1 of the 2 equal parts into which 2 may be divided.

1 is what part of 3? of 5? of 7? of 9? of 8? why?

1 is what part of 19? of 11? of 6? of 15? of 33?

Illustrative Examples.

1. 3 is what part of 10? 1 is $\frac{1}{10}$ of 10, $\therefore$ 3 must be $\frac{3}{10}$ of 10. *Ans.* $\frac{3}{10}$.

2. $\frac{2}{3}$ is what part of 7?

Operation. $\frac{1 \times 2}{7 \times 3} = \frac{2}{21}$, *Ans.*

1 is $\frac{1}{7}$ of 7, $\therefore$ $\frac{2}{3}$ of 1 must be $\frac{2}{3}$ of $\frac{1}{7}$ of 7, or $\frac{2}{21}$ of 7.

3. What part of $\frac{2}{3}$ is 1?

Operation. $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$, $\therefore$ $\frac{3}{3}$ or 1 whole one is $\frac{3}{2}$ of $\frac{2}{3}$. *Ans.* $\frac{3}{2}$.

4. What part of $\frac{7}{9}$ is 2?

Operation. $\frac{2 \times 9}{7} = \frac{18}{7}$, *Ans.*

$\frac{1}{9}$ is $\frac{1}{7}$ of $\frac{7}{9}$, $\therefore$ $\frac{9}{9}$, or 1 is $\frac{9}{7}$ of $\frac{7}{9}$, and 2 must be $2 \times \frac{9}{7}$, or $\frac{18}{7}$ of $\frac{7}{9}$. *Ans.* $\frac{18}{7}$.

5. What part of $\frac{11}{12}$ is $\frac{3}{5}$?

Operation. $\frac{12 \times 3}{11 \times 5} = \frac{36}{55}$, *Ans.*

$\frac{1}{12}$ is $\frac{1}{11}$ of $\frac{11}{12}$, $\therefore$ $\frac{12}{12}$, or 1 is $\frac{12}{11}$ of $\frac{11}{12}$, and $\frac{3}{5}$ must be $\frac{3}{5}$ of $\frac{12}{11}$ of $\frac{11}{12}$, or $\frac{36}{55}$ of $\frac{11}{12}$. *Ans.* $\frac{36}{55}$.

From the above we derive the following

Rule. To ascertain what part one number is of another: *Divide the number expressing the part, by that of which it is a part.*

Examples.

What part of

1. 5 is 3? *Ans.* $\frac{3}{5}$.	7. 8 is $\frac{2}{3}$? *Ans.* $\frac{1}{12}$.	13. $2\frac{3}{4}$ is 1?
2. 8 is 6?	8. 11 is $\frac{22}{23}$? *Ans.* $\frac{2}{23}$.	14. $\frac{11}{15}$ is $\frac{2}{3}$? *Ans.* $\frac{10}{11}$.
3. 10 is 7?	9. 48 is $5\frac{3}{4}$?	15. $\frac{13}{14}$ is $\frac{14}{15}$?
4. 20 is 15?	10. 19 is $12\frac{4}{7}$?	16. $\frac{8}{11}$ is $\frac{11}{16}$?
5. 30 is 75?	11. $\frac{7}{8}$ is 1? *Ans.* $\frac{8}{7}$.	17. $3\frac{4}{5}$ is $2\frac{4}{5}$?
6. 89 is 267?	12. $\frac{9}{12}$ is 1?	

18. If by a pipe a cistern can be filled in 3 hours, what part of the cistern will be filled in 1 hour? in 2 hours?

19. If a piece of work can be performed in 9 days, what part of the work can be performed in 7 days?

20. A can perform a journey on foot in $7\frac{1}{2}$ days; what part of it can perform in $2\frac{1}{2}$ days?

21. Mr. Bailey has \$54, and pays \$18 for a coat; what part of his money does he spend?

22. Charles picks $2\frac{5}{8}$ quarts of blackberries, and Eben $5\frac{2}{8}$ quarts. If Eben's blackberries are worth one dollar, what part of a dollar are Charles's worth?

23. A and B hired a pasture together. A pastured 12 cows, and B 13 cows in it; what part of the price should each pay? $\frac{12}{25}$ $\frac{13}{25}$

24. Four men were hired to work on a farm; A mowed 7 acres; B mowed 5 acres; C, 4 acres, and D, 2 acres. They received \$27. What was each one's share? 10½ 7½ 6 3

25. Mr. Snow, dying, left \$75000 to his wife and three sons. To his wife, \$30,000; to his oldest son just as large a part of the remainder as his wife's portion was of the entire property; to his 2d son $\frac{7}{9}$ of what his eldest received, and to his youngest the rest. What was each son's share? 18000 14000 13000

For Dictation Exercises, see Key.

Multiples of Numbers.

137. A **Multiple** of a number is any number that will contain it without a remainder; thus, 8, 12, 16, and 20, are multiples of 4.

138. A **Common Multiple** of two or more numbers is a number that will contain each of them without a remainder; thus, 20 is a common multiple of 5 and 2.

139. The **Least Common Multiple** of two or more numbers is the least number that will contain each of them without a remainder; thus, 10 is the L. C. M.* of 2 and 5.

Exercises.

Name any 6 multiples of 5. Name 3 multiples of 12. Name all the multiples of 11 up to 140. Name any common multiple of 10 and 6. Of 3, 6, and 5.

140. To find the Least Common Multiple of two or more Numbers.

The common multiple of two or more numbers must contain

* Least Common Multiple.

all the factors of those numbers, and the *least* number that contains all their factors must be the *least common multiple*.

ILL. EX. Find the L. C. M. of 4, 6, 10 and 15.

OPERATION.

$4 = 2 \times 2$
$6 = 2 \times 3$
$10 = 2 \times 5$
$15 = 3 \times 5$
L. C. M $= 2 \times 2 \times 3 \times 5 = 60$, *Ans.*

We find the factors of 4 to be 2 and 2, of 6 to be 2 and 3, of 10 to be 2 and 5, of 15 to be 3 and 5. To contain 4, the L. C. M. must contain the factors 2 and 2, which we note. To contain 6, it must contain 2 and 3; we have already noted 2, so we need introduce only the 3. To contain 10, it must contain 2 and 5; as we have noted 2, we introduce only the 5. To contain 15, it must contain 3 and 5; we have already noted these factors, $\therefore 2 \times 2 \times 3 \times 5 = 60$, must be the L. C. M. Hence the

RULE. To find the L. C. M. of two or more numbers: *Separate the numbers into their prime factors. Find the product of all the different prime factors, taking each factor the greatest number of times it occurs as a factor in any one number.*

EXAMPLES.

Find the L. C. M. of

1. 8, 18, 20, and 21. *Ans.* 2520.
2. 12, 16, and 28. *Ans.* 336.
3. 3, 5, 8, 12, 20, 36, and 45. *Ans.* 360.

NOTE. — When one of the given numbers is a factor of another, it may be disregarded in the operation; thus, in the preceding example, 3, 5, and 12 may be rejected. Why?

Find the L. C. M. of

4. 18, 36. 40, 60, and 72.
5. 12, 16, 42, 56, and 70.
6. 13, 28, 35, 39, and 49.
7. 9, 18, 32, 48, and 52.
8. 8, 16, 28, 35, and 63.
9. Of the nine digits.

When several numbers are prime to each other, what must heir L. C. M. equal?

141. The above is the better method for finding the L. C. M when the numbers are easily separated into their prime factors. For larger and more difficult numbers observe the following method: —

ILL. Ex. Find the L. C. M. of 36, 112, 76, and 60.

OPERATION.

2) 36, 112, 76, 60
2) 18, 56, 38, 30
3) 9, 28, 19, 15
3, 28, 19, 5

L. C. M. $= 2 \times 2 \times 3 \times 3 \times 28 \times 19 \times 5 = 95760$.

Here, by repeated divisions, we take out all the factors that are common, 2, 2, and 3; the least common multiple must contain these factors and those which are not common; $\therefore 2 \times 2 \times 3 \times 3 \times 28 \times 19 \times 5 = 95760$, must be the L. C. M. sought. Hence the

RULE. To find the L. C. M. of two or more numbers: *Divide by any prime factor which is contained in two or more of the numbers without a remainder, writing the quotient and undivided numbers in a line beneath, and thus proceed till no two numbers can be divided by the same prime. The product of all the divisors and the numbers remaining is the L. C. M.*

EXAMPLES.

Find the L. C. M. of the following: —

10. 338, 364, and 448. *Ans.* 75 712.
11. 184, 390, and 552. *Ans.* 35,880.
12. 847, 968, and 1001. *Ans.* 88,088.
13.* 2784, 147, and 472.
14.* 912, 9500, and 855.
15.* 1146, 1936, and 24.
16.* 880, 9680, and 8624.
17.* 539, 573, and 9680.

18. What is the width of the narrowest street, across which stepping stones either 4, 5, or 8 feet long will exactly reach?

19. What is the narrowest box that will exactly pack ribbons either 3, 4, or 5 inches wide?

20. What is the smallest bill that may be paid by using either dimes, three-cent pieces, or quarter dollars?

21. What is the smallest cistern the contents of which may be exactly measured by using either 15, 28, or 36 gallon casks?

☞ For Dictation Exercises, see Key.

142. Reduction of Fractions to Equivalent Fractions having a Common Denominator.

When the denominators of fractions are alike, they are said to have a **Common Denominator.**

Ill. Ex. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to fractions having a common denominator.

We can change these fractions to fractions of any given denominator; but the most convenient denominator for most purposes is that which is the *least common multiple of the denominators of the given fractions;* and, in the following examples, such denominators are always required. In the preceding example, we must first find the L. C. M. of 3, 4, and 6, which is 12; and then reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to twelfths. $1 = \frac{12}{12}$, $\therefore \frac{1}{3} = \frac{1}{3}$ of $\frac{12}{12}$ or $\frac{4}{12}$, and $\frac{2}{3} = \frac{2 \times 4}{12} = \frac{8}{12}$. By the same process we find that $\frac{3}{4} = \frac{9}{12}$, and $\frac{5}{6} = \frac{10}{12}$. *Ans.* $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$. Hence the

Rule. To reduce fractions to equivalent fractions having a common denominator: *Reduce the fractions to their simplest forms; find the least common multiple of the denominators for the common denominator; multiply the numerator of each fraction by the number by which you would multiply its denominator to produce the common denominator.* The respective products will be the numerators of the required fractions.*

Ill. Ex. Reduce $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{2}{9}$ to fractions having the L. C. D.

Entire Operation.

$$8 = 2 \times 2 \times 2 \qquad \frac{5}{8} = \frac{5 \times \overline{3 \times 3}}{72} = \frac{45}{72}.$$
$$12 = 2 \times 2 \times 3 \qquad \frac{7}{12} = \frac{7 \times \overline{2 \times 3}}{72} = \frac{42}{72}.$$
$$9 = 3 \times 3 \qquad \frac{2}{9} = \frac{2 \times \overline{2 \times 2 \times 2}}{72} = \frac{16}{72}.$$
$$\text{L. C. M.} = 2 \times 2 \times 2 \times 3 \times 3 = 72.$$

Here 72 is the L. C. M.; and as $8 = 2 \times 2 \times 2$, it must be multiplied by 3×3 to produce 72, $\therefore \frac{1}{8} = \frac{3 \times 3}{72}$, and $\frac{5}{8} = \frac{5 \times \overline{3 \times 3}}{72} = \frac{45}{72}$.

Show why $\frac{7}{12} = \frac{42}{72}$; why $\frac{2}{9} = \frac{16}{72}$.

when th...
For large...
method:—

* ...number is not readily seen, it may be found by dividing the ...ominator by the denominator of the original fraction.

EXAMPLES.

Reduce the following to fractions having the least common denominator:—

1. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. *Ans.* $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$.
2. $\frac{3}{7}$, $\frac{7}{12}$, and $\frac{5}{21}$. *Ans.* $\frac{36}{84}$, $\frac{49}{84}$, $\frac{20}{84}$.
3. $\frac{5}{13}$, $\frac{3}{26}$, and $\frac{5}{12}$.
4. $\frac{8}{36}$, $\frac{10}{12}$, and $\frac{8}{16}$.†
5. $\frac{3}{15}$, $\frac{21}{28}$, $\frac{5}{12}$, and $\frac{8}{21}$.
6. $\frac{9}{20}$, $\frac{5}{12}$, $\frac{3}{35}$, and $\frac{8}{42}$.
7. $\frac{35}{432}$, $\frac{28}{621}$, and $\frac{32}{504}$.

☞ For Dictation Exercises, see Key.

143. ADDITION OF FRACTIONS.

EXAMPLES.

1. $\frac{2}{5}+\frac{1}{5}=$ what? *Ans.* $\frac{3}{5}$.
2. $\frac{3}{7}+\frac{2}{7}=$ what? *Ans.* $\frac{5}{7}$.
3. $\frac{5}{8}+\frac{2}{8}=$ what?
4. $\frac{6}{11}+\frac{1}{11}=$ what?
5. $\frac{5}{100}+\frac{7}{100}=$ what?
6. $\frac{8}{33}+\frac{2}{33}=$ what?

The above examples are easily performed, as the quantities to be operated upon are like quantities, that is, have the same denominator. In such cases, we have only to add the numerators. When fractions of different denominators are to be added, *they must first be reduced to fractions having a common denominator.*

ILL. EX. — Add $\frac{4}{9}$ and $\frac{5}{12}$.

OPERATION. $\frac{4}{9}=\frac{16}{36}$. $\frac{5}{12}=\frac{15}{36}$. $\frac{16+15}{36}=\frac{31}{36}$, *Ans.*

7. $\frac{1}{6}+\frac{1}{4}+\frac{2}{3}=$? *Ans.* $1\frac{1}{12}$.
8. $\frac{7}{10}+\frac{5}{6}+\frac{4}{5}=$? *Ans.* $2\frac{1}{3}$.
9. $\frac{4}{7}+\frac{8}{10}+\frac{1}{2}+\frac{2}{5}=$?
10. $\frac{4}{9}+\frac{4}{6}+\frac{11}{12}+\frac{5}{18}=$?
11. $\frac{5}{16}+\frac{3}{20}+\frac{1}{4}+\frac{2}{5}=$?
12. $\frac{7}{24}+\frac{4}{9}+\frac{3}{8}=$?
13. $\frac{2}{25}+\frac{5}{30}+\frac{8}{12}=$?
14. $\frac{4}{21}+\frac{7}{15}+\frac{12}{35}=$?
15. $\frac{14}{21}+\frac{7}{18}+\frac{5}{12}=$?
16. $\frac{3}{4}+\frac{5}{8}+\frac{16}{30}=$?
17. $\frac{8}{13}+\frac{4}{39}+\frac{3}{12}=$?
18. $\frac{9}{12}+\frac{8}{81}+\frac{5}{18}+\frac{11}{45}=$?
19.* $\frac{3}{56}+\frac{8}{21}+\frac{9}{32}+\frac{9}{64}=$?
20.* $\frac{28}{71}+\frac{19}{108}+\frac{3}{10}+\frac{7}{23}=$?

NOTE. — Add the whole numbers and fractions of the following, and similar examples, separately.

21. $3\frac{6}{7}+2\frac{5}{8}+7\frac{8}{12}=$?
22. $18\frac{2}{3}+16\frac{1}{2}+28\frac{3}{5}=$?
23. $272\frac{1}{2}+16\frac{1}{4}+18\frac{5}{8}=$?
24.* $104\frac{3}{56}+8\frac{7}{60}+480\frac{51}{98}=$?

† What operation should first be performed upon these fractions?

25. $2007\frac{3}{10} + 1070\frac{5}{8} + 8040\frac{4}{5} = ?$

26.* $\frac{1}{9}$ of $\frac{10}{13} + \frac{1}{3}$ of $\frac{7\frac{1}{2}}{45} + \frac{2}{5}$ of $\frac{3}{36} = ?$

27.* $\frac{7}{10}$ of $2\frac{6}{8} + \frac{1}{5}$ of $\frac{3}{8} + \frac{4}{7}$ of $2\frac{1}{10} = ?$

28.* $2\frac{7}{10} + \frac{1}{2}$ of $\frac{7}{15} + \dfrac{\frac{2}{9}}{3\frac{1}{5}} = ?$ *Ans.* $3\frac{1}{360}$.

29.* $4\frac{1}{30} + 1\frac{9}{11} + \dfrac{18\frac{2}{3}}{15\frac{3}{4}} =$ what? *Ans.* $7\frac{109}{2970}$.

☞ For Dictation Exercises, see Key.

144. Subtraction of Fractions.

Examples.

1. $\frac{5}{7} - \frac{2}{7} = ?$ *Ans.* $\frac{3}{7}$.
2. $\frac{7}{8} - \frac{3}{8} = ?$ *Ans.* $\frac{4}{8} = \frac{1}{2}$.
3. $\frac{7}{10} - \frac{3}{10} = ?$
4. $\frac{5}{100} - \frac{3}{100} = ?$
5. $\frac{12}{161} - \frac{2}{161} = ?$
6. $\frac{38}{139} - \frac{5}{139} = ?$
7. $\frac{7}{12} - \frac{2}{5} = ?$ *Ans.* $\frac{11}{60}$.

Note. — The denominators in Example 7 being unlike, ***the fractions must be reduced to fractions having the same denominator.***

8. $\frac{5}{7} - \frac{2}{3} = ?$ *Ans.* $\frac{1}{21}$.
9. $\frac{7}{9} - \frac{8}{11} = ?$ *Ans.* $\frac{5}{99}$.
10. $\frac{9}{10} - \frac{3}{5} = ?$
11. $\frac{7}{8} - \frac{2}{13} = ?$
12. $\frac{5}{12} - \frac{2}{21} = ?$
13. $\frac{7}{13} - \frac{4}{39} = ?$
14. $2\frac{2}{3} - 1\frac{1}{2} = ?$

Note. — Subtract without changing the mixed numbers to improper fractions.

15. $8\frac{1}{2} - 3\frac{2}{5} = ?$ *Ans.* $5\frac{1}{10}$.
16. $7\frac{1}{8} - 2\frac{1}{10} = ?$ *Ans.* $5\frac{1}{40}$.
17. $10\frac{3}{4} - 5\frac{19}{28} = ?$
18. $18\frac{1}{4} - 15\frac{1}{13} = ?$
19. $17\frac{1}{2} - 12\frac{9}{10} = ?$ *Ans.* $4\frac{3}{5}$.

Note. — As $\frac{9}{10}$ cannot be taken from $\frac{1}{2}$, it will be necessary to reduce 1 of the 17 to halves, making the minuend $16\frac{3}{2}$, when subtraction can be easily performed.

20. $2\frac{1}{2} - 1\frac{2}{3} = ?$ *Ans.* $\frac{5}{6}$.
21. $17\frac{1}{5} - 2\frac{8}{9} = ?$ *Ans.* $14\frac{14}{45}$.
22. $12\frac{1}{2} - \frac{3}{4} = ?$
23. $26\frac{2}{3} - 1\frac{5}{6} = ?$
24. $19 - 2\frac{1}{8} = ?$ *Ans.* $16\frac{7}{8}$.
25. $36 - \frac{3}{5} = ?$
26. $75 - 15\frac{1}{3} = ?$
27. $1 - \frac{1}{2}$ of $\dfrac{15\frac{1}{5}}{16} = ?$
28. $18\frac{2}{3} \div \frac{4}{7} - 1\frac{1}{5}$ of $2\frac{1}{6} =$ what?

☞ For Dictation Exercises, see Key.

145. ADDITION AND SUBTRACTION OF FRACTIONS COMBINED.

Give a rule for the addition of fractions; for subtraction.

EXAMPLES.

1. $\frac{5}{9}+\frac{4}{11}-\frac{5}{12}=?$ *Ans.* $\frac{199}{396}$.
2. $\frac{3}{4}-\frac{2}{5}+\frac{5}{8}=?$
3. $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+\frac{1}{9}-\frac{1}{10}=?$ *Ans.* $\frac{1627}{2520}$.
4. $\frac{3}{4}-\frac{1}{8}+\frac{5}{64}-\frac{5}{128}=$ what? $\frac{85}{128}$
5. $1-\frac{1}{2}-\frac{1}{4}-\frac{1}{8}-\frac{1}{16}-\frac{1}{32}-\frac{1}{64}-\frac{1}{128}=?$ $\frac{1}{128}$
6. $\frac{8}{9}-\frac{1}{3}-\frac{4}{81}-\frac{4}{729}=$ what?
7. $20-5\frac{1}{2}+\frac{8}{11}$ of $\frac{2}{3}=?$
8. $8\frac{2}{27}-2\frac{2}{3}+7\frac{2}{5}=?$
9. $16\frac{7}{8}\times\frac{2}{3}$ of $\frac{6}{7}+10\frac{3}{7}-\frac{1}{10}$ of $\dfrac{2\frac{3}{5}}{\frac{2}{9}}=?$ *Ans.* $18\frac{631}{700}$.
10. $7-(\frac{7}{12}-\frac{9}{20})=?$ *Ans.* $6\frac{13}{15}$.
11. $5-(\frac{3}{8}+\frac{7}{54})=?$ *Ans.* $4\frac{107}{216}$.

12. A man receives $4\frac{1}{2}$ per cent. commission for selling goods; he pays $\frac{7}{8}$ per cent. for storage; what per cent. does he retain?

13. If he receives $6\frac{3}{8}$ per cent. for selling goods, and $1\frac{2}{3}$ per cent. for insuring their sale, and pays $1\frac{3}{5}$ per cent. for storage, and $\frac{7}{12}$ per cent. for auctioneering; what per cent. does he retain?

14. How much will be left of a piece of cloth containing 7 yards, after cutting from it 2 vests and a coat, allowing $\frac{3}{4}$ of a yard for a vest and $4\frac{1}{8}$ yards for a coat?

15. Bought of Mrs. Frye 1 bonnet for \$4.37$\frac{1}{2}$, 2 hats at \$2.12$\frac{1}{2}$ apiece, 4 yards ribbon at \$.16$\frac{2}{3}$ per yard, 2 yards ribbon at 33$\frac{1}{3}$ cents a yard, and gave in payment a ten dollar bill; what should she give me in return?

16. From 8 apple trees I gathered as follows: $2\frac{1}{2}$ barrels, $5\frac{1}{3}$ barrels, $5\frac{5}{6}$ barrels, $4\frac{1}{4}$ barrels, $3\frac{3}{4}$ barrels, $1\frac{2}{5}$ barrels, $3\frac{1}{4}$ barrels, and $2\frac{1}{3}$ barrels. I sold $15\frac{1}{2}$ barrels to one man, and $2\frac{1}{8}$ barrels to another, how many barrels had I left?

17.* To what must you add the difference between $8\frac{3}{5}$ and $36\frac{7}{10}$, that the amount may be $50\frac{2}{3}$?

18.* If $7\frac{5}{8} \times \frac{2}{5} - 2\frac{7}{100}$ is the minuend, and $\frac{3}{40}$ the remainder, what is the subtrahend?

146.* Greatest Common Divisor of Fractions.*

Ill. Ex. Find the greatest common divisor of $\frac{6}{7}$, $\frac{8}{9}$, and $\frac{4}{5}$.

OPERATION.

$$\frac{\text{G. C. D. of 6, 8, and 4} = 2}{\text{L. C. M. of 7, 9, and 5} = 315}, \quad Ans.$$

We find the G. C. D. of the numerators 6, 8, and 4 to be 2. 2 is a divisor of 6, but must be divided by 7 to be a divisor of $\frac{6}{7}$. It must also be divided by 9 to be a divisor of $\frac{8}{9}$, and by 5 to be a divisor of $\frac{4}{5}$. To be at the same time a divisor of these fractions, it must therefore be divided by 7, and 9, and 5, or by their least common multiple. Hence the

Rule. To find the G. C. D. of fractions: *Reduce the fractions to their lowest terms; then divide the G. C. D. of the numerators by the L. C. M. of the denominators.*

Examples.

1. Find the G. C. D. of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$. *Ans.* $\frac{1}{12}$.
2. Find the G. C. D. of $\frac{8}{12}$, $\frac{9}{15}$, and $2\frac{1}{2}$ or $\frac{5}{2}$. *Ans.* $\frac{1}{30}$.
3. Find the G. C. D. of $3\frac{1}{3}$, $\frac{7}{10}$, $\frac{5}{6}$, and $\frac{10}{15}$. $\frac{1}{30}$
4. Find the G. C. D. of $\frac{2}{7}$, $\frac{8}{9}$, and 4. $\frac{2}{63}$

Note.—4 can be regarded as $\frac{4}{1}$.

5. Find the G. C. D. of $\frac{27}{16}$, $\frac{3}{20}$, $\frac{6}{5}$, and 2. $\frac{1}{80}$
6. What is the size of the largest cup which is an exact measure of $1\frac{1}{2}$, $1\frac{2}{3}$, $8\frac{1}{2}$, and $2\frac{5}{6}$ pints?
7. What is the width of the widest carpeting that will fit 4 rooms of the following widths: $13\frac{1}{3}$ feet, 21 feet, $31\frac{1}{2}$ feet, $36\frac{3}{4}$ feet?

☞ For Dictation Exercises, see Key.

147.* Least Common Multiple of Fractions.*

Ill. Ex. Find the least common multiple of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$.

OPERATION.

$$\frac{\text{L. C. M. of 1, 3, and 5} = 15}{\text{G. C. D. of 2, 4, and 6} = 2}, \quad Ans.$$

We find the L. C. M. of the numerators, 1, 3, and 5, to be 15. But we do not wish to ascertain the least number that

* Articles 146 and 147 can be omitted by younger pupils.

will contain 1, 3, and 5, but one that will contain $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{6}$. To contain each of these fractions separately, it might be divided by 2, by 4, or by 6; but to contain them at once, it can be divided only by their G. C. D. Hence the

RULE. To find the L. C. M. of fractions: *Reduce the fractions to their lowest terms, then divide the L. C. M. of the numerators by the G. C. D. of the denominators.*

EXAMPLES.

1. Find the L. C. M. of $\frac{5}{12}$, $\frac{13}{15}$, and $7\frac{1}{3}$. *Ans.* $476\frac{2}{3}$.

2. Find the L. C. M. of $\frac{1\frac{3}{8}}{2\frac{1}{5}}$, $\frac{1}{3}$ of $3\frac{1}{4}$, and 6. *Ans.* 390

3. What is the width of the narrowest cloth that can be cut into strips either $\frac{3}{4}$, $1\frac{1}{2}$, or 4 inches wide? 12

4. What will be the length of the shortest court that can be paved with stones of either of the following lengths, viz., $1\frac{1}{2}$ ft., 2 ft., 4 ft., or $2\frac{2}{3}$ ft.? *Ans.* 24 ft.

5. What must be the width of the narrowest court that will receive either of the same stones widthwise, their widths being 1 ft., $1\frac{1}{2}$ ft., 3 ft., and 2 ft.? 6

6. On a stringed instrument in perfect tune, while C makes 1 vibration, D makes $\frac{9}{8}$, E $\frac{5}{4}$, F $\frac{4}{3}$, G $\frac{3}{2}$, A $\frac{5}{3}$, B $\frac{15}{8}$, and C′ 2. If all are struck at once, in how many vibrations of C will they all again coincide? 180

7. In how many vibrations of C will C, E, G, and C′ coincide? will C and D coincide? C and E? B and C′? C and C′?

☞ For Dictation Exercises, see Key.

QUESTIONS FOR REVIEW.

DEFINITIONS AND PROPERTIES OF NUMBERS.—What is the sign for plus? for minus? for greater than? less than? equal to? multiplied by? divided by? therefore? What does a parenthesis or vinculum signify? What are integral numbers? What are fractional numbers? mixed numbers? What is a prime number? a composite number? What are the factors of a number? What is a prime factor? A composite number equals what product? When are numbers prime to each other? What is a power of a number? What is the square or second power of a number? the fifth power? What is a root of a number? the square root? the cube root? the sixth root?

What is the sign for a power? for a root? What indicates the degree of root? What is an even number? an odd?

DIVISIBILITY OF NUMBERS. — When are numbers divisible by 2? by 3? by 4? by 5? by 6? by 8? by 9? by 10? by 11? by any composite number? How shall we ascertain whether any given number is prime? Describe Eratosthenes' sieve?

FACTORING OF NUMBERS. — What is the simplest way of resolving numbers into their prime factors? What other method can you describe, and when would you use it? Find the factors of 180 by first method, and explain the process. Find the factors of 10296 by second method, and explain the process.

GREATEST COMMON DIVISOR.—What is a divisor of a number? a common divisor of two or more numbers? the greatest common divisor? Find the G. C. D. of three numbers by the first method given. Explain and give the rule. Find the G. C. D. of three numbers by second method. Explain and give the rule. In what cases is the second method the better? When is it necessary to find the G. C. D. of numbers?

FRACTIONS. — What is a fraction? Name and describe its terms. Name the different kinds of fractions of which you have learned. Define a common fraction; a decimal fraction; a proper fraction; an improper fraction; a mixed number; a compound fraction; a complex fraction. Give an example of each. Explain the expression $\frac{7}{8}$. Upon what does the value of a fraction depend? Which of the fundamental rules is indicated by a fraction? What effect does multiplying the numerator of a fraction have upon that fraction? Why? In what other way could you produce the same effect, and why? What effect does dividing the numerator have upon a fraction? Why? In what other way could you produce the same effect, and why? What effect does multiplying both terms of a fraction by the same number have upon it? Why? What effect does dividing both terms of a fraction have upon it? Why?

REDUCTION OF FRACTIONS. — How do you reduce fractions to lower terms? What is cancellation? How do you reduce whole or mixed numbers to improper fractions? How do you reduce improper fractions to whole or mixed numbers?

MULTIPLICATION OF FRACTIONS. — How do you multiply a fraction by a whole number? a mixed number by a whole number? Explain, by an example, the method of multiplying a whole number by a fraction. Multiply a fraction by a fraction; explain and give the rule. How do you multiply a mixed number by a mixed number or a fraction? How

do you reduce compound fractions to simple ones? Can you give one general rule for multiplying fractions, whole or mixed numbers, by fractions?

DIVISION OF FRACTIONS. — How do you divide a fraction by a whole number? a mixed number by a whole number? a whole number by a fraction? Explain, by an example, the method of dividing a fraction by a fraction, and give the rule. Give one general rule for dividing a fraction, a whole or mixed number by a fraction. How do you reduce complex fractions to simple ones? How do you find what part of one number another is?

LEAST COMMON MULTIPLE. — Define a multiple; a common multiple of two or more numbers; the least common multiple. When do you make use of the L. C. M.? Give and explain the first method of finding it; the second. What does the L. C. M. of prime numbers equal?

COMMON DENOMINATOR. — When are fractions said to have a common denominator? In what operations upon fractions do we first reduce them to those having the same denominator? Can we change fractions to those of any denominator? How? (*Ans.* By dividing or multiplying the numerator by the same number by which we divide or multiply the denominator to produce the denominator required.) What denominator is generally chosen? Reduce a simple, a compound, and a complex fraction to those of the same denominator, explain the process, and give the rule.

ADDITION AND SUBTRACTION OF FRACTIONS. — How do you add fractions of different denominators? How do you subtract one fraction from another? How do you add mixed numbers? In subtraction of one mixed number from another, how do you proceed when the fraction in the subtrahend exceeds that in the minuend?

G. C. D. AND L. C. M. OF FRACTIONS.* — How do you find the G. C. D. of fractions? How do you find the L. C. M. of fractions? Find the G. C. D. of $\frac{3}{4}$, $\frac{6}{7}$ and $\frac{12}{13}$, and explain. Find the L. C. M. of $\frac{1}{2}$, $\frac{5}{6}$, and $\frac{7}{12}$, and explain.

148. MISCELLANEOUS EXAMPLES.

1. Into strips of what widths may I cut cloth which is 36 inches wide, that none may be wasted, the width of the strips to be expressed in inches? 1 2 3 4 6 9 12 18

2. How many gallons in the largest vessel which will exactly measure 3 hogsheads, containing severally 128, 94, and 158 gallons? 2 gal

* Optional.

3. What will $16\frac{1}{2}$ yards of cloth cost at \$.53 a yard?

4. What cost $9\frac{1}{3}$ bushels of corn, at \$.$87\frac{1}{2}$ a bushel?

5. What cost $271\frac{2}{3}$ acres of land, at \$$31\frac{3}{8}$ per acre?

6.* $\frac{2}{5}$ of $\frac{3}{4}$ of 56 times what number equals $\frac{128}{235}$?

7. I paid \$.65 for 2 boxes of strawberries; what will be the cost of $45\frac{1}{2}$ boxes at the same rate?

8. What is my bill for 7 pear trees, \$$\frac{7}{8}$ apiece for the trees, and \$2 a dozen for setting?

9. What do I receive per pound by selling $15\frac{5}{8}$ pounds of coffee for \$$3\frac{1}{6}$?

10. $\frac{1}{5}$ of a man's property is in land, and is valued at \$$2324\frac{5}{8}$; what is the value of his whole property?

11. Bought $\frac{2}{7}$ of an acre of land for \$40.75; what would 1 acre cost at the same rate?

12. What costs 3 pieces of calico, $37\frac{1}{2}$ yards in a piece, at $19\frac{1}{4}$ cents per yard?

13. If $32\frac{1}{12}$ acres of land cost \$1100, what costs 1 acre?

14. Sold my house and farm of $47\frac{2}{5}$ acres for \$6150; allowing \$3500 for the house, what did I receive per acre for the land?

15. How long will a barrel of flour last a family of 8 persons, if it lasts 3 persons $4\frac{1}{2}$ months?

16.* What number is that from which if you take $\frac{9}{10}$, the remainder will be $\frac{1}{60}$?

17. What number is that to which if you add $9\frac{7}{8}$, the sum will be $124\frac{5}{6}$?

18. What is that number to which if you add $\frac{3}{4}$ of $26\frac{1}{2}$, the sum will be $147\frac{1}{5}$?

19. Bought $7\frac{1}{2}$ yards broadcloth at \$5 per yard, $14\frac{1}{2}$ yards of kerseymere at \$$1\frac{1}{4}$ per yard, $4\frac{7}{8}$ yards of silk at \$$\frac{3}{4}$ per yard, and $\frac{3}{4}$ yards of doeskin at \$$4\frac{1}{2}$ per yard, for which I gave in payment a \$100 bill. What balance is due me?

20.* I have paving stones 12 inches long and 10 inches wide; what must be the width of a walk which will just receive these stones, laid either lengthwise or widthwise?

21.* What is the smallest sum of money which can be exactly paid in pieces of money worth either \$.$16\frac{2}{3}$ or \$.$12\frac{1}{2}$?

22. How long will 200 pounds of meat last 9 persons at the rate of $2\frac{5}{8}$ pounds a day for each person?

23. What length of time would a man require to travel around the earth, the distance being 25000 miles, if he travel at the rate of $31\frac{1}{2}$ miles per day?

24. If a man can build $2\frac{2}{5}$ rods of wall in a day, how much can he build in $6\frac{1}{2}$ days?

25. What is that number $\frac{1}{4}$ of which exceeds $\frac{1}{5}$ by 2?

NOTE. $\frac{1}{4}-\frac{1}{5}=\frac{1}{20}$; if $\frac{1}{20}$ be 2, $\frac{20}{20}$ will equal $20 \times 2 = 40$, *Ans.*

26. What number is that $\frac{2}{5}$ of which exceeds $\frac{1}{4}$ by $11\frac{3}{4}$?

27. How many bushels of wheat can a man purchase for \$$2724\frac{1}{2}$, at $31\frac{3}{4}$ cents per bushel?

28. What is $\frac{3}{8} \div (\frac{7}{12}$ of $\frac{5}{6}$ of $8\frac{1}{2})$.

29. What is $(\frac{2}{3}$ of $\frac{4}{7}) \div (\frac{1}{3}$ of $\frac{2}{5})$?

30. If I buy 125 bushels of corn at $41\frac{2}{3}$ cents per bushel, and sell it at $52\frac{1}{2}$ cents per bushel, what do I gain?

31. What number divided by $\frac{5}{7}$ equals $125\frac{3}{7}$?

32. What are the contents of 3 floors measuring as follows: $13\frac{4}{5}$ square yards, $32\frac{7}{16}$ square yards, and $49\frac{21}{24}$ square yards?

33. The product of three numbers is $74\frac{1}{5}$; two of them are $8\frac{4}{7}$ and $6\frac{1}{13}$; what is the third?

34. Exchanged 42 tubs of butter, averaging $48\frac{5}{7}$ pounds, at $21\frac{1}{2}$ cents per pound, for 42 barrels of flour, at \$$9\frac{2}{3}$ per barrel, and received the balance in cash; required the balance.

35. I have three boxes of cloth, each containing 12 pieces, each piece containing $4\frac{4}{5}$ yards, weighing $3\frac{3}{4}$ pounds to the yard; what is the weight of the whole?

36. What will $42\frac{7}{8}$ quires of paper weigh at $\frac{3}{5}$ pound per quire?

37. A grocer has five casks of raisins of the following weights: $115\frac{3}{8}$ pounds, $117\frac{3}{4}$ pounds, $99\frac{3}{16}$ pounds, $100\frac{1}{4}$ pounds, and $121\frac{1}{2}$ pounds; what is the average weight per cask?

38. What is the cost of the above at $8\frac{7}{8}$ cents per pound?

39. Owning $\frac{2}{5}$ of a paper-mill, I sold $\frac{3}{4}$ of my share for \$1750; what is the value of the whole mill at the same rate?

68.* If E can do the same work in 7 days, how long would be required for C, D, and E, to do it working together?

Ans. $2\frac{18}{131}$ days.

69.* If A, B, and C can do a piece of work in 6 days, and A and B can do the same work in 8 days, in what time can C do it alone? *Ans.* 24 days.

70.* Shipped to Havre 2000 bbls. of flour, which I sold at $\$7\frac{3}{8}$ per bbl.; received in return $500\frac{2}{3}$ hhds. of wine, worth $\$21\frac{1}{2}$ per hhd.; what sum is still due me?

71.* A merchant owned $\frac{2}{7}$ of a cargo of teas, the whole cargo worth \$65000; he sells $\frac{2}{5}$ of his share for $\$8583.33\frac{1}{3}$; does he gain or lose, and how much?

72.* From a tank containing 184 gallons of water, $20\frac{3}{4}$ gallons were drawn out; if $\frac{3}{8}$ of what then remained was equal to $\frac{5}{8}$ of what afterwards rained in, how much rained in? How much did the tank then contain? *Ans.* $236\frac{57}{80}$ gallons.

73.* I pay \$700 for a piece of land; cut $52\frac{1}{2}$ cords of wood from it, which I sell at \$5.40 a cord; I pay $\$1\frac{5}{8}$ a cord for cutting and hauling the wood, and \$10 for surveying the land; I divide 3 acres of it into house lots of $\frac{1}{4}$ acre each; 4 of these I sell at \$175 each, and the rest at \$162.50 per lot. Reserving 2 acres for myself, valued at \$300, I sell the remainder of the land for \$600, what do I gain? *Ans.* $\$2388.18\frac{3}{4}$.

74.* Messrs. B, D, W, and S, built a drain together, each agreeing to pay his proportion of whatever he occupied. B occupied 20 feet alone, B and D 22 feet, B, D, and W, 140 feet. B, D, W, and S, 18 feet. The drain was built at a cost of $33\frac{1}{3}$ cents per foot; what was each person's share of the cost?

NOTE.—B's share $= 20 \times 33\frac{1}{3} + \frac{22 \times 33\frac{1}{3}}{2} + \frac{140 \times 33\frac{1}{3}}{3} + \frac{18 \times 33\frac{1}{3}}{4}$.

Ans. B, $\$27.38\frac{8}{9}$; D, $\$20.72\frac{2}{9}$; W, $\$17.05\frac{5}{9}$; S, \$1.50.

75.* A, B, C, and D, hired a team together in Boston for a journey north, each agreeing to share the expense for the distance he rode. At Reading, 14 miles from Boston, A got out; at Andover, 8 miles further, B got out; at Lawrence, 4 miles further, C left, and D went on alone 8 miles to Haverhill. Re-

turning, he took up C, B, and A, where he left them, and all rode into Boston. They paid \$8.50 for the use of the team; what was each one's share?

NOTE.—The distance from Boston to Haverhill is 34 miles; the price for 1 mile out and back is $\$\frac{8.50}{34}=\$.25$; D's share is $\frac{14\times25}{4}+\frac{8\times25}{3}+\frac{4\times25}{2}+8\times25$.

Ans. A, \$.87½; B, \$1.54⅙; C, \$2.04⅙; D, \$4.04⅙.

149. GENERAL REVIEW, No. 3.

1. What are the prime factors of 420?
2. Divide 15 × 7 × 12 × 8, by 21 × 10 × 3 × 4.
3. What is the greatest common divisor of 21, 84, and 51?
4. What is the least common multiple of 42, 9, 14, and 12?
5. Reduce $\frac{180}{375}$ and $\frac{56}{656}$ to their lowest terms.
6. Reduce $254\frac{3}{4}$ to an improper fraction.
7. Reduce $\frac{688}{5}$ to a mixed number.
8. Reduce $\frac{3}{15}$ of $\frac{9}{14}$ of $\frac{21}{10}$ of $6\frac{5}{9}$ to a simple fraction.
9. Reduce $\frac{7}{8}$, $\frac{4}{9}$, and $\frac{5}{7}$, to a common denominator.
10. Reduce $\frac{13}{18}$, $\frac{12}{27}$, and $8\frac{5}{6}$, to the least common denominator.
11. Add $\frac{3}{4}$ of $\frac{4}{7}$, $\frac{14}{21}$, and $9\frac{1}{3}$.
12. Add $15\frac{4}{9}$, $3\frac{1}{6}$, and $25\frac{1}{3}$.
13. From $\frac{10}{24}$ of $\frac{7}{15}$ take $\frac{1}{12}$.
14. Subtract $8\frac{11}{12}$ from $10\frac{7}{20}$.
15. Multiply $\frac{6}{13}$ by $3\frac{3}{4}$.
16. Divide $\frac{5}{6}$ of $\frac{3}{14}$ by $10\frac{2}{7}$.
17. Change $\frac{\frac{3}{8}}{4}$, $\frac{\frac{7}{3}}{\frac{2}{9}}$, $\frac{1\frac{3}{5}}{1\frac{1}{2}}$, and $\frac{2}{3\frac{4}{5}}$ to simple fractions.

$\frac{2\frac{1}{3}}{8\frac{2}{5}}$

18.* What part of $8\frac{2}{5}$ is $2\frac{1}{3}$?
19.* What is the greatest common divisor of $\frac{4}{15}$, $\frac{2}{5}$, and $2\frac{2}{3}$?
20.* What is the least common multiple of $\frac{4}{27}$, $\frac{3}{16}$, and $\frac{7}{96}$?
21.* How many fourths of $\frac{5}{7}$ of 40 in $3\frac{4}{7}\times\frac{8}{9}\div\frac{2}{3}$ of $\frac{20}{27}$?

☞ For changes, see Key.

COMPOUND DENOMINATE NUMBERS.

150. Numbers are either **Simple** or **Compound.**

151. A **Simple Number** is a number expressed in units of one denomination; as, 5 *books*, 7 *pens*.

152. A **Compound Number** is a number expressed in units of two or more denominations, but of the same nature; as, *5 pounds 6 ounces of sugar, 3 years 2 months 4 days of time.*

153. **Reduction** is the process of changing the denomination of numbers without altering their value.

154. **Reduction Descending** is the process of changing numbers to numbers of equal value in lower denominations; thus, 1 *dollar* = 100 *cents*.

155. **Reduction Ascending** is the process of changing numbers to numbers of equal value in higher denominations; thus, 100 *cents* = 1 *dollar*.

156. Compound numbers express *Currency*, *Weight*, and *Measure*.

CURRENCY.

Every nation has its own currency. That of the United States has already been given (Art. 68), but the table will be inserted here for the sake of uniformity.

157. FEDERAL MONEY.

The denominations are *eagles, dollars, dimes, cents*, and *mills*. The legal coins in circulation are as follows:

GOLD.			SILVER.		
Double Eagle	=	$20.00.	Dollar	=	$1.00.
Eagle	=	10.00.	Half Dollar	=	.50.
Half Eagle	=	5.00.	Quarter Dollar	=	.25.
Quarter Eagle	=	2.50.	Dime	=	.10.
Three Dollar piece	=	3.00.	Half Dime	=	.05.
One Dollar piece	=	1.00.	Three Cent piece	=	.03.

Copper and nickel Cent, and Two Cent pieces.

NOTE.— The gold coin is hardened by an alloy of $\frac{1}{10}$ copper and silver (the silver not to exceed the copper). The silver coin is hardened by $\frac{1}{10}$ copper. The cent coined since 1856 has 88 parts of copper to 12 of nickel. The two-cent piece, coined 1864, has 95 parts copper to 5 of tin and zinc.

TABLE.

10 mills (m.)	= 1 cent,	marked	c. or ct.
10 c.	= 1 dime,	"	d.
10 d.	= 1 dollar,	"	\$.
\$10.	= 1 eagle,	"	E.

NOTE.—Mill is derived from the Latin *mille*, one thousand, because 1000 mills = 1 dollar, *the unit of computation;* cent from Latin *centum*, one hundred, because 100 cents = 1 dollar; dime from the French *dime*, a tenth, as a dime is one tenth of a dollar; dollar from the German *thaler*, dollar, dollars having been first coined in Germany.

EXERCISES.

1. Write 3 E. \$2. 7 d. 5 c. 2 m. as it is usually written.
Ans. \$32.752.

2. Write 162 E. \$8. 3 d. 9 c. 8 m. as it is usually written.
Ans. \$1628.398.

Write in the same manner,

3. 128 E. 3 d. 8 m.
4. 19 E. \$6. 3 c. 2 m.
5. 68 E. \$8. 2 m.
6. \$7. 2 c. 5 m.
7. \$5. 6 d. 8 c. 3 m.
8. 3984 E. 7 d. 4 c. 8 m.

9. Add the answers of the last six examples, and give the amount in mills. *Ans.* 42,017,798 mills

158. ENGLISH MONEY.

The denominations are *pounds, shillings, pence,* and *farthings.*

TABLE.

4 farthings (qr. or far.)	= 1 penny,	marked	d.
12 d.	= 1 shilling,	"	s.
20 s.	= 1 pound,	"	£.

NOTE.—The *guinea* of 21 s., and the *crown* of 5 s., are also used. The coin which represents the £ value is called a *sovereign*.

159. Reduction Descending.

Ill. Ex. Reduce 3 £ 11 s. 8 d. 2 far. to farthings.

Operation.
3 £ 11 s. 8 d. 2 far.
20
71 s.
12
860 d.
4
3442 farthings, *Ans.*

As 20 s. = 1 £, we shall have 20 times as many s. as £. (20 × 3) s. = 60 s.; 60 s. + 11 s. = 71 s. As 12 d. = 1 s., we shall have 12 times as many d. as s.; (12 × 71) d. = 852 d.; 852 d. + 8 d. = 860 d. As 4 far. = 1 d., we shall have 4 times as many far. as d.; (4 × 860) far. = 3440 far; 3440 far. + 2 far. = 3442 farthings. Hence the

Rule for Reduction Descending. *Multiply the number of the highest denomination by the number which it takes of the next lower denomination to make one of that higher, and to the product add the given number of the next lower denomination. Multiply that sum in like manner, and thus proceed till the number is reduced to the required denomination.*

Examples.

1. Reduce 7 £ 8 s. 3 d. 3 far. to farthings. *Ans.* 7119 far.
2. Reduce 30 £ 2 s. 0 d. 2 far. to farthings. *Ans.* 28898 far.
3. Reduce 8 £ 0 s. 3 d. to farthings.
4. Reduce 9 s. 1 d. 2 far. to farthings.
5. Reduce 368 £ 17 s. 2 d. to pence.
6. Reduce 25 crowns 3 s. 2 d. to farthings.
7. Reduce 43 crowns 4 s. 8 d. to pence.
8. Reduce 209 guineas to pence.
9. What will be the number of farthing candles that may be bought for 2 s. 6 d.?

160. Reduction Ascending.

Ill. Ex. Reduce 3579 farthings to an equivalent value in higher denominations.

OPERATION.

$$
\begin{array}{l}
4\,)\,\underline{3579} \\
12\,)\,\underline{894}\ \text{d.} + 3\ \text{qr.} \\
20\,)\,\underline{74}\ \text{s.} + 6\ \text{d.} \\
\quad 3\ £\ 14\ \text{s.}\ 6\ \text{d.}\ 3\ \text{qr.},\ \textit{Ans.}
\end{array}
$$

As 4 qr. = 1 d., we shall have $\frac{1}{4}$ as many pence as farthings, or 894 d. and 3 qr. remaining; as 12 d. = 1 s., we shall have $\frac{1}{12}$ as many shillings as pence, or 74 s. and 6 d. remaining. As 20 s. = 1 £, we shall have $\frac{1}{20}$ as many £ as s., or 3 £ and 14 s. remaining, making the entire result 3 £ 14 s. 6 d. 3 qr. Hence the

RULE FOR REDUCTION ASCENDING. *Divide the given number by the number which it takes of its denomination to equal one of the next higher, and note the remainder. Divide the quotient thus obtained as before, and thus proceed till the required denomination is attained. The last quotient, with the several remainders, will be the required result.*

PROOF. *As reduction ascending is the converse of reduction descending, either process may be proved by the other.*

EXAMPLES.

1. Reduce 3681 farthings to an equivalent value in higher denominations. *Ans.* 3 £ 16 s. 8d. 1 qr.

In the same manner reduce,

2. 36875 farthings. *Ans.* 38 £ 8 s. 2 d. 3 far.
3. 4328 pence. *Ans.* 18 £ 0 s. 8.
4. 39818 shillings.
5. 86347 farthings.
6. 298721 farthings.

161. COMPARISON OF ENGLISH AND FEDERAL CURRENCY.

1 £ = $4.84.

How many $ in		How many £ in	
1.	36 £? *Ans.* $174.24	4.	$39.43? *Ans.* $8\frac{71}{484}$ £
2.	49 £?	5.	$43.76?
3.	$64\frac{1}{2}$ £?	6.	$78.39?

WEIGHT.

162. Troy Weight.

Gold, silver, and precious stones are weighed by this system.

The denominations are *pounds*, *ounces*, *pennyweights*, and *grains*.

Table.

24 grains (gr.)	= 1 pennyweight,	marked	pwt.
20 pwt.	= 1 ounce,	"	oz.
12 oz.	= 1 pound,	"	lb.

Ill. Ex. Reduce 2 lb. 9 oz. 18 pwt. 3 gr. to grains.

Operation.

2 lb. 9 oz. 18 pwt. 3 gr.
12
———
33 oz.
20
———
678 pwt.
24
———
2715
1356
———
16275 gr., *Ans.*

Ill. Ex. Reduce 16275 gr. to numbers of higher denominations.

Operation.

24) 16275 gr.
2|0) 67|8 pwt. + 3 gr.
12) 33 oz. + 18 pwt.
2 lb. + 9 oz.
Ans. 2 lb. 9 oz. 18 pwt. 3 gr.

Examples.

1. Reduce 18 lb. 11 oz. 5 pwt. 17 gr. to grains. *Ans.* 109097 gr.
2. Reduce 48 lb. 2 oz. 0 pwt. 3 gr. to grains. 277443
3. Reduce 1 oz. 23 gr. to grains. 513
4. Reduce 3681 lb. 9 oz. 1 pwt. to pennyweights. 883.621

Reduce to equivalent values in higher denominations,

5. 928641 pwts. *Ans.* 3869 lb. 4 oz. 1 pwt.
6. 3786541 grs.
7. 9042028 grs.
8. What is the value of 2 lb. 8 oz. of gold, at $16 00 an

512

9. What is the value of 1 lb. 3 oz. 7 pwt. of gold, at 4 cents per grain?

10. What will 20 silver dollars weigh, each dollar weighing 412½ grains?

163. Apothecaries' Weight.

Apothecaries use this weight for mixing medicines; but they buy, and generally sell, by Avoirdupois weight.

The denominations are *pounds*, *ounces*, *drachms*, *scruples*, and *grains*.

Table.

20 grains (gr.)	= 1 scruple,	marked	sc.	or ℈.
3 ℈	= 1 drachm,	"	dr.	or ʒ.
8 ʒ	= 1 ounce,	"	oz.	or ℥.
12 ℥	= 1 pound,	"	lb.	or ℔.

Ill. Ex. Reduce 2 ℔. 3 ℥, 2 ʒ, 1 ℈, 5 gr. to grains.

Operation.

2 ℔
12
———
27 ℥
8
———
218 ʒ
3
———
655 ℈
20
———
13105 gr., *Ans.*

Ill. Ex. Reduce 68321 grains to numbers of higher denominations.

Operation.

2|0) 6832|1 gr.
3) 3416 ℈ + 1 gr.
8) 1138 ʒ + 2 ℈.
12) 142 ℥ + 2 ʒ.
11 lb. + 10 ℥.

Ans. 11 lb. + 10 ℥, 2 ʒ, 2 ℈, 1 gr.

Examples.

1. Reduce 5 ℔. 7 ℥, 7 ʒ, 2 ℈, 12 gr. to grains.
Ans. 32632 gr.

2. Reduce 3 ℔. 0 ℥, 7 ʒ, 1 ℈, 9 gr. to grains.

3. Reduce 258481 grains to pounds, ounces, &c.
Ans. 44 ℔. 10 ℥, 4 ʒ, 0 ℈, 1 gr.

4. Reduce 36845 ℈ to pounds, ounces, &c.

5. Reduce 987326 gr. to pounds, ounces, &c. 171-4-7-1-1
6. Reduce 28 ℔. 3 ℥, 1 ʒ, 2 ℈, 5 gr. to grains. 162,825 gr

164. Avoirdupois Weight.

This weight is used for weighing almost all articles, except gold, silver, and precious stones.

The denominations are *tons*, *hundred weight*, *quarters*, *pounds*, *ounces*, and *drams*.

Table.

16 drams (dr.)	= 1 ounce,	marked	oz.
16 oz.	= 1 pound,	"	lb.
25 lb.	= 1 quarter,	"	qr.
4 qr.	= 1 hundred weight,	"	cwt.
20 cwt.	= 1 ton,	"	T.

Note. — The long ton of 2240 lbs., which gives 28 lbs. to the qr., is sometimes used for weighing gross articles, as iron and coal, and is the ton recognized by the United States Government.

Ill. Ex. Reduce 2 cwt. 3 qr. 8 lbs. to pounds.

Operation.

```
 2 cwt. 3 qrs. 8 lbs.
 4
 —
11 qr.
25
 —
63
22
 —
283 lb., Ans.
```

Ill. Ex. — Reduce 186421 dr. to numbers of higher denominations.

Operation.

```
16) 186421 dr.
    ———————
16) 11651 oz. + 5 dr.
    ———————
25) 728 lb. + 3 oz.
    ———
 4) 29 qr. + 3 lb.
    ——
    7 cwt. + 1 qr.
Ans. 7 cwt. 1 qr. 3 lb. 3 oz. 5 dr.
```

Examples.

1. Reduce 5 cwt. 3 qr. 24 lbs. to pounds. *Ans.* 599 lbs.
2. Reduce 23 T. 4 lbs. to ounces. 736064

Reduce to equivalent values in higher denominations,

3. 9328 lbs. *Ans.* 4 T. 13 cwt. 1 qr. 3 lb.
4. 86842 oz.
5. 193256 lbs.
6. 8236548 dr.
7. 9654321 dr. 18-17-0-12

8. Reduce 4 T. 3 cwt. 2 qr. 0 lb. 8 oz. to ounces. 133,608
9. How many ounces in 1 cwt.? in 1 T.? 1600, 32000
10. How many pounds in one ton? 2000 lb
11. How many more pounds in a long ton than in a short ton? 2
12. At the rate of 3 lb. a day, how many hundred weight of flour will a family consume in a year, or 365 days? 10-3-2

165. COMPARISON OF WEIGHTS.

TROY.		APOTH.		AV.
1 lb.	=	1 ℔.	=	$\frac{144}{175}$ lb.
1 oz.	=	1 ℥	=	$\frac{192}{175}$ oz.
1 gr.	=	1 gr.	=	$\frac{1}{7000}$ lb.
7000 gr.	=	7000 gr.	=	1 lb.

EXAMPLES.

1. Reduce 364 lbs. Troy to Avoirdupois weight.
Ans. 299$\frac{81}{175}$ lbs. Av.
2. Reduce 36 lbs. Troy to Avoirdupois weight.
3. Reduce 5 lbs. Avoirdupois to grains in Apothecaries' weight; to units of higher denominations. *Ans.* 6 lb. 0 ℥, 7 ʒ, 1 ℈.
4. Reduce 375 lbs. Avoirdupois to Apothecaries' weight. 455
5. Reduce 73 lbs. Avoirdupois to Troy weight. 88-8-11

MEASURES OF EXTENSION.

166. LONG MEASURE.

The denominations are *miles, furlongs, rods, yards, feet, inches* and *lines.*

TABLE.

12 lines (l.)	= 1 inch,	marked	in.
12 in.	= 1 foot,	"	ft.
3 ft.	= 1 yard,	"	yd.
5½ yd. or 16½ ft.	= 1 rod,	"	r. or rd.
40 r.	= 1 furlong,	"	f. or fur.
8 f.	= 1 mile,	"	m.

69⅙ miles nearly = 1 degree (°) of longitude at the Equator,
360 of which degrees = the distance round the earth.
3 miles = 1 land league.
1 mile = 320 rods = 5280 feet.

ILL. EX. Reduce 2 m. 5. f. 13 r. 4 yd. 2 ft. to inches.

OPERATION.

```
  m. f.  r.  y. ft.
  2. 5. 13. 4. 2.
  8
  —
  21 f.
   40
  ——
  853 r.
    5½
  ———
  426½
 4269
 ————
 4695½
    3  yd.
 ————
14088½ ft.
   12
 ————
169062 in., Ans.
```

ILL. EX. Reduce 1543514 inches to miles, furlongs, etc.

OPERATION.

```
        12 ) 1543514 in.
              ———————
         3 ) 128626 ft. + 2 in.
              ——————
 5½ = 11/2 ) 42875 y. + 1 ft.
                 2
              —————
        11 ) 85750 halves of yd.
              ————
      4|0 ) 779|5 r. + 5/2 yd.   5/2 yds. = 2 yd.
              ———                    1 ft. 6 in.
          8 ) 194 f. + 35 r.
              ———
              24 m. + 2 f.

        24 m. 2. f. 35 r. 2 yd. 1 ft. 2 in.
                                1 ft. 6 in.
       ————————————————————————————————————
   Ans. 24 m. 2 f. 35 r. 2 yd. 2 ft. 8 in.
```

EXAMPLES.

1. Reduce 3 m. 7 f. 14 r. to yards. *Ans.* 6897 yds.
2. Reduce 3 f. 11 r. 2 yd. 1 ft. 7 in. to inches. *Ans.* 26029 in.
3. Reduce 1590 inches to rods, etc. *Ans.* 8 rd. 0 yd. 0 ft. 6 in.
4. Reduce 5 m. 6 f. 3 r. 3 yd. 2 ft. 5 in. to lines. 4. 380 ,12
5. Reduce 16906 inches to numbers of higher denominations.
6. Reduce 1291968 lines to miles, furlongs, etc. 1 - 5 - 5
7. How many miles round the earth?
8. How many miles through the earth from pole to pole, the distance being 41704788 feet?
9. Find the cost per mile for grading a road, at 90 cents per rod. *Ans.* $288.
10. What will it cost to fence both sides of a road, 26 r. 2 yd. long, at $.65 per yd.? *Ans.* $188.50.
11. How many furrows, each, 10 in. wide, will be made in ploughing a lot of land lengthwise, which is 6 r. 1 ft. wide? 120

167. Surveyors' Measure.

The denominations are *miles, chains, rods, links,* and *inches.*

TABLE.

$7\frac{92}{100}$ inches	= 1 link,	marked	l.
25 l.	= 1 rod,	"	r.
4 r.	= 1 chain,	"	ch.
80 ch.	= 1 mile,	"	m.

1 chain = 4 rods = 66 feet = 100 links = 792 inches.

NOTE. — Rods are seldom used by surveyors, the distances being generally taken in chains and links.

ILL. Ex. Reduce 4 m. 75 ch. 32 l. to links.

OPERATION.

4 m. 75 ch. 32 l.
80
395 ch.
100
39532 l., *Ans.*

ILL. Ex. Reduce 763218 links to miles, chains, etc.

OPERATION.

100) 763218
8|0) 763|2 ch + l.
95 m. + 32 ch.

Ans. 95 m. 32 ch. 18 l.

EXAMPLES.

1. Reduce 3 m. 35 ch. 8 l. to links. *Ans.* 27508 l.
2. Reduce 5 m. 78 ch. 2 l. 5 in. to inches. 378,596
3. 13845 links to miles, chains, etc. *Ans.* 1 m. 58 ch. 45 l.
4. 259248 inches to miles, chains, etc.
5. In 1 m. 46 ch. 2 r. how many rods? 566 rds
6. In 9584 feet, how many chains? 145 ch 24 l

168. Mariners' Measure.

The denominations for short distances are *cable-lengths, fathoms,* and *feet.*

TABLE.

6 feet (ft.)	= 1 fathom,	marked	fath.
120 fath.	= 1 cable-length,	marked	c. l.
$7\frac{1}{3}$ c. l.	= 1 common mile,	"	m.

Longer distances are estimated in nautical or geographical miles, each mile being $\frac{1}{60}$ of a degree measured on a great circle

of the earth, and averaging 6086.34 ft.* or 1.15 + common miles
3 nautical miles = 1 sea league.

EXAMPLES.

1. How many feet in 7 c. l. 32 fath.? *Ans.* 5232 ft
2. How many feet in 5 c. l. 4 ft.?
3. How many cable-lengths in 672 fath.? *Ans.* 5 c. l. 72 fath.
4. Reduce 3684 feet to units of higher denominations.

169. CLOTH MEASURE.

Cloth is measured by its length, without regard to its width. The yard is considered the unit of measure, and is divided into *halves*, *quarters*, *eighths*, and *sixteenths*.

170. SQUARE MEASURE.

This measure is used for determining the area or contents of surfaces.

The denominations are *square miles*, *acres*, *roods*, *square rods*, *square yards*, *square feet*, and *square inches*.

TABLE.

144 square in. (sq. in.)	= 1 square foot,	marked	sq. ft.
9 sq. ft.	= 1 square yard,	"	sq. yd.
$30\frac{1}{4}$ sq. yd. or $272\frac{1}{4}$ sq. ft.	= 1 square rod,	"	sq. r.
40 sq. r.	= 1 rood,	"	R.
4 R.	= 1 acre,	"	A.
640 A.	= 1 square mile,	"	sq. m.

ILL. EX. Reduce 2 sq. m. 87 A. 3 R. 19 r. to rods.

OPERATION.

```
   2 sq. m. 87 A. 3 R. 19 r.
 640
────
1367 A.
   4
────
5471 R.
  40
──────
218859 r., Ans.
```

* Topographical Bureau, Washington, 1864.

ILL. EX. Reduce 386060 sq. in. to square rods, yards, &c.

OPERATION.

$$\begin{array}{l}
144\,)\,386060 \text{ sq. in.} \\
\quad 9\,)\,\overline{2680} \text{ sq. ft.} + 140 \text{ sq. in.} \\
30\tfrac{1}{4}\,)\,\overline{297} \text{ sq. yd.} + 7 \text{ sq. ft.} \\
\;\;4 \qquad 4 \\
\overline{121}\,)\,\overline{1188} \\
\qquad\qquad 9 \text{ sq. rd.} + 24\tfrac{3}{4} \text{ sq. yd.} \\
\qquad\qquad \tfrac{3}{4} \text{ sq. yd.} = 6 \text{ sq. ft. } 108 \text{ sq. in. (Art. 198.)} \\
9 \text{ sq. rd. } 24 \text{ sq. yd. } 6 \text{ sq. ft. } 108 \text{ sq. in.} \\
\qquad\qquad\qquad 7 \text{ sq. ft. } 140 \text{ sq. in.} \\
\hline
\end{array}$$

Ans. 9 sq. rd. 25 sq. yd. 5 sq. ft. 104 sq. in.

1. Reduce 3 sq. m. 35 A. to square yards. *Ans.* 9462200 yd.
2. Reduce 19 A. 2 R. 5 sq. rd. to square inches.

Reduce to numbers of higher denominations,

3. 9687 sq. rd. *Ans.* 60 A. 2 R. 7 r.
4. 5652 sq. yd.
5. 32865 sq. ft.
6. 84791 sq. in.
7. 932485 sq. in.
8. What are 2 A. 3 R. 15 r. 3 y. 8 ft. of land worth at 5 cents a foot?

171. A **rectangle** is a figure whose opposite sides are equal, and whose angles are right angles. (Art. 191.)

172. A **square** is a rectangle whose sides are all equal.

173. The **area** of a rectangle is found by *multiplying its length by its breadth.*

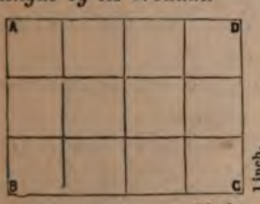


ILLUSTRATION I. Suppose the length of the figure A, B, C, D, to be 4 inches, and its breadth 3 inches. By dividing the line B C into 4 equal parts, and C D into 3 equal parts, and drawing lines from the points of division as in the figure, it will readily be seen that the entire figure is divided into 4 × 3, or 12 equal parts, each part containing 1 square inch.

ILLUSTRATION II. A figure 1 in. long and 1 in. wide contains 1 sq. in. A figure 4 inches long and 1 inch wide must contain 4 times as many sq. inches, or 4 sq. inches. A figure 4 inches long and 3 inches wide must contain 3 times as many sq. inches as if it were only 1 inch wide, or 4×3 sq. inches.

The area of a rectangle being found by multiplying its length by its breadth, it follows that

When the area and one dimension of any rectangle are given, the other dimension may be found by dividing the area by the given dimension.

NOTE. — In performing this operation, express the dividend in the superficial denomination corresponding to the linear denomination of the divisor; that is, if the divisor is expressed in *feet*, the dividend must be expressed in *square feet;* if in *yards*, the dividend must be expressed in *square yards*, &c.

174. EXAMPLES.

1. If one side of a rectangular field is 16 r. 7 ft., and the other 12 r. 5 ft., how many square feet does it contain?

2. If one side of a square field is 4 r. 8 ft., how many square feet does it contain?

3. If a rectangular field measures 24 r. 2 ft. in length, and 17 r. 4 yd. in breadth, how many square yards does it contain?

4. If a floor contains 36 square yards, and its length is 18 ft., what is its width? *Ans.* 18 ft.

5. If a ceiling contains $306\frac{1}{4}$ sq. ft., and its width is $17\frac{1}{2}$ ft., what is its length?

6. A garden containing $\frac{3}{4}$ of an acre measures on one side 192 feet; required the length of the other side.

7.* How many square feet and inches does the top of a table contain, which measures 3 ft. $2\frac{1}{2}$ in. by 4 ft. 8 in.?

8. How many square yards of carpeting will be required to cover a floor 17 feet in length by 13 feet in width?

9. What is the cost of oil-cloth to cover a floor 12 feet by $16\frac{1}{2}$ feet, at 75 cents per square yard?

173. Solid or Cubic Measure.

This measure is used in finding the contents of solid bodies or space, *i. e.*, of anything that has length, breadth, and thickness, height or depth.

The dimensions are *cubic yards*, *cubic feet*, and *cubic inches*.

Table.

1728 cubic inches (cu. in.)	= 1 cubic foot,	marked cu. ft.
27 cubic feet,	= 1 cubic yard,	" cu. yd.

Note. — The denomination ton is sometimes used, but its value is variable, a greater number of feet being assigned to the ton for light bulky articles than for the heavier.

In measuring firewood and some other merchandise, the denomination **cord** is used. A pile of wood 4 feet wide, 4 feet high, and 8 feet long, contains 1 *cord*. A pile 4 feet wide, 4 feet high, and 1 foot long, contains 1 *cord foot*. Hence,

16 cu. ft.	= 1 cord foot,	marked cd. ft.
8 cd. ft. or 128 cu. ft.	= 1 cord,	" cd.

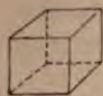
1 cord. ½ cord. 1 cd. ft.

Examples.

1. In 3 cu. yd. 18 cu. ft. 136 cu. in. how many inches?
Ans. 171,208 cu. in.
2. Reduce 5 cu. yds. 8 cu. ft. 736 cu. in. to inches.
3. Reduce 368742 cu. in. to cubic yards, feet, &c.
Ans. 7 yd. 24 ft. 678 in.
4. Reduce 3427948 cu. in. to cubic yards, feet, &c.
5. How many cord feet in 36 c. 5 cd. ft.?
6. How many cords in 54328 cu. ft.? 424-56

424 - 3 - 8 -

176. A solid bounded by six equal squares is called a **Cube.** The squares are called the **faces** of the cube, and, together, make its **surface.** The bounding lines are called **edges.** If its edges are 1 inch long, it contains 1 cu. in.; if 1 foot long, it contains 1 cu. ft., &c.



177. A solid that is bounded by rectangles is called a **Rectangular Parallelopiped;** rectangular, because its faces are rectangles, and parallelopiped, because its opposite faces are parallel.

178. The **Solidity of a Parallelopiped** *equals the product of its three dimensions.*

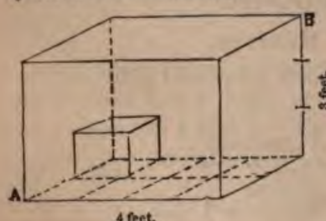


ILLUSTRATION. Let the figure A B represent a parallelopiped 4 feet long, 2 feet wide, and 3 feet high. If it is 4 feet long and 2 feet wide, its lower face or base must contain $4 \times 2 = 8$ square feet. If upon these square feet the solid extends 1 foot high, it will contain 8 cubic feet resting upon the base. But the solid is 3 feet high, and must, therefore, contain three times as many cubic feet as if it were only 1 foot high, or $3 \times 4 \times 2$ cubic feet $= 24$ cubic feet.

EXAMPLES.

1. If a solid is 3 ft. long, 5 ft. wide, and 2 ft. high, how many cubic feet does it contain? *Ans.* 30 cu. ft.

2. How many cubic inches in a block 3 in. wide, 4 in. high, and 1 ft. 2 in. long? *Ans.* 168 cu. in.

3. How many cords in a woodpile 40 ft. long, 4 ft. wide, and 4 ft. high? 5 cords

179. If the solidity of a parallelopiped equals the product of its three dimensions, it follows that

When the solid contents and two dimensions are given, the third can be found by dividing the contents by the product of the two given dimensions.

When the solid contents of a block and the area of its base are given, how do you find its height? When its contents and height are given, how can you find the area of its base?

4. How high must a box be made, to contain 24 cu. ft., the length of the box being 4 ft. and its width 3 ft.? *Ans.* 2 ft.

5. How high must it be, if its length is 8 ft. and its breadth 3 ft.?

6. If its height is $2\frac{2}{5}$ feet, what must be the area of its base?

7. How long must a pile of wood be, which is 4 ft. wide, 3 ft. 6 in. high, to contain a cord?

8. There are 144 square inches on one side of a block containing a cubic foot; what is the length of the edge of the block?

9. There being $112\frac{1}{2}$ cubic feet in a stick of timber which is $1\frac{1}{2}$ feet square at the end, what is the length? *Ans.* 50 ft.

MEASURES OF CAPACITY.

180. LIQUID MEASURE.

The denominations are *gallons*, *quarts*, *pints*, and *gills*.

TABLE.

4 gills (gi.)	= 1 pint,	marked pt.
2 pts.	= 1 quart,	" qt.
4 qts.	= 1 gallon,	" gall.

NOTE. — The denominations *tierce*, *barrel*, *hogshead*, *pipe*, *butt*, and *ton*, are sometimes used, but their size is variable. Barrels generally contain $31\frac{1}{2}$ or 32 gall.; hogsheads, 63 gall.

Casks are generally gauged and marked accordingly. They are called *hogsheads*, *pipes*, *butts*, or *tuns*, without distinction.

EXAMPLES.

1. Reduce 3 gall. 3 qt. 1 pt. 2 gi. to gills. *Ans.* 126 gi.
2. Reduce 5 gall. 1 qt. 0 pt. 3 gi. to gills. 171 gi
3. Reduce 23684 gills to gallons. *Ans.* 740 gall. 1 pt.
4. Reduce 984324 gills to hogsheads.
5. What will 27 gall. 3 qt. of milk cost at 4 cents per qt.?

181. DRY MEASURE.

The denominations are *bushels*, *pecks*, *quarts*, *pints*, and *gills*.

TABLE.

4 gills (gi.)	= 1 pint,	marked pt.
2 pts.	= 1 quart,	" qt.
8 qts.	= 1 peck,	" pk.
4 pks.	= 1 bushel,	" bu.

EXAMPLES.

1. Reduce 5 bu. 3 pk. 3 qt. 1 pt. to pints. *Ans.* 375 pt.
2. Reduce 2641 pt. to bu., etc. *Ans.* 41 bu. 1 pk. 0 qt. 1 pt.
3. Reduce 10 bu. 1 pk. 2 qt. 0 pt. 3 gi. to gills.
4. Reduce 8765432 gi. to bu., etc. 34239 – 3 – 7
5. What will 4 bu. 1 pk. 2 qt. of cherries cost at 8 cts. per quart?
6. Sold 3 bu. 3 pk. 5 qt. of peaches for $7.50; what did I receive per quart?

182. COMPARISON OF LIQUID AND DRY MEASURES.

Liq. Meas.	Dry Meas.		Cu. in.
1 qt.		=	$57\frac{3}{4}$.
1 gall.		=	231.
	1 qt.	=	$67\frac{1}{5}$.
	1 bu.	=	$2150\frac{2}{5}$.

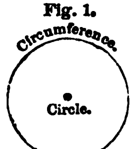
EXAMPLES.

1. I have a dish that contains 2 cu. ft.; how many quarts of blackberries will it hold? *Ans.* $51\frac{3}{7}$.
2. How many quarts of water? *Ans.* $59\frac{65}{77}$ qt.
3. How many gallons of water will a cistern hold that is 3 ft. long, 3 ft. wide, and $2\frac{1}{2}$ ft. high? 168
4. How many bushels of apples can be put into a bin 8 ft. long, 3 ft. 2 in. wide, and 2 ft. high?

CIRCULAR OR ANGULAR MEASURE.

183. This measure is used principally in *astronomy, geography, navigation,* and *surveying.*

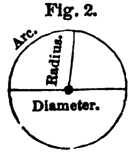
Fig. 1.



184. A **Circle** is a plane surface bounded by a line, every part of which is equally distant from a point within called the centre.

185. The bounding line is called the **Circumference** of the circle. Any part of the circumference is called an **Arc.**

Fig. 2.



186. A straight line passing from the centre of the circle to the circumference, is called a **Radius** (plural, *radii*).

187. A straight line passing from one point in the circumference, through the centre, to an opposite point, is called a **Diameter.**

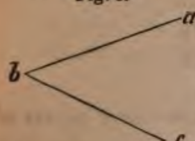
188. The circumference of any circle is supposed to be divided into 360 equal parts, called **Degrees**, each degree into 60 **Minutes**, and each minute into 60 **Seconds**.

TABLE.

60 seconds ($''$)	$=$ 1 minute,	marked $'$.
$60'$	$=$ 1 degree,	" $^\circ$.
360°	$=$ 1 circumference,	" circ.

189. A **Semi-circumference** is half a circumference, a **Quadrant** one fourth, and a **Sextant** one sixth. **A Sign,** used only in astronomy, equals 30°.

Fig. 3.



190. An **Angle** is the opening between two lines which meet each other. The point of meeting is called the **Vertex** of the angle. The angle in the annexed figure may be read, "the angle *a b c*," or simply "the angle *b*." An angle is measured by that part of the circumference of a circle included between its sides, the centre of the circle being at the vertex of the angle; thus,

Fig. 4.

In fig. 4, the angle *d e f* is measured by the arc *m n;* that is, if the arc *m n* contains 70°, the angle *d e f* is an angle of 70°.

191. An angle which includes 90°, or $\frac{1}{4}$ of a circumference, is a **Right Angle,** the sides of which are said to be perpendicular to each other; in fig. 4, the angle *g e h* is a right angle. An angle greater than a right angle is an **Obtuse** angle. An angle less than a right angle is an **Acute** angle; *h e d* is an acute angle and *g e d* is an obtuse angle.

NOTE. — As arcs are measurements of angles, the table for angular measure is the same as the table for circular measure.

192. EXAMPLES.

1. Reduce 148° 54′ 18″ to seconds. *Ans.* 536058″.
2. Reduce 354° 0′ 16″ to seconds.

3. Reduce 53684″ to numbers of higher denominations.

Ans. 14° 54′ 44″

4. Reduce 359° 59′ 59″ to seconds.
5. Reduce 1 quadrant to seconds.
6. How many seconds in 1 sextant?
7. How many minutes in a sign?
8. Reduce 35467″ to numbers of higher denominations.

Time Measure.

193. The length of an **Astronomical** or **Sidereal Day** is the time the earth takes to turn once upon its axis; the length of a **Solar Day** is the time the earth takes to turn so as to bring the sun to the same meridian again. The solar day is divided into 24 hours, each hour into 60 minutes, and each minute into 60 seconds.

The denominations of time are *centuries, years, months, weeks, days, hours, minutes* and *seconds.*

Table.

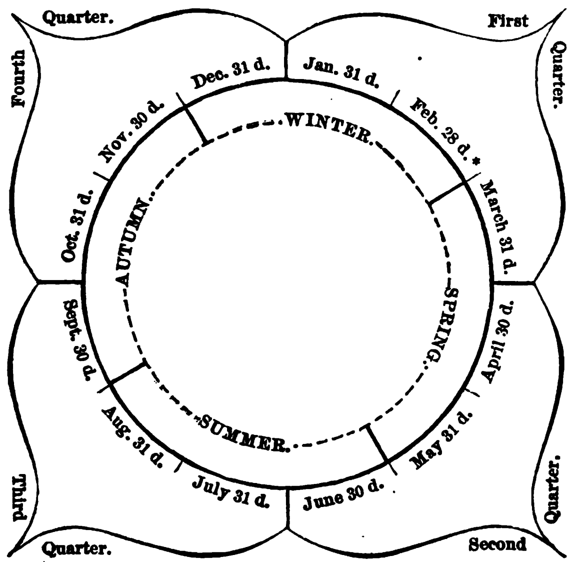
60 seconds (s.)	= 1 minute,	marked	m.
60 m.	= 1 hour,	"	h.
24 h.	= 1 day,	"	d.
7 d.	= 1 week,	"	w.
52 w. 1 d. or 365 d.	= 1 common year,	"	c. y.
366 d.	= 1 leap year,	"	l. y.
$365\frac{1}{4}$ d.	= 1 Julian year,	"	J. y.
100 y.	= 1 century,	"	C.

194. The time which the earth takes to revolve around the sun is 365 d. 5 h. 48 m. 50 s. nearly. The common year (365 days) thus loses nearly one day in 4 years. Hence the leap year of 366 days was established, which occurs once in 4 years. But this adds too much by about $11\frac{1}{6}$ m. a year, which in 100 years amounts to nearly $18\frac{2}{3}$ h. To balance this error, every 100th year is not regarded as a leap year. But this drops too much by a little more than $5\frac{1}{3}$ h., which in 4 centuries amounts to nearly 1 d. Hence every four-hundredth year is a leap year. This leaves ɑ *error* which is less than 1 d. in 3600 years. Hence the

Rule for ascertaining when any year is a leap year. *When the number denoting the year is divisible by* 4, *and not by* 100, *it is a leap year; and any year that is divisible by* 400 *is a leap year.*

195. A year is divided into four seasons, of three calendar months each, and commences with January, the second winter month.

The *succession of the seasons*, *quarters* and *months*, and the number of days in each month, are shown by the following diagram:



"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except February alone,
To which we twenty-eight assign,
Till leap year gives it twenty-nine."

Note. In the following examples, common and leap years are understood unless the Julian is specified.

* Leap year, 29 d.

ILL. EX. Reduce 2 y. 7 w. 4 d. 4 h. 33 m. to minutes.

OPERATION.

7 w. 4 d. = 53 d.
2 y. 53 d. 4 h. 33 m.
365

783 d.
24

3132
1566

18796 h.
60

1127793 m.

ILL. EX. Reduce 5387294 minutes to numbers of higher denominations.

OPERATION.

6|0) 538729|4 m.

24) 89788 h. + 14 m.

365) 3741 d. + 4 h.

10 y. + 91 d.
2

89 d.

As 2, at least, of the 10 years must be leap years, 2 days should be taken from the 91 days remaining, which leaves 89 days.

Ans. 10 y. 89 d. 4 h. 14 m.

196. EXAMPLES.

1. Reduce 8 y. 3 w. 19 d. 7 h. to hours, allowing for 2 leap years. *Ans.* 71095 h.

NOTE. 2 d. + 3 w. 19 d. = 42 d., ∴ the example may be stated, Reduce 8 c. y. 42 d. 7 h. to hours.

2. Reduce 13 y. 8 w. 2 d. 3 h. 18 m. to minutes, allowing for 3 leap years.

3. Reduce 180739 hours to numbers of higher denominations. *Ans.* 20 y. 225 d. 19 h.

4. Reduce 5683762 minutes to numbers of higher denominations.

5. How many minutes in the 1st century? *Ans.* 52594560 m.

6. How many hours in 10 y. 36 d., beginning with Jan. 1st, 1852? *Ans.* 88536 h.

7. How many seconds in the 3 summer months?

8. How many days from April 12th, 1831, to May 3d, 1832?

NOTE.—From April 12, 1831, to April 12, 1832 = 366 days; to May 3, 21 days more. *Ans.* 387 days.

9. How many days from Jan. 1st, 1832, to Jan. 1st, 1863?

10. How many days from March 1st, 1850, to Jan. 1st, 1864?

ow many seconds in 10 years, 36 minutes, allowing 365¼ year?

197. Miscellaneous Table.

Numbers.

12 units or single things	= 1 dozen.
12 dozen	= 1 gross.
12 gross	= 1 great gross.
20 units or single things	= 1 score.

Paper.

24 sheets of paper	= 1 quire,
20 quires	= 1 ream.

A book formed of sheets folded
- in 2 leaves, is a folio.
- in 4 leaves, is a quarto.
- in 8 leaves, is an octavo.
- in 12 leaves, is a duodecimo or 12mo.
- in 16 leaves, is a 16mo.
- in 18 leaves, is an 18mo.
- in 24 leaves, is a 24mo.
- in 32 leaves, is a 32mo.
- in 64 leaves, is a 64mo.

Height of Animals.

3 in. = palm.
4 in. = hand.
9 in. = span.

Capacity.

1 barrel of flour = 196 lbs.
1 barrel of pork = 200 lbs.

Examples.

1. How many rows of buttons, 6 in a row, are there in a great gross of buttons? 288 rows

2. In 3 score and 6 years how many days? 24,106 d

3. How many sheets of paper in 3 reams, 7 quires, 21 sheets?

4. How high must a doorway be for a horse to pass freely under that is $15\frac{1}{2}$ hands high? 5 ft 2 in

5. How many loaves of bread can be made from a barrel of flour, allowing $12\frac{1}{4}$ oz. to the loaf? 259 loaves

6. If pork is worth $18.75 a bbl., what is it worth per lb.?

SUGGESTION. The pupil may now write from memory and present for inspection, or repeat forward and backward, the table of Federal Money; of English Money; of Troy Weight; of Apothecaries' Weight; of Avoirdupois Weight; of Dry Measure; of Liquid Measure; of Long Measure; of Mariners' Measure; of Surveyors' Measure; of Square Measure; of Cubic Measure; of Circular Measure; of Time.

☞ For Dictation Exercises in Reduction, see Key.

FRACTIONAL APPLICATIONS.

198. REDUCTION OF A FRACTION OF ONE DENOMINATION TO WHOLE NUMBERS OF LOWER DENOMINATIONS.

ILL. EX., I. Reduce $\frac{2}{3}$£ to shillings, &c.

OPERATION.

$\frac{2}{3}£ = \frac{2}{3}$ of 20 s. $= \frac{40}{3}$ s. $= 13\frac{1}{3}$ s.
$\frac{1}{3}$ s. $= \frac{1}{3}$ of 12 d. $= 4$ d. *Ans.* 13 s. 4 d.

ILL. EX., II. Reduce $\frac{5}{7}$ cwt. to quarters, pounds, &c.

OPERATION.

$\frac{5}{7}$ cwt. $= \frac{5}{7}$ of 4 qr. $= \frac{20}{7}$ qr. $= 2\frac{6}{7}$ qr.
$\frac{6}{7}$ qr. $= \frac{6}{7}$ of 25 lb. $= \frac{150}{7}$ lb. $= 21\frac{3}{7}$ lb.
$\frac{3}{7}$ lb. $= \frac{3}{7}$ of 16 oz. $= \frac{48}{7}$ oz. $= 6\frac{6}{7}$ oz.
$\frac{6}{7}$ oz. $= \frac{6}{7}$ of 16 dr. $= \frac{96}{7}$ dr. $= 13\frac{5}{7}$ dr.
Ans. 2 qr. 21 lb. 6 oz. $13\frac{5}{7}$ dr.

Or, expressing the work in an abbreviated form,

$\frac{5}{7}$ cwt. $= \frac{20}{7}$ qr. $= 2\frac{6}{7}$ qr.
$\frac{6}{7}$ qr. $= \frac{150}{7}$ lb. $= 21\frac{3}{7}$ lb.
$\frac{3}{7}$ lb. $= \frac{48}{7}$ oz. $= 6\frac{6}{7}$ oz.
$\frac{6}{7}$ oz. $= \frac{96}{7}$ dr. $= 13\frac{5}{7}$ dr.

Hence the

RULE. To reduce a fraction of one denomination to whole numbers of lower denominations: *Multiply the fraction by the number which it takes of the next lower denomination to make one of that; reduce the fraction thus obtained to a whole or mixed number, if possible. If a fraction remain, proceed with it as before, and thus continue as far as desired.*

EXAMPLES.

Reduce to whole numbers of lower denominations,

1. $\frac{5}{6}$ of 1 £. *Ans.* 16 s. 8 d.
2. $\frac{7}{8}$ of 1 lb. Troy. *Ans.* 10 oz. 10 pwt.
3. $\frac{4}{5}$ of 1 lb.
4. $\frac{5}{9}$ of 1°.
5. $\frac{7}{9}$ of 1 cwt.
6. $\frac{2}{9}$ of 1 c. y.
7. $\frac{5}{32}$ of 1 gallon.
8. $\frac{3}{5}$ of 1 bu.
9. $\frac{7}{11}$ of 1 mile.
10. $\frac{5}{9}$ of 1 furlong.
11. $\frac{7}{12}$ of 1 chain.
12. $\frac{3}{4}$ of 1 league.
13. $\frac{7}{22}$ of 1 sq. mile.
14. $\frac{7}{18}$ of 1 cu. yard.
15. $\frac{5}{12}$ of 1 cord.
16. $\frac{3}{5}$ of 1 J. y.
17. $\frac{1}{3}$ of $\frac{4}{5}$ of 1 A.

☞ For Dictation Exercises, see Key.

199. REDUCTION OF WHOLE NUMBERS OF LOWER DENOMINATIONS TO THE FRACTION OF A HIGHER DENOMINATION.

ILL. EX., I. Reduce 5 s. 3 d. 3 qr. to the fraction of a £.

OPERATION.

$$3\text{ qr.} = \tfrac{3}{4}\text{ d.}$$

$$3\tfrac{3}{4}\text{d.} = \tfrac{15}{4}\text{ d.} = \tfrac{15}{4}\text{ of }\tfrac{1}{12}\text{ s.} = \frac{\overset{5}{\cancel{15}}}{4 \times \underset{4}{\cancel{12}}} = \tfrac{5}{16}\text{ s.}$$

$$5\tfrac{5}{16}\text{ s.} = \tfrac{85}{16}\text{ s.} = \tfrac{85}{16}\text{ of }\tfrac{1}{20}\text{ £} = \frac{\overset{17}{\cancel{85}}}{16 \times \underset{4}{\cancel{20}}}\text{ £} = \tfrac{17}{64}\text{ £, } \textit{Ans.}$$

ILL. EX., II. Reduce 7 oz. 6 pwt. 16 gr. to the fraction of a lb.

OPERATION.

$$16\text{ gr.} = \tfrac{2}{3}\text{ pwt.}$$

$$6\tfrac{2}{3}\text{ pwt.} = \tfrac{20}{3}\text{ pwt.} = \frac{\cancel{20}}{3 \times \cancel{20}}\text{ oz.} = \tfrac{1}{3}\text{ oz.}$$

$$7\tfrac{1}{3}\text{ oz.} = \tfrac{22}{3}\text{ oz.} = \frac{\overset{11}{\cancel{22}}}{3 \times \underset{6}{\cancel{12}}}\text{ lb.} = \tfrac{11}{18}\text{ lb., } \textit{Ans.}$$

Hence the

RULE. To reduce whole numbers of lower denominations to the fraction of a higher denomination: *Reduce the number of the lowest denomination to a fraction of the next higher. Annex it to the number of that higher denomination, and change the mixed number thus obtained to an improper fraction. Reduce as before, and thus continue as far as desired.*

EXAMPLES.

Reduce

1. 6 s. 3 d. to the fraction of a £. *Ans.* $\frac{5}{16}$ £.
2. 3 p. 6 qt. $1\frac{1}{3}$ pt. to the fraction of a bu. *Ans.* $\frac{23}{24}$ bu.
3. 1 qt. 0 pt. 1 gi. to the fraction of a gall.
4. 1 ℥, 2 ʒ, 2 ℈, to the fraction of a ℔.
5. 5 cwt. 1 qr. 16 lb. $10\frac{2}{3}$ oz. to the fraction of a T.
6. 6 fur. 2 r. 2 y. 1 ft. to the fraction of a m.
7. 4 y. 0 ft. $4\frac{1}{2}$ in. to the fraction of a r.
8. 2 r. 1 l. to the fraction of a ch.

What part of

9. 1 A. is 2 R. 1 r. 24 sq. y. 6 sq. ft. 108 sq. in.?
10. 1 cu. yd. is 13 cu. ft. 864 cu. in.?
11. 1 cd. is 5 cd. ft. 4 cu. ft. 576 cu. in.?
12. 1 c. y. is 162 d. 5 h. 20 m.?
13. 1 l. y. is 146 d. 9 h. 36 m.?
14. 1 J. y. is 350 d. 15 h. 21 m. 36 s.?
15. If 1 £ is worth \$4.84, what is the value of 4 s. 6 d.?

SOLUTION. 4 s. 6 d. $= \frac{9}{40}$ £. 1 £ $=$ \$4.84, $\therefore \frac{9}{40}$ £ $= \frac{9}{40}$ of \$4.84 $=$ \$1.089.

What cost

16. 3 pk. 2 qt. of meal at \$.60 a bu.?
17. 2 qt. 1 pt. of kerosene oil at \$.52 a gall.?
18. 62 lb. 8 oz. soap at \$7.50 per cwt.?
19. 2 ℥, 1 ʒ, 4 gr., quinine at \$4.00 per ℥?
20. How long will it take a man to travel 9 miles at the rate of 3 m. 6 f. 26 r. 3 yd. 2 ft. an hour?
21. At \$60.00 an acre, what cost 2 A. 3 R. $13\frac{1}{4}$ sq. rd.?
22. At \$9.00 a ton, what cost 1 T. 5 cwt. 2 qr. 14 lb. of coal? (Long ton.)
23. At \$198 a lb., what cost 10 oz. 10 pwt. 10 gr. of gold?
24. The weight of a cubic foot of water being $62\frac{1}{2}$ lbs., how many pounds of water will a tank contain which measures 9 ft. 6 in. by 8 ft. 8 in., and is 6 ft. 9 in. deep?
25. A cubic foot of granite weighs 163 lbs. 5 oz.; what is the weight of a block 3 ft. $2\frac{2}{5}$ in. by 2 ft. 4 in. and 1 ft. 3 in. thick?

For Dictation Exercises, see Key.

200. Addition.

Addition of Compound Numbers is the process of finding a number equal in value to two or more given compound numbers. The process is similar to addition of simple numbers.

Ill. Ex. What is the sum of 3 £ 11 s. 6 d. 3 qr., 4 £ 7 s. 8 d. 2 qr., 7 s. 6 d. 2 qr., and 9 £ 18 s.?

Operation.

£.	s.	d.	qr.
3	11	6	3
4	7	8	2
	7	6	2
9	18		

Ans. 18 £ 4 s. 9 d. 3 qr.

Writing the numbers, pounds under pounds, shillings under shillings, &c., we commence by adding the numbers in the farthings' column, and find the amount = 7 qr. = 1 d. + 3 qr. Writing 3 in the farthings' place, we add the 1 d. with the column of pence, and have for the amount, 21 d. = 1 s. + 9 d. Writing 9 in the pence' place, we add the 1 s. with the shillings, and have 44 s. = 2 £ + 4 s. Writing 4 in the shillings' place we add the 2 £ with the column of pounds, and have for an answer, 18 £ 4 s. 9 d. 3 qr. Hence the

Rule for Addition of Compound Numbers. *Write the numbers of like denominations in the same column, and commence in adding with the numbers of the lowest denomination. Divide the amount by the number it takes of that denomination to make one of the next higher, write the remainder under the column, and add the quotient with the numbers of the next higher denomination. Add the next column in the same manner, and thus continue till all the numbers are added.*

201. Examples.

Add the following numbers:

CURRENCY.

1.

$
49.703
8.47
.882
4.369

2.

	£.	s.	d.	qr.
	5	8	4	2
	7	15	3	3
		19	0	2
	16	4	6	
Ans.	30	7	2	3

3.

£.	s.	d.	qr.
206	18	4	3
29	14	9	2
118	7	10	
	13	7	1

WEIGHTS.

4.

lb.	oz.	pwt.	gr.
3	6	7	2
5	0	3	8
6	9	16	21
5	3	15	

5.

lb.	oz.	pwt.	gr.
5	3	2	23
5	10	18	13
4	11	17	15
5	3	0	7

6.

T.	cwt.	qr.	lb.	oz.
5	16	3	20	8
4	0	2	17	3
	18	0	5	4
15	6	3	4	14

7.

T.	cwt.	qr.	lb.	oz.	dr.
18	5	1	22	14	8
	7	3	4	15	7
9	16	0	15	6	14

8.

lb.	℥.	ʒ.	℈.	gr.
5	9	7	2	18
3	5	2	1	5
18	4	7	2	17

9.

lb.	℥.	ʒ.	℈.	gr.
38	11	6	0	4
	6	7	1	13
5	9	4	2	15

MEASURES OF LENGTH.

10.

m.	f.	r.	yd.	ft.	
8	4	32	4	2	
7	7	38	0	1	
5	3	19	4	2	
17	0	10	$3\frac{1}{2}$	2	
			$\frac{1}{2}$=1		6 in.
17	0	10	4	0	6 in.

11.

deg.	m.	f.	r.	yd.	ft.	in.	l.
3	28	7	36	4	0	7	9
8	59	4	18	3	2	5	7
20	17	5	37	4	1	9	11
32	$36\frac{5}{6}$	2	13	1	1	11	3
	$\frac{5}{6}$=	6	26	3	2		
32	37	0	39	5	0	11	3

12.

m.	f.	r.	yd.	ft.	in.
5	4	36	2	2	7
39	7	28	3	1	9
40	6	17	2	0	11

56 - 3 - 2 - [illegible] - 1

13.

deg.	m.	f.	r.	yd.	ft.
58	59	7	30	2	2
8	47	6	31	3	1
9	64	0	14	4	2

77 - 23 - 4 - [illegible] - 1 - [illegible]

14.

m.	ch.	r.	l.	in.
3	2	3	20	5
8	73	2	19	4
5	13	0	22	8

Ans. m. ch. r. l. $1\frac{16}{100}$ in.

15.

m.	ch.	r.	l.	in.
1	39	3	24	7
3	0	1	18	3
2	3	0	21	5

16.			17.		
c. l.	fath.	ft.	c. l.	fath.	ft.
10	12	2	9	96	5
4	94	4	12	102	3
11	37	5	8	86	2

V MEASURE OF SURFACE. V

18.					19.				
A.	R.	sq. r.	sq. y.	sq. ft.	m.	A.	R.	sq. r.	sq. ft.
3	3	33	13	8	2	28	3	29	147
15	2	16	12	7	3	520	2	36	208
22	1	27	4	6	5	361	3	22	168

in

41 - 3 - 37 - 1 - 0 - 108 11-271-[illegible]-8 250-1

CUBIC MEASURE.

20.			21.		
cu. yd.	cu. ft.	cu. in.	cd.	cd. ft.	cu. ft.
320	20	1000	18	6	13
29	24	968	27	7	14
500	0	728	36	5	15

MEASURES OF CAPACITY.

22.				23.			
gall.	qt.	pt.	gi.	bu.	pk.	qt.	pt.
18	3	1	3	185	3	7	1
5	3	0	2	39	2	5	1
6	0	1	3	98	0	6	0
8	2	1	1	102	3	1	1

CIRCULAR MEASURE. V TIME.

24.			25.				
°	′	″	y.*	d.	h.	m.	s.
31	4	18	2	328	18	26	31
37	30	27	5	27	7	24	45
27	24	54	3	79	6	58	39
128	44	58		281	23	41	23

* Art. 195. Note.

26.

	y.	w.	d.	h.
	7	2	8	5
	2	47	3	0
	3	39	2	2
	13	37	6	7
			1	
Ans.	13	37	5	7

27.

	y.	w.	d.	h.	m.
	17	18	5	13	48
	3	49	8	7	0
	4	39	6	7	18
Ans.	26	4	4	4	6

NOTE. — As it takes 52 w. + 1 d. to equal one y., for every year that is added to the column of years, 1 day must be taken from the amount of days.

28. Required the contents of 3 hogsheads containing respectively 58 gall. 3 qt., 67 gall. 2 qt., and 89 gall. 3 qt.

29. Mr. Rice bought cranberries as follows: 7 bu. 3 pk 2 qt., 2 bu. 1 pk. 7 qt., 5 bu. 2 pk. 6 qt.; required the amount.

30. How far will Mr. Brown travel in four days, if he travels on the 1st day, 25 m. 2 f. 7 r. 3 yd., on the 2d, 18 m. 7 f. 38 r. 2 yd., on the 3d, 23 m. 2 f. 4 r. 6 yd., and on the 4th, 31 m. 5 f. 12 r.?

31. How much land have I in 4 pastures, the 1st containing 7 A. 2 R. 3 r. 31 yd., the 2d, 15 A. 3 R. 26 r., the 3d, 22 A. 1 R. 12 r. 18 yd., and the 4th, 5 A. 0 R. 9 r. 2 yd.?

☞ For Dictation Exercises, see Key.

202. SUBTRACTION.

Subtraction of Compound Numbers is the process of finding a number equal to the difference between two given compound numbers.

ILL. EX. Subtract 2 bu. 3 pk. 7 qt. from 5 bu. 2 pk. 2 qt.

OPERATION.

bu.	pk.	qt.
5	2	2
2	3	7

Ans. 2 bu. 2 pk. 3 qt.

As we cannot take 7 qt. from 2 qt., we must reduce 1 of the 2 pk. in the minuend to quarts, which = 8 qt.; 8 qt. + 2 qt. = 10 qt.; 7 qt. from 10 qt. = 3 qt., which we write under the column of quarts. There is but one peck left in the minuend. As we cannot take 3 pk. from 1 pk., we must reduce 1 of the 5 bu. to pk., which = 4 pk.; 4 pk. + 1 pk. = 5 pk.; 3 pk. from 5 pk. = 2 pk., which we write under the column of pecks. 2 bu. from 4 bu. = 2 bu. The answer is 2 bu. 2 pk. 3 qt. Hence the

Rule for Subtraction of Compound Numbers. *Write the subtrahend beneath the minuend, so that numbers of the same denomination shall be in the same column. Commence with the lowest denomination; subtract each number from that immediately above it, writing the difference beneath. If any upper number is less than the lower, increase it by adding to it one of the next higher denomination reduced to that denomination, and then subtract, bearing in mind, in the next operation, that the upper number is less by the one reduced.*

Proof. The proof is the same as in simple subtraction.

203. Examples.

1.

	£.	s.	d.	qr.
From	18	7	6	2
Subtract	5	8	9	3
Ans.	12	18	8	3

2.

lb.	oz.	pwt.	gr.
9	6	18	13
2	9	5	18

3.

℔.	℥.	ʒ.	℈.	gr.
15	7	6	0	9
5	11	4	2	11

4.

m.	f.	ch.	r.	l.
1	1	8	3	17
	3	9	3	23

5.

°	′	″
35	47	28
19	54	48

6.

cd.	cd. ft.	cu. ft.
50	3	12
25	7	15

7.

y.*	d.	h.	m.	s.
2	248	7	0	19
1	324	18	36	27

8.

y.*	w.	d.	h.	m.	s.
7	37	4	2	12	18
5	50	3	0	18	42

9.

gal.	qt.	pt.	gi.
9	2	1	2
4	3	0	3

10.

bu.	pk.	qt.	pt.
7	2	0	0
3	5	2	1

11.

T.	cwt.	qr.	lb.	oz.
3	0	0	0	0
			1	7

12.

T.	cwt.	qr.	lb.	oz.	dr.
20	3	0	2	13	
	17	3	8	9	10

13.

cu. yd.	cu. ft.	cu. in.
1860	0	17
58	2	1720

* Art. 195. Note.

14.

rd.	yd.	ft.	in.
3	2	1	9
2	2	2	2
	$4\frac{1}{2}$	2	7
	$\frac{1}{2}=$	1	6
Ans.	5	1	1

15.

m.	f.	r.	yd.	ft.	in.
2	6	30	3	2	3
	7	31	4	1	6

16.

sq.m.	A.	R.	sq.r.	sq.y.	sq.ft.	sq.in.
6	5	1	18	8	5	18
2	68	1	27	19	4	116

17.

sq.m.	A.	R.	sq.r.	sq.y.	sq.ft.
3	7	2	0	0	0
			18	28	8

18.

c. l.	fath.	ft.
36	18	5
9	97	$3\frac{1}{2}$
26	41	$1\frac{1}{2}$

19.

cd.	cd. ft.	cu. ft.	cu. in.
394	4	7	59
15	7	13	410
378	4	9	1377

204. Ill. Ex. Required the time from June 5, 1862, to Jan. 1, 1863.

y.	mo.	d.
1863	1	1
1862	6	5
Ans.	6 m.	26 d.

Jan. being the 1st m. and June the 6th, we subtract 1862 y. 6 m. 5 d. from 1863 y. 1 m. 1 d., allowing 30 days for one month and 12 months for a year, and have for the answer 6 m. 26 d.

Note. — This method is not so exact as that illustrated in Art. 196, but is one in common use.

20. Required the time from the landing of the Pilgrims, Dec. 22, 1620, to the Declaration of Independence, July 4, 1776.

21. From the Declaration of Independence to the present time.

22. Nathaniel Bowditch was born March 26, 1773, and died March 16, 1838; what was his age?

23. Horace Mann was born May 4, 1796, and died Aug. 2, 1859; what was his age?

24. Lafayette was born Sept. 6, 1757, and died May 19, 1834; what was his age? 76 yr. 8 mo 13 d

25. Required the time between the births of Tycho Brahe, *Dec. 4, 1546,* and La Place, March 23, 1749. 202 - 3 - 19

26. Required the time between the births of Benjamin Franklin, Jan. 17, 1706, and George Washington, Feb. 22, 1732. 26 – 1 – 5

27. How old was Washington when he died, Dec. 14, 1799?

28. How old was Lafayette when that event took place? 67 – 9 – 2

☞ For Dictation Exercises, see Key.

205. Addition and Subtraction combined.

1. 3 qr. 9 lb. 8 oz. + 7 cwt. 9 lb. 2 oz. + 28 cwt. 2 qr. = ? 36

2. 9 ℔, 9 ℥, 1 ʒ, 1 ℈, 9 gr. — 2 ℔, 11 ℥, 4 ʒ, 2 ℈, 18 gr. = ?

3. 17 £. 9 s. 6 d. 3 qr. + 2 £. 12 s. 8 d. 2 qr. — 7 £. 15 s. 8 d. 3 qr. = ? 12 £ 6 s 6 d 2 qr

4. 28 cd. 2 cd. ft. 14 cu. ft. — 19 cd. 6 cd. ft. 2 cu. ft. — 4 cd. 5 cd. ft. 14 cu. ft. + 18 cd. 3 cd. ft. 9 cu. ft.? 22 cd 2 cd ft 7 cu ft

5. 1848 cu. yd. 9 cu. ft. 1700 cu. in. — (118 cu. yd. 7 cu. ft. 176 cu. in. + 50 cu. yd. 960 cu. in.) = ? 1680 cu yd 2 cu ft 564

6. 19 lb. 5 oz. 12 pwt. 19 gr. + 9 lb. 3 oz. 18 pwt. 2 gr. + 7 lb. 3 oz. 17 pwt. 22 gr. — (13 lb. 3 oz. 12 pwt. 18 gr. — 2 lb. 12 pwt. 16 gr.) = ? *Ans.* 25 lb. 3 oz. 8 pwt. 17 gr.

7. 7 T. 5 cwt. 2 qr. 20 lb. + 2 T. 8 cwt. 3 qr. 7 lb. — (2 T. 7 cwt. 18 lb. — 1 T. 19 cwt. 24 lb. 10 oz.) = ?

8. 3° 7′ 18″ + 9° 12′ 40″ + 40° 12′ + 90° — 7° 2′ 8″ — (19° 3′ 10″ + 12° 3′ 13″) = ? 9 T 6 cwt 2 qr 8 lb 10 oz

9. 9 y. 22 w. 3 d. 10 h. — 4 y. 7 w. 5 d. 3 h. + 8 y. 45 w. 3 d. 12 h. — 2 y. 25 w. 6 d. 2 h. = ? 11 y 34 wk 2 d 17 h

10. 8 m. 7 f. 12 r. 4 y. 1 ft. + 4 f. 3 r. + 4 m. 5 f. 30 r. 2 y. — 3 m. 7 f. 38 r. 5 y. 1 ft. 4 in. = ? 1 m 4 fur 7 rd 3 ft 9 in

11. 3 sq. m. 204 A. 5 sq. r. 9 sq. yd. + 98 A. 3 R. 2 sq. rd. 4 sq. yd. — (140 A. 2 R. 34 sq. rd. 28 sq. yd. + 278 A. 1 R. 39 sq. yd.) = ? 2 s m 523 A 3 R 11 r 6 yd 4 ft 72 in

12. 384 cu. yd. 19 cu. ft. 1700 cu. in. — (207 cu. yd. 2 cu. ft. 18 cu. in. — 116 cu. yd. 18 cu. ft. 394 cu. in.) + 504 cu. yd. 24 cu. ft. 89 cu. in. = ? 799 cu yd 6 cu ft 437 cu in

13. Mr. Day having 49£ ready money, pays to one man a debt of 5£. 7 s. 8 d.; to another, 10£. 15 s. 6 d. to the third, 18£. 12 s. 9 d.; how much money has he left? 14 £ 4 s 1 d

206. TABLE OF LATITUDES AND LONGITUDES.*

Place.	State.	Longitude from Greenwich. ° ′ ″	Latitude. ° ′ ″
Albany,	N. Y.,	W. 73 44 39	N. 42 39 50
Boston,	Mass.,	W. 71 3 30	N. 42 21 27
Canton,	China,	E. 113 14	N. 23 7
Calcutta,	India,	E. 88 19 2	N. 22 35 5
Cape Horn,	S. America,	W. 67 16 8	S. 55 58 40
Cape of Good Hope,	Africa,	E. 18 29	S. 32 24 3
Charleston,	S. C.,	W. 79 55 38	N. 32 46 33
Chicago,	Ill.,	W. 87 37 47	N. 42 0 0
Cincinnati,	Ohio,	W. 84 27	N. 39 5 54
Constantinople,	Turkey,	E. 28 59	N. 41 0 16
London,	England,	W. 5 48	N. 51 30 48
Mexico,	Mexico,	W. 103 45 30	N. 19 25 45
Montreal,	L. C.,	W. 73 35	N. 45 31
New Orleans,	La.,	W. 90	N. 29 57 30
New York,	N. Y.,	W. 74 0 3	N. 40 42 43
Paris,	France,	E. 2 20 22½	N. 48 50 12
Philadelphia,	Pa.,	W. 75 9 54	N. 39 58 24
Portland,	Me.,	W. 70 14 34	N. 43 39 54
Quebec,	L. C.,	W. 71 12 18	N. 46 49 12
San Francisco,	Cal.,	W. 122 26 48	N. 37 47 53
St. Petersburg,	Russia,	E. 30 19	N. 59 56 30
Washington,	D. C.,	W. 77 0 15	N. 38 53 20

14. What is the difference of latitude between Boston and Charleston? *Ans.* 9° 34′ 54″.

15. Between Washington and London?

16. Between Philadelphia and Paris? 8°-51′-48″

17. Between Quebec and St. Petersburg? 13°-7′-18″

18. Between Portland and Cape Horn?† 99° 38′ 34″

19. Between Cape Horn and Cape Good Hope? 23 34 37″

20. What is the difference of longitude between Boston and Montreal? 2°-31-30″

21. Between New Orleans and New York? 15° 59′ 57″

22. Between Mexico and Paris?† 106° 5′ 52½″

* From the American Almanac and New American Cyclopedia.

† The difference of latitude between places on opposite sides of the equator is found by *adding* the latitudes. The difference of longigitude between places on opposite sides of the first meridian is found by *adding* the longitudes. If their sum exceeds 180°, the difference of longitude equals 360° minus that sum.

23. Between Constantinople and Chicago? 116° 26′ 43″

24. Between Canton and San Francisco? 124° 19′ 12″

☞ For Dictation Exercises, see Key.

207. ADDITION AND SUBTRACTION OF DENOMINATE FRACTIONS.

ILL. EX., I. $\frac{2}{3}$ cwt. + $\frac{3}{4}$ qr. = what?

These fractions, being of different denominations, must first be reduced to the same denomination. The fraction $\frac{2}{3}$ cwt. may be changed to quarters, or the $\frac{3}{4}$ qr. may be changed to parts of a cwt., and then addition can be performed. In the first case, the answer will be in qr.; in the second, in cwt.

Another excellent method is first to reduce both fractions to integers of lower denominations, if necessary (Art. 198), and then add. Thus,

	qr.	lb.	oz.
$\frac{2}{3}$ cwt. =	2	16	$10\frac{2}{3}$
$\frac{3}{4}$ qr. =		18	12
	3	10	$6\frac{2}{3}$, *Ans.*

ILL. EX., II. $\frac{2}{3}$ £ − $\frac{4}{5}$ *s.* = what?

OPERATION.

$\frac{2}{3}$ £ =	13*s.*	4*d.*
$\frac{4}{5}$ *s.* =		$9\frac{3}{5}$
	12	$6\frac{2}{5}$, *Ans.*

EXAMPLES.

Perform the following examples, and give the answers, as far as possible, in whole numbers of lower denominations.

1. $\frac{3}{5}$ ch. + $\frac{6}{11}$ rd. = ? *Ans.* 2 rd. $23\frac{7}{11}$ l.
2. $\frac{2}{9}$ y. + $\frac{2}{9}$ d. = ?* *Ans.* 81 d. 8 h.
3. $\frac{7}{30}$ y. + $\frac{5}{36}$ d. = ? *Ans.* 85 d. 7 h. 20 m.
4. $\frac{1}{9}$ ℔. + $\frac{3}{8}$ ℥ = ? *Ans.* 1 oz. 5 dr. 2 sc.
5. $\frac{7}{18}$ cu. yd. + $\frac{1}{2}$ cu. ft. = ? *Ans.* 11 cu. ft.
6. $\frac{2}{3}$ cd. − $\frac{1}{4}$ cd. ft. = ? *Ans.* 5 cord ft. 1 cu. ft. 576 cu. in.
7. $\frac{7}{8}$ ℔. Troy − $\frac{2}{9}$ oz. = ? 10 oz 5 pwt 13$\frac{1}{3}$ gr
8. $\frac{5}{6}$ bu. + $\frac{1}{4}$ qt. = ? 3 pk 2 qt 1 pt 3$\frac{1}{5}$ gi
9. $\frac{9}{32}$ gall. − $\frac{3}{5}$ qt. = ? 1 pt $\frac{4}{5}$ gi
10. $\frac{7}{11}$ m. − $\frac{3}{4}$ rd. = ? 5 fur 2 rd 4 yd 2 ft 7
11. $\frac{7}{22}$ sq. m. + $\frac{45}{88}$ A. = ?

* Art. 195, Note.

13. 30 y. 35 d. 7 h. 20 min. 35 sec. ÷ 29 = ?

14. How far must a bird fly in one minute to fly 55 miles in an hour?

15. If 37 bu. 4 pk. of rye be divided between 7 men, what will each man receive?

16. If 65 A. 2 R. 18 sq. rd. 10 sq. yd. $1\frac{1}{2}$ sq. ft. be divided into 55 house-lots, what is the size of each?

17. How long will it take to travel 1 mile, at the rate of 75 miles in 10 h. 18 min. 12 s.?

18. Among how many men may 624 gall. 3 qt. be divided, that each man may receive 12 gall. 3 qt.? 49 men

NOTE. — Reduce each of the above to quarts before dividing.

19. How many bins, each containing 5 bu. 3 pk., will be required to hold 885 bu. 2 pk. of potatoes?

20. If a man walks 3 m. 6 fur. 26 rd. in one hour, how long will it take him to walk 23 m. 7 fur. 9 rd.?

☞ For Dictation Exercises, see Key.

210. LONGITUDE AND TIME.

As the earth turns upon its axis once in 24 hours, it follows that $\frac{1}{24}$ of 360°, or 15° of longitude, must pass under the sun in 1 hour, and $\frac{1}{60}$ of 15°, or 15′, must pass under the sun in 1 min. of time, and $\frac{1}{60}$ of 15′, or 15″ must pass under the sun in 1 sec. of time; or, in a

TABULAR FORM.

15° of longitude	make	a difference	of	1 hour in time.
15′	"	"	"	1 minute in time.
15″	"	"	"	1 second in time.

Hence, to find the difference of longitude between any two places: *Multiply the difference of time between the two places, expressed in hours, minutes and seconds, by* 15. *The product will express the number of degrees, minutes and seconds required.*

NOTE. — As the earth turns from west to east, sunrise occurs earlier in places east and later in places west of any given point. Hence the time is later in all places east, and earlier in all places west, of any given point than it is at that point.

EXAMPLES.

NOTE.—For table of latitude and longitude of places, see Art. 206, page 136.

1. The time in Pittsburg is 35 m. 54 s. earlier than in Boston; what is the difference of longitude between the two places?

Ans. 8° 58′ 30″.

2. What is the longitude of Pittsburg? *Ans.* 80° 2′ W.

3. The time at St. Paul's is 1 h. 16 m. 19⅖ s. earlier than in New York; what is the longitude at St. Paul's? 93° 4′ 54″ W

4. The time in Copenhagen is 50 m. 19⅘ s. later than in Greenwich; what is its longitude? 12° 34′ 57″ E

5. The time in Naples is 5 h. 41 m. 14⅕ s. later than in Boston; what is its longitude? *Ans.* 14° 15′ 3″ E.

211. From Art. 210 we also derive the following

RULE. To find the difference of time between any two places: *Divide the difference in longitude, expressed in degrees, minutes and seconds, by* 15. *The quotient will be the number of hours, minutes and seconds required.*

1. What is the difference of time between Albany and Boston?

Ans. 10 m. 44⅗ s.

2. Between Paris and St. Petersburg?
3. Between Montreal and Mexico? 2 h 0 min 58 ⅗ s
4. Between Cape Horn and Cape Good Hope?

Ans. 5 h. 43 m. 0$\frac{8}{15}$ s.

5. Between Charleston, S. C., and Calcutta?
6. Between Canton and San Francisco?

7. When it is 8 o'clock P. M. in Washington, what is the time in London? *Ans.* 1 o'clock, 7 m. 37⅘ s. A. M. of the next day.

8. At 2 A. M., Jan. 1, 1864, at Paris, what was the time in New Orleans? Dec 31, 1863 7h 50m 38½ s PM

212. QUESTIONS FOR REVIEW.

1. When are denominate numbers simple? when compound? Give examples of each. May abstract numbers be compound? What is REDUCTION as applied to compound numbers? What is

REDUCTION DESCENDING?—REDUCTION ASCENDING? Rule reduction descending. Write, perform, and explain an example, illustrating it. Rule for reduction ascending. Illustrate it. How can you prove examples in reduction descending? in reduction ascending?

2. What are the denominations in UNITED STATES MONEY? the coins? How are the gold coins hardened? the silver? Give the table of U. S. or Federal Money. From what are the names of the denominations derived? Which place from the decimal point do the mills occupy? How will you write forty-five thousand three hundred twenty-five mills? How many dollars in the above number of mills? how many cents?

Give the table of ENGLISH MONEY. Name the denominations. What is a guinea? a crown? a sovereign? value of a pound English money in Federal money?

3. Name the tables of WEIGHT. Repeat the table of Troy Weight; of Apothecaries' Weight; of Avoirdupois Weight. Which is in most common use? By which would you buy and sell coal? iron? silver? salt? quinine? fish? emeralds? flour? gold? opium? What is a long ton? Which is the most, a lb. Avoirdupois or a lb. Troy or Apothecaries' Weight? an oz. Avoirdupois, or an oz. Troy or Apothecaries' Weight? 1 lb. Avoirdupois = how many grs. Troy or Apothecaries' Weight?

4. Name the table of EXTENSION IN LENGTH. Repeat the table of Long Measure. Draw a line that you think to be 1 inch long. Divide it into lines. Mark off one foot on your slate or paper. Measure the distance from your home to the school-house. What is a land league? How many English miles = 1° on the earth's surface? how many geographical? How is cloth usually measured? Give the denominations of Surveyors' Measure. Repeat the table. Denominations of Mariners' Measure;—the table? Which is longer, a land or sea league?

5. What are the denominations of SQUARE MEASURE? Repeat the table. Draw a square 1 inch each way;—$\frac{1}{2}$ inch each way. What part of the first square is the second? Difference between *5 square* inches and 5 inches square? Define a rectangle; a

9 3/10 gall Liquid measure in one Bushel

square. Show why you multiply the length of a rectangle by the breadth to obtain the surface. Can you multiply feet by yards? When the length of one side of a rectangle is given in feet, and the other in rods, how do you find the surface? When the square contents and one dimension are given, how do you find the other? Find the area of the top of your desk.

6. Give the denominations of CUBIC MEASURE;—the table Give the table for Wood Measure. Define a parallelopiped. How do you find its contents? Illustrate. When the solid contents and two dimensions are given, how do you find the third? How many faces has a cube? How many edges? How many cubic feet of air would your school-room contain if there were nothing else in it? *Suppose the average number of pupils who attend your school were shut up in the school-room without any means of ventilation, how long before they would all die, if each person should render* 20 *cubic feet of air per hour unfit to sustain life?*

7. Repeat the denominations of LIQUID MEASURE;—the table. What is the common size of barrels and hogsheads?

8. Repeat the denominations of DRY MEASURE;—the table. Which is the larger, 1 quart Liquid or 1 quart Dry Measure? 1 bushel Dry Measure = how many gallons Liquid Measure?

9. Where is CIRCULAR MEASURE used? Define circle; circumference; arc; radius; diameter; degree; minute; second; semi-circumference; quadrant; sextant; sign. Give the table. Define an angle; the vertex. Read the annexed angle, $e{<}^{d}_{f}$. How is an angle measured? Does the size of an angle depend at all upon the length of its sides? What is a right angle? How are its sides in regard to each other? What is an obtuse angle? an acute angle? How many right angles can you have at the centre of a circle? Draw a right angle; an obtuse angle; an acute angle; a circle; a semi-circle; an arc; a radius; a diameter; a sextant; a quadrant.

10. What is an astronomical day? a solar day? Which is the longer? How is the solar day divided? Give the denominations of *time*. Repeat the Table. Give the reason for leap

year; the rule. What are the seasons of the year, and how divided? Give the number of days in each calendar month. Repeat the helping lines. Repeat the table of numbers under the head of Miscellaneous; of Paper; of Books. What measures are sometimes used for animals?

11. How do you reduce a *fraction* of one denomination to whole numbers of lower denominations? How do you reduce whole numbers of lower denominations to the fraction of a higher?

12. What is COMPOUND ADDITION; SUBTRACTION; MULTIPLICATION; DIVISION? How do these operations differ from similar operations upon simple numbers? Give an example in each, and repeat the rule. How can you prove these operations?

13. How do you find the *difference of latitude* between two places upon the same side of the equator? upon different sides? Give, in your own words, a rule for finding the *difference of longitude* between any two places. Suppose two places are in different longitudes, and in your operation the number of degrees between them is found to exceed 180°, what will you do?

14. How do you *add or subtract denominate fractions*?

15. When the *difference of time* between any two places is given, how do you find the *difference of longitude*? When the *difference of longitude* is given, how do you find the *difference of time*? For places east of any given point, must the difference of time be added or subtracted to give the true time? For places west, what must be done?

213. MISCELLANEOUS EXAMPLES.

1. If coal is worth $\$9\frac{1}{2}$ a ton, what is the expense of a coal fire for a week, allowing it consumes 25 lbs. a day, coal being sold by the long ton?

2. What will be the cost of freighting 50 bbls. of flour, at \$.382 per bbl.?

3. A quantity of gold weighed 4 lb. 10 oz. 3 pwt. before refining, and 3 lb. 11 oz. 2 pwt. 9 gr. afterwards. What was lost in the process?

4. If £69 12s. be paid for 6 cwt. of tobacco, what is the price per pound?

5. An apothecary mixed 3 ℔, 10 ℥, 2 ʒ, 2 ℈, 14 gr.
1 ℔, 4 ℥, 1 ʒ, 2 ℈, 17 gr.
2 ℔, 7 ℥, 6 ʒ, 1 ℈, 13 gr.
and divided the mixture into 100 equal parts. What was the weight of each part? 7dr 1 sc 13 $\frac{6}{25}$ gr

6. What is the duty on 6 lb. 4 oz. of essence of lemon, at \$.50 per pound? \$3.12 $\frac{1}{2}$

7. What is the cost of 137 gall. 2 qts. of molasses at 12½ cts per quart? ~~\$3.12 $\frac{1}{2}$~~ \$68.75

8. If a bird fly 1° in 1 h. 8 m. 15 s., in what time will it fly round the world at the same rate? 2 w. 3 d. 1 h. 30 m.

9. How many times will a wheel 3 ft. 4 in. in circumference turn in crossing a bridge that is 40 rd. 1 yd. 2 ft. long? 199 $\frac{1}{2}$ times

10.* What will be the cost of 125 pieces of delaine, averaging 33 yards in length and 22 inches in width, at 25 cts. per yard in length, and 2 cts. per sq. yard for duties? *Ans.* \$1081.66⅔.

11. How many bushels will a bin contain which is 10 ft. long, 8 ft. wide, and 5 ft. deep? 321 $\frac{3}{5}$ bu

12. At \$6.00 a cord, what cost a pile of wood 33 ft. long, 8 ft. 10 in. high, and 4 ft. wide? \$54.66 $\frac{5}{8}$

13. Divide an arc of 15° 12′ 3″ by 7½. 2° 1′ 36 $\frac{2}{5}$″

14. Reduce ⅗ of a great gross to integers of lower denominations. 7 gr 2 doz 4 $\frac{4}{5}$ units

15. What will be the cost of fencing a lot of land 20 rods by 260 rods, at 12½ cts. per foot? \$1155

16. A farmer divided one half of his estate of 350 A. 3 R. 20 rd. equally between his two daughters, and the balance, after setting off 17¾ A., equally between his two sons. What was the share of each son and daughter? S = 78 A 3 R 15 rd D: 75

17. How many cords of wood in 25 loads, each measuring 1 cd. 1 cd. ft. 12 cu. ft.? 30 cd 3 cd ft 12 cu ft

18. What would be the cost of the above at \$4.50 per cord? 137.10

19. How many yards of carpeting 1½ yd. wide will cover a floor 18 ft. sq.? 24 yd

20. If a cotton mill can make 1200 yds. of cloth per hour, how many yards could be made by working 10 hours a day from July 7th to *January 4th, allowing* for 26 Sabbaths? 1,860,000 yd

21. Change 15 lb. 8 oz. Av. to pounds and ounces Troy.

22.* How many cakes of ice $1\frac{1}{2}$ ft. sq. by 1 ft. thick may be contained in a building measuring in the inside 105 ft. long, 60 ft. wide, 31 ft. 3 in. high? 87,508 cakes

23. How many bricks 8 in. by 4 in. will cover a court 75 ft. by 50 ft.? 16 875 bricks

24. How many sq. ft. does the surface of a box contain, which is 3 ft. long, 2 ft. wide, and 6 ft. deep? 72 sq ft

25. What is the price of 250 tons of lead at \$0.11 per pound?

26.* I have imported 1 T. 5 cwt. 1 qr. 15 lbs. of black lead, which cost me at New York \$200.00 a ton, and on which I have also paid \$10.00 a ton for duties; for what must I sell it per pound to gain \$150.00 on the lot, if I buy and import by the long ton? *Ans.* $\$.14\frac{14809}{22744}$.

27. At \$6.50 for two dozen pints of olive oil, what cost 1 qt.?

28.* What is the difference between 37 f. 8 rd. 0 yd. 1 ft. 3 in., and 37 f. 7 rd. 5 yd. 2 ft. 9 in.? None

29. I have sold $7\frac{1}{2}$ tons of chalk for \$75.30. What do I receive per pound? 4 $\frac{1}{50}$ mills

30.* A regiment of troops that enlisted for 9 months was not discharged till July 20th, 1863, which was 1 mo. 26 d. after the term of service had expired. When did they enlist? Aug 24, 1862

31. Divide 60 miles by 7, carrying out the quotient to the lowest denomination.

32. From a pile of wood 48 ft. long, 4 ft. high, and 4 ft. wide, was sold at one time 3 cd. 5 cd. ft.; at another 2 cd. 32 cu. ft.; what is the remainder worth at \$4 per cord?

33. What is the value of 7 cd. 7 cd. ft. and 5 cd. 112 cu. ft. of wood, at \$7 per cord? \$76.25

34. If a man saves 1 hour 50 minutes a day by habits of order, 1 hour 30 minutes by promptness in business, and half an hour by early rising, how much time is saved in 25 years of $365\frac{1}{4}$ days each? 3 yr 362 d 17 h

35. A floor 30 ft. by 12 ft. is to be covered with carpeting $\frac{5}{8}$ of a yard wide. Required the number of yards. 64 yd

36. Bought $7\frac{1}{2}$ tons of coal for \$75.30; what was the cost per ct.? 50 $\frac{1}{5}$ ct

37.* A single block of quartz in Australia is said to have yielded $32000 worth of gold. At $16 an oz. Troy, what was the weight of the metal in Avoirdupois? 1 cwt 1 qr 12 lb 2 oz 4 4/7 dr

38. Bought 5 oz. 7 pwt. 12 gr. of gold-foil at $30 per ounce; 3 oz. 15 pwt. of gold-plate at $18.75 per ounce; what was the amount of my bill? $231.56 1/4

39. How many cubic feet in the hold of a vessel which contains 2000 bushels of grain? 2488 8/9 cu ft

40.* Southampton is in longitude 1° 30′ W. New York is about 74° W. Would a passenger on arriving at New York from Southampton find his watch too fast or too slow, and by how much, if right for Southampton time? 4 h 50 min fast

41. Two vessels are 100° apart, and sailing toward each other; one sails 2° 50′ 2″ in a day, and the other 3° 10′ 45″ in the same time. How far apart will they be at the end of 10 days? 39° 50′

42. A steam frigate, sailing at the rate of $15\frac{2}{3}$ miles an hour, gives chase to a pirate vessel, $5\frac{2}{5}$ miles ahead, sailing at the rate of $14\frac{1}{2}$ miles an hour; in what time will the frigate overtake the pirate? *Ans.* $4\frac{4}{5}$ h.

43. How many feet of lumber in a piece of square timber 10 inches wide, 6 inches thick, and 9 feet long? *Ans.* 45 ft.

NOTE. — Lumber is considered 1 inch in thickness.

44. In 50000 feet of lumber how many cords, cord feet, and cubic feet? *Ans.* 32 cd. 4 cd. ft. $6\frac{2}{3}$ cu. ft.

45. How many cords and cord feet of wood can be put into a shed 8 ft. by 18 ft. 7 in., and 10 ft. 5 in. high? 12 cd 12

46. How many cords, and what will be the cost at $4.56 per cord, of wood in a pile $40\frac{1}{2}$ ft. long, 6 ft. high, and 8 ft. wide?

47. A man purchased 75 cords of wood for $360; he sold the following lots, 10 cd. 64 cu. ft., 15 cd. 80 cu. ft., and $11\frac{3}{4}$ cd., all at $5 per cord; what did he gain on what he sold? 57 1/2 dollars

48. What would be the cost of sawing the remainder of the 75 cords, if it is worth 25 cents to saw 2 cd. ft.? 34.12 1/2

49. How many barrels of 31 gallons each will be contained in a water tank 3 ft. square and 4 ft. 3 in. deep?

50. What was the cost per pound for lead, 5 lbs. to the sq ft, to line the above tank, if the whole cost $38⅔? $.12

51.* What part of 1 m. 5 f. is 2 m. 7 f.?

52. What is the area of a lot of land 25 chains long and 17 rods wide? 10 A. 2 R. 20 rd.

53. How many cubic inches in 2 bu. $1\frac{1}{2}$ pk. $2\frac{1}{2}$ qt. dry measure, and 5 gall. liquid measure? 6034 $\frac{1}{5}$ cu. in.

54. What is the bill for $\frac{1}{2}$ dozen silver spoons, each weighing 2 oz. 9 pwt. 12 gr. at $1.50 per ounce, and 2 T. 3 cwt. 2 qr. of iron at $3 per cwt.? 152.77$\frac{1}{2}$

55. When it is noon at London, what is the time in Lawrence, Mass., 71° 20′ W.? 7 H 15 min 3 $\frac{1}{3}$ sec A. M.

56.* What will it cost to paper a room 16 ft. 6 in. by 14 ft., and 7 ft. high, with paper ⅝ yds. wide, 8 yds. in a roll, $.75 a roll, bordering at the top of the wall being 3 cents a yard, and overlapping the paper by the width of the border, no allowance being made for windows and doors? *Ans.* $7.72⅔.

57.* How many rolls of paper, 20 inches wide and 8 yards long, will paper the walls of a room 18 ft. by 16 ft. and 10 ft. high, in which are two doors, each 6 ft. by $2\frac{1}{2}$ ft., and 4 windows, each 6 ft. by $2\frac{1}{2}$ ft.? 14$\frac{7}{9}$

58.* How many cubic yards of earth must be removed to dig a ditch 3 ft. wide and $2\frac{1}{2}$ ft. deep outside of a lot of land 40 rods by 38 rods, $10\frac{1}{2}$ ft. *Ans.* $724\frac{1}{6}$ yds.

NOTE.—In order that the ditch may entirely surround the land, twice its width, or 6 ft., must be added to either the length or width of the land. Adding it to the width, we have for the entire length of the ditch 2×40 rd. $+ 2 \times 39$ rd. $= 158$ rd. The pupil will see this more clearly by making a drawing of the land and ditch.

59.* How many bricks in the walls of a building 29 ft. long by 24 ft. wide and 30 ft. high, the walls being 2 ft. thick, and the bricks 8 in. by 4 in. by 2 in.? *Ans.* 158,760 bricks.

60. How much carpeting ¾ yd. wide will cover a block 3 ft. long, 8 inches wide, and 6 inches high?

61. Add $\frac{3}{7}$ of the month of February, 1860, to ⅛ of the days from February 25, 1861, to May 6, 1861.

62.* Which will cost more, and how much more, 15 times 2 cwt. 24 lbs. of lead at 2£ 4d. a cwt., or 15 lbs. $1\frac{1}{8}$ oz. silver at 5s. 9d. per ounce? Lead by 15£. 13s. 8d. $3\frac{1}{10}$ far.

63. Suppose a boat to be moved forward through a strait 1 mile in length, by steam 100 ft. a minute, by sail 25 ft. a minute, and by the current 30 ft. a minute; how long will it be in going the length of the strait? 34 m. $8\frac{27}{31}$ sec.

64. How long, if it were moving in the opposite direction, propelled only by steam? 7 m. $26\frac{5}{7}$ sec

65.* What is the difference between $\frac{1}{4}$ of \$10.50 and $\frac{1}{3}$ of $\frac{1}{4}$ of 7£ 6s. 10d.? (Ans. in \$.) \$. 336

66.* The Fitchburg railroad, 67 m. 6 f. 24 rd. $5\frac{28}{100}$ yd. long, was built for \$3540000; what was the cost per mile? \$52.190.

67.* How many paving-stones 6 in. by 8 in. will be required to pave a street 27 rods long by 50 ft. wide? 66825 stones

68.* A druggist bought 8 lbs. Dover's powder at \$2 per lb. Av., and sold it in separate powders, 7 grs. to a powder, at the rate of 4 for 6 cts.; what did he gain? *Ans.* \$104.

69.* An apothecary mixed 5 ℥, 1 ʒ, 2 ℈ of aloes, for which he paid \$1 a pound, with 7 ℥, 6 ʒ, 1 ℈, 12 gr. of rhubarb, for which he paid \$4 a pound, and made of the mixture into pills, which he sold in boxes, 75 grains in each box, for 25 cents a box; what does he gain? *Ans.* $\$17.80\frac{1}{24}$.

70.* In how many days will a locomotive, which makes two trips from Boston to Providence daily (the distance from B. to P. being 61 m. 6 f. 16 r.), run 5592 m. 1 f. 4 r.? $2\frac{12289}{18716}$ days

71.* How many cords of wood can be put into a building measuring on the outside 40 ft. by 31 ft. and 15 ft. high, the walls being 6 in. thick? 127 cd. 0 cd. ft. 14 cu. ft.

72.* What will be the cost, at $18\frac{1}{2}$ cents per cubic yard, for removing the earth to build a cellar 12 feet deep whose measurement inside of the wall, which is 3 ft. 4 in. thick, is 27 ft. long by 15 ft. wide? $\$49.47\frac{53}{81}$

73. What is the average width of a board whose edges are straight, the width being 1 ft. 7 in. at one end, and 1 ft. 9 in. at the other? 1 ft. 8 in.

74. What is the length of a board 1 ft. 8 in. wide, which contains $38\frac{1}{4}$ sq. ft.? 22 ft 11 $\frac{2}{5}$ in

75. Estimate the cost of feeding a pair of oxen through the winter of 1863 and 1864, if 1 ox weighed 1772 lbs. and the other 1431 lbs., and hay was $13.75 per ton, and the oxen were allowed $\frac{1}{50}$ of their weight in hay each day.

76. What is the length of a stick of timber which is 17 inches square, and contains 154 ft. 120 in. cu. measure? 76 ft 9

77. If a druggist sells 1 gross 2 doz. papers of bitters a day, how many will he sell from the 19th of Dec., 1859, to 15th Mar., 1860, deducting 12 Sundays? 7 gr. gr. 3 gross 6 doz

78. A man sold a sheep for $1\frac{1}{2}$£, a calf for $\frac{4}{5}$£, and a fowl for $\frac{5}{8}$s. $\frac{3}{4}$d.; what did he receive for them all? 2 £. 6 s. 5 d. 1 q.

79.* What is my tax on silver, consisting of a lot of spoons weighing 3 lbs. 10 oz. 9 dr. Avoirdupois; 1 doz. forks weighing 2 lbs. 8 oz. 5 dr., basket and other articles of plate weighing 2 lbs. 5 oz. 2 dr., at 3 cts. per oz. Troy, 40 oz. Troy being exempt?

Ans. 2.51\frac{3}{8}$.

80.* What will be the cost of bricks, at $7 per M., to construct the walls of a building 100 ft. by 40 ft., 36 ft. high, the walls being 15 in. thick, in which are two doors each 7 ft. by 3 ft. and 24 windows 3 ft. 3 in. by 6 ft. 6 in., the bricks being of the usual size, and no allowance made for mortar? $209.17

81.* What will the bricks cost to construct the walls and bottom of a cistern whose inside dimensions are 8 ft. by 8 ft. and 6 ft. deep, the walls and bottom to be 1 ft. thick?

82.* When snow is uniformly 6 inches deep, how many cubic feet are there on one acre of land? 21,780 cu ft

83.* What must be the depth of a ditch around a garden outside, 5 rods by 2 rods, the ditch 1 ft. wide, that the earth taken from the ditch may raise the surface 2 inches? *Ans.* 1 ft. $11\frac{8}{47}$ in.

84.* How many square feet in a walk around a garden inside and next to the fence, the garden being $27\frac{1}{2}$ rods long, $20\frac{1}{4}$ rods wide, the walk being 4 feet wide? *Ans.* 6239 ft.

85. When it is 7 o'clock P. M. at Boston, what is the time at St. Petersburg?

86.* Suppose a vessel to go up stream by the power of steam at the rate of 16 miles an hour, by sail at the rate of 4 miles, and to be set back by the current at the rate of 2 miles an hour; in what time will another, which is propelled forward at the rate of 25 miles an hour and is set back the same as the former, overtake her, if she starts 3 hours later? $10\frac{4}{5}$ h.

DUODECIMALS.*

214. Duodecimal Fractions, or **Duodecimals,** are fractions whose denominators are 12 or some integral power of 12. They may also be considered as a kind of compound numbers, the values of whose denominations vary by a uniform scale of 12.

Duodecimals are sometimes used in computing lengths, surfaces, and solids; but all examples in *mensuration* can be performed by the use of common or decimal fractions.

215. The denominations are feet, primes ($'$), seconds ($''$), thirds ($'''$), fourths ($''''$), fifths (^{v}), &c. The foot is considered the unit; primes are 12ths of feet; seconds 12ths of primes or 144ths of feet; thirds 12ths of seconds, 144ths of primes, or 1728ths of feet, &c. Hence, in length, inches are represented by primes, in surface by seconds, and in solids by thirds.

The marks which indicate the degree of the denominations are called **Indices.**

216. TABLE.

1 foot $= 12'$.	$1''' = 12''''$.
$1' = 12''$.	$1'''' = 12^{v}$.
$1'' = 12'''$.	etc. etc.

Units $\times$ primes, seconds, &c. $=$ primes, seconds, &c.

Primes ($\frac{1}{12}$s.) $\times$ primes ($\frac{1}{12}$s.) $=$ seconds ($\frac{1}{144}$s.)

Primes ($\frac{1}{12}$s.) $\times$ seconds ($\frac{1}{144}$s.) $=$ thirds ($\frac{1}{1728}$s.)

Primes ($\frac{1}{12}$s.) $\times$ thirds ($\frac{1}{1728}$s.) $=$ fourths ($\frac{1}{20736}$s.)

Seconds ($\frac{1}{144}$s.) $\times$ seconds ($\frac{1}{144}$s.) $=$ fourths ($\frac{1}{20736}$s.)

etc. etc.

* Optional.

217. *Duodecimals may be added, subtracted, multiplied, and divided like compound numbers, it being borne in mind that a unit of any denomination is* 12 *times one of the next lower denomination, and* $\frac{1}{12}$ *of one of the next higher.*

Illustrative Examples.

ADDITION.

$$\begin{array}{rrrr} 2 & 3' & 8'' & 4''' \\ 5 & 3' & 9'' & 2''' \\ 7 & 10' & 2'' & 10''' \\ \hline \end{array}$$

Ans. 15 ft. 5′ 8″ 4‴

SUBTRACTION.

$$\begin{array}{rrrr} 18 & 2' & 4'' & 3''' \\ 3 & 11' & 8'' & 4''' \\ \hline \end{array}$$

Ans. 14 ft. 2′ 7″ 11‴

MULTIPLICATION.

$$\begin{array}{rrr} 25 & 3' & 2'' \\ & & 7 \\ \hline \end{array}$$

Ans. 176 ft. 10′ 2″

DIVISION.

$$8\,)\,365 \quad 10' \quad 8'' \quad 4''' \quad 8''''$$

Ans. 45 ft. 8′ 10″ 0‴ 7⁗

Examples.

1. What is the sum of the contents of 3 blocks of granite containing severally 92 ft. 11′ 7″ 6‴, 484 ft. 1′ 9″, 472 ft. 6′? *Ans.* 1049 ft. 7′ 4″ 6‴.

2. If from a board measuring 31 ft. 7′, there be cut 19 ft. 11′ 4″ $9\frac{3}{5}$‴, what will remain? *Ans.* 11 ft. 7′ 7″ $2\frac{2}{5}$‴.

3. Required the contents of 5 blocks of marble, each containing 4 ft. 3′ 9″. *Ans.* 21ft. 6′ 9″.

4. There being 679 ft. 7′ 6″ of glass in 29 windows of equal size, how much glass does one window contain? *Ans.* 23 ft. 5′ $2\frac{20}{29}$″.

5. There are 3049 ft. 3′ 0″ 8‴ of glazing in my dwelling-house and two equal green-houses. My dwelling-house contains 679 ft. 7′ 6″ 9‴. What is the quantity of glass in each green-house? *Ans.* 1184 ft. 9′ 8″ 11‴ 6⁗.

218. It only remains to multiply and divide duodecimals by duodecimals. These operations can be easily performed, if we observe that *the index of the product of any two terms equals the sum of the indices of the terms themselves, and the index of the quotient of one term divided by another, equals the difference of*

the indices of the dividend and divisor. Thus, $2' \times 4'' = 8'''$; $12'''' \div 3''' = 4'$.

219. Multiplication.

Ill. Ex. What is the area of a floor measuring 12 ft. 3′ in length and 10 ft. 6′ in breadth?

Operation.	
12ft. 3′	
10ft. 6′	
6 1′ 6″	
122 6′	
128ft. 7′ 6″ *Ans.*	

$6' \times 3' = 18'' = 1' + 6''$. We write the 6″ and reserve the 1′ to add with the primes. $6' \times 12 = 72'$, which, with the 1′ reserved $= 73' = 6$ ft. $+ 1'$, which we write in their proper places. $10 \times 3' = 30' = 2$ ft. $+ 6'$. We write the 6′ and reserve the 2 ft. to add with the feet. 10×12 ft. $= 120$ ft.; 120 ft. $+ 2$ ft. $= 122$ ft. Adding the partial products we obtain for the answer 128 ft. 7′ 6″. Hence the

Rule for Multiplication of Duodecimals. *Beginning with the lowest denomination, multiply all the terms of the multiplicand by each term of the multiplier separately; divide each product by* 12 *(except when the product is feet); write the remainder, and reserve the quotient to add to the next product. Give to every term thus obtained an index equal to the sum of the indices of its two factors. The sum of the partial products will be the entire product.*

Examples.

1. Multiply 7 ft. 4′ by 5 ft. 2′. *Ans.* 37 ft. 10′ 8″.

2. Multiply 4 ft. 8′ 5″ by 3 ft. 4′. *Ans.* 15 ft. 8′ 0″ 8‴.

3. How many feet of boards will be required to construct 50 boxes 2 ft. 3′ long, 2 ft. 3′ wide, 1 ft. 11′ high, making no allowance for thickness of boards? *Ans.* 1368 ft. 9′.

4. Which will contain more, and how much more, a box 3 ft. 9′ by 1 ft. 6′ by 2 ft. 3′, or a box 2 ft. 6′ each way?
Ans. The latter by 2 ft. 11′ 7″ 6‴.

5. What will be the cost of polishing a piece of marble on one side and all the edges at $\$.33\frac{1}{3}$ per square foot, the marble being 3 ft. 7′ by 1 ft. 9′ and 1′ thick? *Ans.* $\$2.38\frac{71}{108}$.

6. How many cubic feet of masonry in a wall 16 rods long, 8 ft. 9′ high, and 2 ft. 2′ thick? *Ans.* 5005 ft.

7. What is the cost of laying two floors, each 16 ft. 8′ by 12 ft. 6′, at 18 cts. per sq. yd.? *Ans.* \$8.33$\frac{1}{3}$.

8. Find the price, at \$24 per thousand ft., of 3 boards measuring as follows: 17 ft. 11′ by 1 ft. 2′, 19 ft. 4′ by 1 ft. 11′, and 22 ft. 8′ by 1 ft. 9′. *Ans.* \$2.343.

9. How many feet, board measure, in 6 planks 2 in. thick, each 25 ft. 9′ long, 6′ wide? (See Art. 213, Ex. 43, *note.*)

Ans. 154$\frac{1}{2}$ ft.

220. Division of Duodecimals.

Ill. Ex. A plat of ground contains 65 ft. 0′ 7″; its width is 6 ft. 4′; what is its length?

Operation.

```
6 ft. 4′ ) 65 ft. 0′ 7″(10 ft. 3′ 3″, Ans.
           63     4′
           ----------
            1     8  7″ = 20′ 7″
                          19′ 0″
                          ----------
                           1′ 7″ = 19″
                                   19″ 0‴
                                   ------
                                    0  0
```

6 ft. in 65 ft. = 10, 10 × 6 ft. 4′ = 63 ft. 4′, which, subtracted from the dividend, gives a remainder of 1 ft. 8′ 7″ = 20′ 7″; 6 ft. in 20′ = 3′ 3′ × 6 ft. 4′ = 19′ 0″, which subtracted from 20′ 7″ leaves 1′ 7″; 1′ 7″ = 19″; 6 ft. in 19″ = 3″; 3″ × 6 ft. 4′ = 19″. Hence the

Rule for Division of Duodecimals. *Divide the highest term in the dividend by the highest term in the divisor; the quotient will be the first term in the answer. Multiply the entire divisor by that term, and subtract the product from the dividend. Divide as before, and thus proceed till all the terms of the dividend are divided. Should there be a remainder, it may be reduced to numbers of lower denominations and divided, or annexed to the quotient in a fractional form, having for its denominator the divisor expressed in units.*

Examples.

1. Divide 54 ft. 7′ 4″ 6‴ by 4 ft. 1′. *Ans.* 13 ft. 4′ 6″.

2. What is the width of a table, 4 feet 3′ long, which contains 14 ft. 2′? *Ans.* 3 ft. 4′.

3. How many feet of joist, 4 inches wide and 3 inches thick, allowing nothing for waste by sawing, can be made from a piece of timber 44 ft. 5′ long, 1 ft. 3′ wide, and 1 ft. 4′ thick?

Ans. 888 ft. 4′.

4. How many blocks of stone containing 1 ft. 11′ 5″ 6‴ can be sawed from a block containing 11 ft. 8′ 9″? *Ans.* 6 blocks.

5. What is the thickness of a block of granite, one of whose surfaces contains 75 ft. 10′ 8″, and whose solid contents are 107 ft. 6′ 1″ 4‴? *Ans.* 1 ft. 5′.

221. General Review, No. 4.

1. Reduce 7 £ 3 s. 6 d. to farthings.

2. Reduce 4876 gr. to lb., etc., Troy.

3. 4 ℔, 9 ℥, 7 ʒ, 2 ℈, 8 gr. + 3 ℔, 6 ℥, 2 ʒ, 2 ℈, 8 gr. = ?

4. 3 T. 1 cwt. 2 qr. 1 lb. — 1 T. 2 cwt. 3 qr. 7 lb. 8 dr. = ?

5. 1 m. — 6 f. 16 r. 3 yd. 1 ft. 8 in. = ?

6. Multiply 2 m. 30 ch. 12 l. by 8.

7. Multiply 5 y. 212 d. 10 h. 15 m. by 20. (365¼ days to the year.)

8. Divide 4 A. 3 R. 24 r. by 9.

9. In $\frac{5}{6}$ c. l. how many feet?

10. What part of 1 A. is 3 R. 13 r. 5½ ft.?

11. Reduce $\frac{5}{8}$ cu. yds. to feet and inches.

12. Reduce 8′ 53⅓″ to the fraction of a degree.

13. What cost 12 bu. 2 pks. of plums at $.06 a pint?

14. What cost 2 qts. 1⅓ pts. oil at $1.12 per gallon?

15. Required the number of square feet in a garden 4 rds long and 1 rd. 15 ft. wide.

16. How many cu. ft. of space in a cellar measuring on the inside of the wall 5 yd. 1 ft. in length, 4 yds. in width, and 10 ft in depth?

17. What is the difference of time in two places whose longitudes differ 7° 8′ 4″?

18. When the difference of time is 3 h. 4 m. 6 s., what is the difference of longitude between two places? 46° 1′ 30″

19. How many days from Jan. 5, 1864, to March 3, 1865? 42

☞ For changes, see Key.

DECIMAL FRACTIONS.

222. As by the Decimal System of representing numbers (Art. 23), each lower denomination is one tenth of the next higher, one ten being one tenth of one hundred, one unit one tenth of one ten, so one unit may be divided into ten equal parts, or tenths, one tenth into ten equal parts, or hundredths, etc. Thus we have fractional numbers descending from the unit by a *scale of tens.* Represented as common fractions, the denominators of these numbers are 10, 100 (10^2), 1000 (10^3), etc. Hence,

223. A **Decimal Fraction** is a fraction whose denominator is some integral power of ten.

224. Decimal Fractions are generally written like whole numbers; they are distinguished from whole numbers by having the decimal point placed at their left.

225. Decimal Fractions are read like whole numbers, the denomination being always given; this is determined by the place of the right hand figure in reference to the decimal point; thus,

.5	is read		5 tenths.
.05	"	"	5 hundredths.
.748	"	"	748 thousandths.
.0748	"	"	748 ten-thousandths.
7.48	"	"	7 and 48 hundredths.

226. Numeration Table.

7th.	Millions.	4
6th.	Hundred-thousands.	9
5th.	Ten-thousands.	7
4th.	Thousands.	6
3d.	Hundreds.	5
2d.	Tens.	4
1st.	Units.	2
	Decimal point.	.
1st.	Tenths.	8
2d.	Hundredths.	3
3d.	Thousandths.	0
4th.	Ten-thousandths.	4
5th.	Hundred-thousandths.	7
6th.	Millionths.	1
7th.	Ten-millionths.	5
8th.	Hundred-millionths.	9
9th.	Billionths.	2
10th.	Ten-billionths.	8
11th.	Hundred-billionths.	7
12th.	Trillionths.	6
	Etc.	

INTEGERS. FRACTIONAL NUMBERS.

EXERCISES UPON THE TABLE.

1. Which place at the right of the decimal point is occupied by tenths? by thousandths? by millionths? by billionths? by trillionths? by hundredths? by ten-thousandths? by hundred-thousandths? by ten-millionths? by hundred-millionths? by hundred-billionths?

2. What denomination occupies the second place at the right of the point? the third? the fourth? the fifth? the sixth? the first? the seventh? the ninth? the eighth? the eleventh? the twelfth? the fifteenth?

227. To read decimal fractions, observe the following

RULE. *Read the decimal fraction as if it were a whole number, giving it the denomination of the right hand figure.*

EXERCISES.

Read or write in words the following: —

1. .9.
2. .469.
3. .0599.
4. .05099.
5. .095009.
6. .37$\frac{1}{2}$.
7. .0345706.
8. .00080007.

Read the following, first as mixed numbers, then as improper fractions: —

9. 27.5.
10. 2.75.
11. 885.47533.
12. 7000.0005.
13. .7005.
14. 175.87$\frac{1}{2}$.
15. 250.0055$\frac{1}{7}$.
16. 2505.00$\frac{1}{3}$.

NOTE. — The word *units* may be placed after the 7000 in Ex. 12, in reading it as a mixed number, to distinguish it from the 7 thousand ten-thousandths in Ex. 13. Read thus in all similar cases of ambiguity.

Name the terms in the above examples, beginning at the left.

Ans. (Ex. 1) 9 tenths; (Ex. 2) 4 tenths, 6 hundredths, 9 thousandths; etc.

228. To write Decimal Fractions, observe the following

RULE. *Write the figures as in whole numbers, putting the decimal point so that the right hand figure shall be in the place of the denomination named in the decimal fraction, supplying vacant places, if there be any, with zeros.*

Exercises.

Write the following in figures: —

1. Sixty-four *hundredths.*
2. Nine hundred forty-two *thousandths.*
3. Nine hundred forty-two *ten-thousandths.*
4. Eight thousand three hundred twenty-five *ten-thousandths.*
5. Seventy-five *hundred-thousandths.*
6. Seven thousand five *hundred-thousandths.*
7. Fifty and four hundred eighty-two *thousandths.*
8. One hundred fifty-five *millionths.*
9. One hundred units, and fifty-five *millionths.*
10. Three hundred thousand eight *billionths.*
11. Three hundred thousand units, and eight *billionths.*
12. Forty million eight hundred four thousand and twenty-five, and three hundred four thousand eight hundred seventy-five *hundred-millionths.*
13. Seven million units, and one *ten-millionth.*
14. Seven million and one *ten-millionths.*
15. Thirty and six *tenths.*
16. Three hundred six *tenths.*
17. Three hundred seventy and $\frac{2}{5}$ *ten-thousandths.*
18. Four hundred seven thousand eight hundred seventy-five and $\frac{3}{7}$ *ten-billionths.*

Note. — Zeros may be annexed or omitted at the right of a decimal fraction without altering the value of the fraction, for both numerator and denominator are thereby multiplied or divided by the same number. (Art 119, Prop. iii., iv.) Thus, $.50\ (\frac{50}{100}) = .5\ (\frac{5}{10}) = .500\ (\frac{500}{1000})$.

Fundamental Operations.

229. Decimal Fractions may be written and operated upon like common fractions, the same principles being applicable to both; but as they increase and decrease, like whole numbers, by a *scale of tens,* they can also be treated in all respects like whole numbers. Close attention must be given to placing the decimal point.

230. Addition.

To add Decimal Fractions, observe the following

Rule. *Place the figures of the same denomination under each other; then add as in whole numbers, observing to place the decimal point in the amount under those in the example.*

Examples.

1.	2.	3.
6782.2	8.752	5.3125
298.98	975.84	807.06848
4400.64	35.075	9.0875
3034.05	780.136	975.00625
Ans. 14515.87	1799.803	1796.47473

4.	5.	6.
1482.9	.594	875325.075
29.7868	8.594	8753.25075
668.47	3.75	87.5325075
4872.001	.674	875.325075
8569.8456	600.044	8.75325075
762.4	600.00449	39.07528
6847.9773	85.8585	39075.28
9320.7685	30.5	3907.528
32654.1492	1330.81879	928071.51966393

7. 8.75 + 90.095 + 840.6007 + 4 + 67304.745 + 190075.40007 + 4006.87 + 475.44 = what? 262,805.90077

8. 4 hundred, and 847 *thousandths* + 9 thousand 875 and 4 *thousandths* + 3 hundred 7 and 3 hundred 7 *ten-thousandths* + 6 thousand 200 units, and 62 *ten-thousandths* = what? 16782.8874

9. 9 hundred units, and 9 *hundredths* + 9 thousand 874 and 9 thousand 874 *ten-thousandths* + 987, and 49 thousand 874 *hundred-thousandths* + 9, and 8 million 749 thousand 874 *ten-millionths* + 98 thousand 749, and 874 *thousandths* + 62 thousand units and 62 *thousandths* = ? 172622.38871274

10. 205 *thousandths* + 1 thousand, and 1 thousand 5 *ten-thousandths* + 9 hundred 4 *hundred-thousandths* + 9 million 407 thousand units, and 327 *hundred-thousandths* + 3 thousand 27, and 4 *hundredths* = ? 9411027.55781

231. Subtraction.

To subtract decimal fractions, observe the following

Rule. *Write the subtrahend beneath the minuend, units under units, tenths under tenths, etc.; subtract as in whole numbers, placing the decimal point in the remainder under those in the minuend and subtrahend.*

Examples.

	1.	2.	3.
From	756.875	56.8507	6.005
Take	97.486	38.193	.02983
	Ans. 659.389	18.6577	*Ans.* 5.97517

Note. — In the 3d example, as there are no hundred-thousandths to subtract from, and no ten-thousandths, we reduce one of the thousandths to ten-thousandths, and one of the ten-thousandths to hundred-thousandths, and subtract.

4. From 132.0064 take 123.887. 8.1194
5. Find the difference between 30.801 and 308.01. 277.209
6. 275.87 — 37.15956 = ? 238.71044
7. From 2 hundred units and 5 thousandths, take 209 thousandths. 199.796
8. The subtrahend being 784, and 20 thousand 456 hundred thousandths, and the minuend 906, and 34 hundredths, required the remainder. 122.13544
9. From two thousand take two thousandths. 1999.998
10. Subtract 24073 thousandths from 24, and 73 thousandths. 0
11. What is the value of 45 million, minus 45 millionths? 44 999 999.999955

232. Addition and Subtraction.

1. The difference between two numbers is 67.97, the less being 9874.08; what is the greater? 9942.05
2. The difference between two numbers is 29.875, the larger being 1909; required the smaller. 1879.125
3. The compound interest of a certain sum being $1.4416, exceeds the simple interest by $.91375; what is the simple interest? .52785

4. From 64.0125 + .09778 take 64.0125 — .09778. .19556

☞ For Dictation Exercises, see Key.

233. Multiplication and Division by 10, 100, 1000, etc.

Since numbers increase from right to left by a *scale of tens*, and decrease from left to right in the same manner, it follows that

Any decimal number, whether a fraction or a whole number, may be multiplied by 10, 100, *or any power of* 10, *by removing the decimal point as many places towards the right as there are zeros in the multiplier.*

Thus, $.3 \times 10 = 3.$; $.3 \times 100 = 30.$; $.225 \times 100 = 22.5$.

234. It follows, also, that

Any decimal number, whether a fraction or a whole number, may be divided by 10, 100, *or any power of* 10, *by removing the decimal point as many places towards the left as there are zeros in the divisor.*

Thus, $7 \div 10 = .7$; $.7 \div 100 = .007$; $78.4 \div 1000 = .0784$.

Illustrative Examples.

Multiply 50.7 by 10; 4.75 by 100; 13.57 by 1000; .375 by 10; and give the sum of the products.

Operation.

$$\begin{aligned} 50.7 \times 10 &= 507. \\ 4.75 \times 100 &= 475. \\ 13.57 \times 1000 &= 13570. \\ .375 \times 10 &= 3.75 \\ \hline \text{Sum of Products, } &14555.75 \end{aligned}$$

Divide 58 by 10; 4.7 by 100; 83.2 by 1000; 18470 by 1000; and give the sum of the quotients.

Operation.

$$\begin{aligned} 58. \div 10 &= 5.8 \\ 4.7 \div 100 &= .047 \\ 83.2 \div 1000 &= .0832 \\ 18470. \div 1000 &= 18.470 \\ \hline \text{Sum of Quotients, } &24.4002 \end{aligned}$$

235. Examples.

Add the following:

1. 4.75×100; 5.84×10; 463×10. *Ans.* 5163.4.
2. $.031 \times 1000$; 76.218×100; 4.0005×1000; $.000987 \times 100000$. *Ans.* 11752.
3. $74.7 \div 10$; $16.75 \div 10$; $87 \div 100$; $1324 \div 1000$. *Ans.* 11.339.

4. $756.7 \div 1000$; $20.09 \div 100$; $1800 \div 100$; $175.005 \div 10$; $397000 \div 10000$; $.5 \div 1000$.

NOTE. — Retain the separate results in each of the following examples, and find their sum.

5. Divide 182 by 10; multiply that result by 100; divide that by 1000; multiply that by 10, and that by 10.

OPERATION.

$$182 \div 10 = 18.2;\ 18.2 \times 100 = 1820.$$
$$1820 \div 1000 = 1.82;\ 1.82 \times 10 = 18.2;\ 18.2 \times 10 = 182.$$
$$18.2 + 1820. + 1.82 + 18.2 + 182 = 2040.22, \textit{Ans.}$$

6. Divide 796 by 10; divide that result by 100; multiply that by 10. Sum, 88.356.

7. Divide 8394 by 10; take $\frac{1}{100}$ of the quotient; 1000 times that result; $\frac{1}{100}$ of this product; $\frac{1}{10}$ of this; divide this by 1000, and take 100 times this quotient. Sum, 9334.975794.

8. Multiply .648 by 100; divide the product by 10; multiply the quotient by 1000; take $\frac{1}{2}$ of $\frac{1}{10}$ of that product, and $\frac{1}{3}$ of $\frac{1}{100}$ of this. Sum, 6876.36.

236. MULTIPLICATION.

ILL. EX., I. Multiply 1.87 by .5.

OPERATION.

```
     1.87
       .5
Ans. .935
```

If 1.87 be multiplied by 5, the product will be of the same denomination as the multiplicand, or 9.35; but since the multiplier is 5 tenths, a number but one tenth as large as 5, the product will be but one tenth as large, and the decimal point must be put one place farther to the left, making the answer .935.

ILL. EX., II.

```
Multiply  .012
by        .13
Ans.  .00156
```

.012 multiplied by 13 = 156 thousandths (.156); .012 multiplied by .13, a number one one-hundredth as large as 13, will give a product one one-hundredth as large, or .00156.

From these illustrations we derive the following

RULE. *Multiply as in whole numbers, and point off as many places for decimal fractions in the product as there are places of decimal fractions in both the factors. If there are not figures enough in the product prefix zeros.*

Examples.

1. $.92 \times 5.6 = ?$ *Ans.* 5.152.
2. $9.72 \times .87 = ?$ *Ans.* 8.4564.
3. $.687 \times .038 = ?$ 026106
4. $95.874 \times 4.007 = ?$ 384.167118
5. $308. \times .0063 = ?$ 1.9404
6. $.000001 \times 1000000 = ?$ 1.
7. $4.02 \times 400.02 \times 402.01 =$ what? 646464.401804
8. What is the cost of whitewashing the ceiling of a room 18.75 yds. long, 10.82 yds. wide, at \$.015 per sq. yd.? 3.043
9. What cost 11 thousand bricks, at \$12.3175 per M.? 135.492
10. What cost 635 laths, at \$.276 per hundred? 1.7526
11. What is the cost of tiling a roof 289 feet long and 54 feet wide, at \$8.25 per hundred feet. 1287.495
12. What will be the cost of shingling the above roof with shingles which cost \$6.50 per thousand feet, the shingles lying one third to the weather? 304.317

☞ For Dictation Exercises, see Key.

237. Division.

Ill. Ex., I. Divide 3.864 by 12.

```
OPERATION.
12 ) 3.864
Ans.  .322
```

Since, in the above example, we divide 3864 ~~3684~~ thousandths into 12 equal parts, it is evident that the quotient will be thousandths, and require three decimal places. Therefore, *when the divisor is a whole number, the quotient must be of the same denomination as the dividend.*

Ill. Ex., II. Divide 1.224 by .36.

```
      OPERATION.
.36.)1.22.4 ( 3.4, Ans.
     1 08
       144
       144
       000
```

Here the divisor is not a whole number, but hundredths. It may be made a whole number by removing the decimal point two places to the right. If we also remove the decimal point in the dividend two places to the right, the divisor and dividend will be equally multiplied, and the quotient resulting from the division will be the same as if no alteration had been made (Art. 119, Prop. 111). The dividend now being divided by a whole number, the quotient must be of the same denomination as the altered dividend, or tenths.

From the above illustrations we derive the following

RULE. To divide decimal fractions: *Divide as in whole numbers. If the divisor is a whole number, point off as many decimal places in the quotient as there are decimal places in the dividend. If the divisor is not a whole number, make it a whole number before dividing, by removing the decimal point to the right. Remove the decimal point in the dividend as many places to the right; divide, and point off as many decimal places in the quotient as there are in the altered dividend.*

NOTE I. — When there is a remainder after all the figures in the dividend are exhausted, zeros may be annexed, and the division continued. In pointing off, the annexed zeros must be considered as places in the dividend.

NOTE II. — In the examples in this book, when there is a remainder, the quotient mav be continued to the fifth decimal place, if no other direction is given.

EXAMPLES.

1. 14.91 ÷ 7 = ? *Ans.* 2.13.
2. .072 ÷ 6 = ? *Ans.* .012.
3. 8.25 ÷ 1.5 = ? *Ans.* 5.5.
4. 3.24 ÷ .81 = ? 4.
5. .00468 ÷ .013 = ? .36
6. 5446.776 ÷ 8 = ?
7. 180.375 ÷ 1.625 = ? 111.
8. 579 ÷ .075 = ? *Ans.* 7720.
9. 6.9705 ÷ .45 = ?
10. .0033 ÷ .011 = ? .3
11. 1.29 ÷ .32 = ? *Ans.* 4.03125.
12. .705 ÷ 7.5 = ? *Ans.* .094.
13. 3 ÷ 29.9 = ? *Ans.* .10033+.
14. 20 ÷ .013 = ? *Ans.* 1538.46153+.
15. 4066.2 ÷ .648 = ?
16. 68077 ÷ 71.66 = ? *Ans.* 950.
17. .880351 ÷ 897 = ?
18. .1706 ÷ 4.2368 = ?
19. 56.28 ÷ .0056 = ?
20. 10588.1 ÷ .4606 = ?
21. .417196 ÷ 58.76 = ?
22. .08 ÷ 1.611 = ?
23. 24000 ÷ 1.1713 = ?
24. 1.3 ÷ 197.59 = ?
25. 828.45 ÷ 26.3719 = ?
26. 25.25 ÷ 42993.78 = ?
27. 1203.488 ÷ 28.6 = ?
28. 49.2654756 ÷ .0759 = ?
29. 2464.176 ÷ 57.2 = ?
30. 164.6156 ÷ 1334 = ?
31. .07991997 ÷ 83497 = ?
32. 20339.82009 ÷ 1.07001 = ?

33.* Divide 93.75 by 3265096.575, and give three significant figures in the quotient.

34.* Find the product of the quotients of the following to 6 places: $.65084958 \div 3.69$; $40 \div 5000$.

35.* What is the quotient of $1.497 \div (260.401 - 13.02)$?

36.* Required the product of the quotients of the following: 1021 ten millionths $\div$ 107 ten thousandths; 2012 millionths $\div$ 1.006.

37.* Divide 600 by .006, multiply the quotient by .05, and by that product divide .005.

38.* $(1 \div .002) \times (.2 \div 50) = ?$ 2.

39.* $(80.481825 \div 89.325) \times (9617.5168 \div .47896) = ?$

40.* Required the product of the sum and difference of the following: $856494 \div 839.7$; $.0094658 \div 9.4$. (To 6 places.)

41.* Divide the difference of the above quotients by their sum.

☞ For Dictation Exercises, see Key.

238. Reduction of Common Fractions to Decimal Fractions.

Ill. Ex. Reduce $\frac{7}{8}$ to a decimal fraction.

Operation.	
8) 7.000	$\frac{7}{8} = \frac{1}{8}$ of 7; $\frac{1}{8}$ of 7 = no whole ones, with a remainder of 7, which reduced = 70 tenths (7.0); $\frac{1}{8}$ of 70 tenths = .8 with a remainder of .6; .6 = 60 hundredths; $\frac{1}{8}$ of .60 = .07 with a remainder of
.875, *Ans.*	

.04; .04 = 40 thousandths; $\frac{1}{8}$ of .040 = .005; $\therefore \frac{7}{8} = .875$. Hence the

Rule. To reduce a common fraction to a decimal fraction: *Annex zeros to the numerator, and divide it by the denominator. Point off as many decimal places as there are zeros annexed.*

Examples.

Reduce to decimals,

1. $\frac{3}{8}$. *Ans.* .375.
2. $\frac{7}{20}$. *Ans.* .35.
3. $\frac{3}{125}$. *Ans.* .024.
4. $\frac{1}{3125}$.
5. $\frac{49}{16}$. *Ans.* 3.0625.
6. $5\frac{1}{8}$. *Ans.* 5.125.
7. $\frac{641}{32}$.
8. $1\frac{5}{64}$.
9. $8\frac{3}{4}$. *Ans.* 8.75.
10. $17\frac{15}{128}$.
11. $1.00\frac{3}{25}$. *Ans.* 1.0012.
12. $.0\frac{1}{80}$. *Ans.* .00125.
13. $\frac{1}{6}$. *Ans.* .166+.
14. $\frac{8}{33}$.
15. $\frac{5}{13}$.

16. Reduce to decimals, and add, $\frac{7}{4}$, $\frac{8555}{15625}$, $3\frac{1}{400}$.

17. Reduce to seven places, and add, $1.82\frac{3}{40}$, $.009\frac{7}{16}$, and $.10\frac{15}{256}$.

☞ For Dictation Exercises, see Key.

239. Reduction of Decimal Fractions to Common Fractions.

Ill. Ex. Reduce the following to common fractions: .75; .0125 and 6.25

Operation.

$$.75 = \tfrac{75}{100} = \tfrac{3}{4}, \textit{ Ans.}$$
$$.0125 = \tfrac{125}{10000} = \tfrac{1}{80}, \textit{ Ans.}$$
$$6.25 = 6\tfrac{25}{100} = 6\tfrac{1}{4}, \textit{ Ans.}$$

Hence the

Rule. To reduce a decimal fraction to a common fraction: *Represent the decimal fraction in the form of a common fraction having for its denominator* 1 *with as many zeros annexed as there are decimal places in the decimal fraction, and reduce the common fraction to its lowest terms.*

Examples.

Reduce to common fractions,

1. .0625. *Ans.* $\frac{1}{16}$.	4. 4.0875. *Ans.* $\frac{327}{80}$.	7. $3.1\frac{1}{4}$.
2. .0025.	5. $.08\frac{1}{3}$. *Ans.* $\frac{1}{12}$.	8. $1.0\frac{5}{12}$.
3. .00064.	6. $.15\frac{1}{25}$.	9. $1.0068933\frac{1}{7}$.

☞ For Dictation Exercises, see Key.

240. To add or subtract decimal fractions terminated by common fractions: *Reduce all the decimals to the same denomination; then add or subtract as by Art.* 143 *and* 144; thus, $.3\frac{1}{3} + .83\frac{1}{2} =$ what? $.3\frac{1}{3} + .83\frac{1}{2} = .33\frac{2}{6} + .83\frac{3}{6} = 1.16\frac{5}{6}$, *Ans.*

Examples.

1. Add $.087\frac{1}{2}$, $9.0\frac{1}{3}$, $.7\frac{1}{2}$, $275\frac{3}{8}$, and $.0\frac{1}{11}$. *Ans.* $285.2549\frac{8}{33}$.
2. Add $19.37\frac{1}{2}$, $10.0\frac{8}{15}$, and $.0416\frac{2}{3}$.
3. Subtract $.05555\frac{5}{9}$ from $.3333\frac{1}{3}$.
4.* Subtract $1.2076244\frac{4}{7}$ from $1\frac{6}{13}$.
5.* $.3\frac{1}{3} + .6\frac{2}{3} + .83\frac{1}{3} + .2857142\frac{2}{7} + .5714285\frac{5}{7} + .63\frac{7}{11} = ?$

241.* Circulating Decimals.

If the denominator of a common fraction (when the fraction is in its lowest terms) contains any prime factor besides 2 and 5, the fraction is not capable of being entirely reduced to a decimal form.

In reducing such fractions, if the division be continued, the same figures will recur again and again in the decimal fraction. These fractions are called **Repeating** or **Circulating Decimals.** The figures which repeat are called a **Repetend.**

A Repetend is distinguished by two *dots* written over the first and last of the figures that repeat; thus, $\frac{11}{37}=.297297+=.\dot{2}9\dot{7}$.

242. Examples.

Reduce to decimal fractions,

1. $\frac{1}{3}$. *Ans.* $.3\frac{1}{3}$ or $.\dot{3}$.	4. $\frac{2}{7}$, $\frac{5}{9}$, $\frac{11}{12}$. $\frac{2}{7}=.\dot{2}8571\dot{4}$
2. $\frac{2}{3}$. *Ans.* $.6\frac{2}{3}$ or $.\dot{6}$.	5. $\frac{4}{7}$, $\frac{7}{18}$, $\frac{1}{22}$. $\frac{5}{9}$ $.\dot{5}$
3. $\frac{5}{6}$. *Ans.* $.83\frac{1}{3}$ or $.8\dot{3}$.	6. $\frac{7}{11}$, $\frac{1}{24}$, $\frac{2}{13}$. $\frac{11}{12}=.91\dot{6}$

243. Reduction of Circulating Decimals to Common Fractions.

It can be proved that the Repetend of a Circulating Decimal equals a fraction whose numerator is the repetend, and whose denominator is as many 9's as there are places in the repetend. Hence the

Rule. To reduce a Circulating Decimal to a common fraction: *Express the repetend as a common fraction having as many 9's for the denominator as there are figures in the repetend, and reduce. If any part of the decimal fraction does not repeat, annex the reduced repetend to it, and change the complex fraction thus obtained to a simple fraction.*

Note. — Circulating decimals may be added, subtracted, multiplied, and divided, by first reducing them to common fractions. Other processes might here be given, but the reasoning is too abstruse for an elementary treatise.

Ill. Ex., I. Reduce $.\dot{0}\dot{9}$ to a common fraction.

Operation. $$.\dot{0}\dot{9}=\frac{9}{99}=\frac{1}{11},\ Ans.$$

Ill. Ex., II. Reduce $.1\dot{6}$ to a common fraction.

Operation. $$.1\dot{6}=\frac{1\frac{6}{9}}{10}=\frac{1\frac{2}{3}}{10}=\frac{1}{6},\ Ans.$$

Examples.

Reduce the following to common fractions:

1. $.\dot{6}$.	*Ans.* $\frac{2}{3}$.	4. $.\dot{4}2857\dot{1}$. $\frac{3}{7}$
2. $.8\dot{3}$.	*Ans.* $\frac{5}{6}$.	5. $.\dot{7}1428\dot{5}$. $\frac{5}{7}$
3. $.\dot{1}88\dot{1}$		6. $.2\dot{1}4285\dot{7}$. $\frac{3}{14}$

244. To Reduce Compound Numbers to Decimal Fractions of Higher Denominations.

Ill. Ex., I. Reduce 2 d. 3 qr. to the decimal of a shilling.

OPERATION.

$$\begin{array}{r|l} 4 & 3.00 \text{ qr.} \\ \hline 12 & 2.75000 \text{ d.} \\ \hline & .2291\dot{6} \text{ s., } Ans. \end{array}$$

Since 4 qr. equal 1 d., there will be $\frac{1}{4}$ as many d. as qr., or $\frac{3}{4}$ d., which equals .75 d.; this, with the 2 d. given, equals 2.75 d.; since 12 d. equals 1 shilling, there will be $\frac{1}{12}$ as many shillings as d., &c.

Ill. Ex., II. What is the value of 3 rds. 4 yds. 2 ft. in the decimal of a rod?

OPERATION.

$$\begin{array}{r|l} 3 & 2.00000 \text{ ft.} \\ \hline 5\frac{1}{2} & 4.66666 + \text{yd.} \\ 2 & \quad 2 \\ \hline 11 & 9.33333 + \text{half yd.} \\ \hline & 3.84848 + \text{rods, } Ans. \end{array}$$

Since 3 ft. equal 1 yd., there will be $\frac{1}{3}$ as many yds. as feet, or $\frac{2}{3}$ yds., which equals $.\dot{6}$ yds.; this, with the 4 yds. given, equals $4.\dot{6}$ yds.; since $5\frac{1}{2}$ yds. equals 1 rod, there will be $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$ as many rods as yds., &c.

From the above, we deduce the following

Rule. To reduce compound numbers to decimal fractions of higher denominations: *Divide the number of the lowest denomination by what it takes of that denomination to make one of the next higher; place the quotient as a decimal fraction at the right of that higher; so continue till all the terms are reduced to the denomination required.*

Examples.

1. Reduce 7 d. 3 qr. to the decimal of a £.

Ans. £.03229+

2. Reduce 3 da. 22 h. 4 m. 48 sec. to the decimal of a week.

Ans. .56 wk.

3. Reduce 5 cwt. 3 qr. 10 lb. to the decimal of a ton. .2925 ton
4. Reduce 5 cord ft. 12 cu. feet to the decimal of a cord. .7187
5. Reduce 10 oz. 5 pwt. 12 gr. to the decimal of a pound. .8562
6. Reduce 80 cu. ft. to the decimal of a cord. .625 cord

7. What is the value of 2 fur. 7 rd. 10 ft. expressed in the decimal of a mile? .273769 - mile

8. What part of a ream is 15 quires 12 sheets? .775 ream

9. What part of an acre is 3 R. 15 rd. 6 yd. $8\frac{3}{4}$ ft.? .84619

10. Reduce 7 S. 8° 5′ 38″ to the decimal of a great circle. .60582 - gr. cir.

☞ For Dictation Exercises, see Key.

245. To Reduce Decimal Fractions of Higher Denominations to Whole Numbers of Lower Denominations.

Ill. Ex. Reduce .13125 lbs. Troy to oz., &c.

Operation.

```
lb. .13125
        12
----------
oz. 1.575
       20
----------
pwt. 11.5
       24
----------
gr. 12.
```

Since 12 oz. = 1 lb., there will be 12 times as many ounces as pounds, = 1.575 oz.; since 20 pwt. = 1 oz., there will be 20 times as many pwt. as ounces, = 11.5 pwt.; since 24 gr. = 1 pwt., there will be 24 times as many grains as pwt., = 12 gr. *Ans.* 1 oz., 11 pwt., 12 gr. Hence the

Rule. To reduce decimal fractions of higher denominations to whole numbers of lower denominations: *Multiply the decimal fraction by what it takes of the next lower denomination to make a unit of the denomination of the given decimal, pointing off as in multiplication of decimals; so continue till the number is reduced as low as is required.*

Examples.

Reduce to whole numbers of lower denominations,

1. .8975 of a week. *Ans.* 6 d. 6 h. 46 m. 48 s.
2. 5.624 £.
3. .0074623 lb.
4. .7587565 hhd.
5. .375 of a fathom.
6. $1.0004\frac{1}{2}$ of a bushel.
7. $.319\frac{3}{8}$ of a bbl. (31 gall.)
8. .578 cord.
9. .0756 of a degree.
10. 2.834 of 1 solid yard.
11. .086 of a Julian year.

☞ For Dictation Exercises, see Key.

246. Questions for Review.

What are Decimal Fractions? How are they generally written? how read? How distinguished from whole numbers? Which figure indicates the denomination? What is the name of the first place at the right of the point? of the second? third? fourth? fifth? sixth?

Which is the place of thousandths? of millionths? of billionths? of trillionths?

Give the rule for reading a decimal fraction.

Read 7.05 as a mixed number; as an improper fraction.

Read .20 and .21 so that they may be distinguished. Read .504 and 500.004.

Is the value of a decimal fraction altered by annexing ciphers? What is changed? Why does the value remain the same? What is the effect of placing a cipher between the decimal fraction and the point?

Give the rule for writing decimal fractions. Rule for Addition; for Subtraction; for multiplying by 10, 100, 1000, &c.; for dividing by 10, 100, &c.; general rule for multiplication.

Illustrate the rule by an example, and give the reason for pointing off.

Give the rule for division of decimals. Perform an example to illustrate the rule, and explain. When the dividend does not contain the divisor what must be done?

Rule for reducing common fractions to decimals. Illustrate and explain.

Rule for reducing a decimal to a common fraction. Illustrate and explain.

What fractions cannot be reduced wholly to the decimal form? What are they called?

What are the repeating figures called? How is a repetend distinguished?

Rule for reducing circulating decimals to common fractions.

Rule for reducing a compound number to decimals of higher denominations. Illustrate.

Rule for reducing decimals to whole numbers of lower denominations. Illustrate.

247. Miscellaneous Examples.

1. What is the amount of 3.75 tons, .085 tons, 1.17$\frac{5}{8}$ tons, and 7 cwt. 3 qr.? 5.39876 tons

2. What will be the interest on \$585, for 6 days, if the interest on \$1 be \$.001? \$.685

3. What is the amount of \$75823 for 7 y. 3 m. 15 d., if the amount of \$1 for the same time be \$1.510416$\frac{2}{3}$? 114,524.322916

4. At \$1.33$\frac{1}{3}$ a pair, how many cases of shoes, of 63 pairs each, can be bought for \$936? 11$\frac{1}{7}$

5. How many acres of land in a lot which is 105 rd. 4 yd. 1$\frac{1}{5}$ ft. long, and 100.356 rd. wide?

6. Required the price of three boards at \$.03$\frac{1}{2}$ per sq. ft., the boards being of the following dimensions: 17.75 ft. by 1 ft. 3 in.; 15 ft. 10 in. by 1.37$\frac{1}{2}$ ft., and 13.5 ft. by .916$\frac{2}{3}$ ft.

7. What is the amount due for the following?

5200	ft. of boards at		\$20 per M.	
700$\frac{1}{2}$	ft. "	"	"	22.50 per M.
94	ft. "	"	"	36 "

8. If 3 hhd. 42 gall. 2$\frac{2}{5}$ qt. of molasses cost \$92.64, what is the price per hhd.?

9. What is the cost of board for 7 y. 10 m. 18 d., at \$200 per year?

What cost

10. 9 gall. 3.4 qt. of vinegar, at \$.12$\frac{1}{2}$ per gall.?
11. 30 ch. 1 rd. 15 l. of a canal, at \$3550 per mile?
12. 5 bu. 2 pk. 3 qt. of wheat, at \$1.25 per bu.?
13. 47 gross, 10 doz. pens, at 4 s. 6d. per gross?
14. 5 lb. 7 oz. 6 pwt. 7 gr. of gold, at \$16.30 per oz.?
15. 17$\frac{3}{4}$ yds. of ribbon, at \$.19 per yd.?
16. 12$\frac{1}{2}$ doz. chairs, at \$1.90 apiece?
17. A road 9 m. 3 fur. 12$\frac{1}{2}$ rd. long, at \$2475 per mile?
18. 12520 oranges, at \$2$\frac{3}{8}$ per hundred?
19. 3 cwt. 40 lb. herring, at 12 s. 6d. per cwt.?
20. At \$4 per bu. how many bu., pk. and qt. can be bought for \$15.37$\frac{1}{2}$?

21. At 1 s. 9 d. per lb. what cost 3580.5 lb. hides? $138.53.10

22. If 5 lb. 5 oz. of beef cost $\$.564\frac{29}{64}$, what is the price per lb.? $.10 $\frac{4}{5}$

23. Required the cost of 19 gall. 3 qt. 1 pt. of oil, at 2 s. 6 d. per gall. 2£ 9s. 8d. 1qr

24 If 25375 feet of boards cost \$240.555, what is the price per M.? $9.48

25. What is the cost of 7 ℥, 5 ʒ, 2 ℈, of medicine, at \$.96 per lb.? .61

26. How many cords in a load of wood, 6.5 ft. long, 4.8 ft. wide, and 3.2$\frac{1}{2}$ ft. high? .7921 $\frac{3}{8}$ cords

27. How many casks gauging 10.485 gall. can be filled from a hogshead gauging 83.88 gall.? 8 casks

28.* What will be the cost per sq. yd. if \$157.675 are paid for laying 4 pieces of sidewalk, measuring as follows: 40$\frac{2}{3}$ ft. by 4 ft., 75 ft. by 7.84 ft., 8 ft. 10 in. by 4.5 ft., and 100 ft. by 18.37$\frac{1}{2}$ ft.? $.64+.

PRACTICE.

248. **Practice** is the process of finding the value of a quantity by operating upon an assumed value, or by combining the values of convenient parts.

Ill. Ex., I. What cost 5750 lbs. tea, at 37$\frac{1}{2}$ cts. per pound?

OPERATION.

5750 lbs. at \$1 per lb. will cost		\$5750.
" " " \$.25 " " "	$\frac{1}{4}$ of 5750 =	1437.50
" " " .12$\frac{1}{2}$ " " "	$\frac{1}{2}$ of 1437.50 =	718.75
		\$2156.25, *Ans.*

OR,

5000 lbs. at .37$\frac{1}{2}$ will cost	5000 × .375 =	\$1875.
500 " " " " "	$\frac{1}{10}$ of 1875 =	187.50
250 " " " " "	$\frac{1}{2}$ of 187.50 =	93.75
5750 lbs. at .37$\frac{1}{2}$ per lb.,	=	\$2156.25, *Ans.*

ILL. EX., II. What is the price of 17 A. 3 R. 25 rds. of land, at \$200 per acre?

OPERATION.

If the price of 1 acre is		\$200.
the price of 17 acres will be	$17 \times 200 =$	3400.
" " " 2 roods " "	$\frac{1}{2}$ of $200 =$	100.
" " " 1 " " "	$\frac{1}{2}$ of $100 =$	50.
" " " 20 rods " "	$\frac{1}{2}$ of $50 =$	25.
" " " 5 " " "	$\frac{1}{4}$ of $25 =$	6.25
" " " 17 A. 3 R. 25 rds.	=	\$3581.25, *Ans.*

ILL. EX., III. What cost 5 T. 13 cwt. 1 qr. 10 lbs. of hay, at \$16.67 per ton?

OPERATION.

If the price of 1 ton	=	\$16.67
the price of 5 tons will be	$5 \times 16.67 =$	83.35
" " " 10 cwt. " "	$\frac{1}{2}$ of $16.67 =$	8.335
" " " 3 " " "	$\frac{3}{10}$ of $8.335 =$	2.500+
" " " 1 qr. " "	$\frac{1}{4}$ of $\frac{1}{10}$ of $8.335 =$	.208+
" " " 10 lbs. " "	$\frac{2}{5}$ of $.208 =$	.083+
" " " 5 T. 13 cwt. 1 qr. 10 lbs.	=	\$94.476+, *Ans.*

ILL. EX., IV. What cost 8.96 bbls. of flour, at \$9.87½ per barrel?

OPERATION.

The cost of 8.96 bbls. of flour, at \$1	per bbl. =	\$8.96
" " " " " " " " \$10	" " =	89.60
" " " " " " " " \$.12½	" " =	1.12
" " " " " " " " \$9.87½	" " =	\$88.48, *Ans.*

ILL. EX., V. What cost 17 yds. of velvet, at 3£ 5s. 10d. per yard?

OPERATION.

		£	s.	d.
17 yds., at 3£, will cost	$17 \times 3£ =$	51	0	0
" " " 5s., " "	$17 \times \frac{1}{4}£ =$	4	5	0
" " " 10d., " "	$17 \times \frac{5}{6}s. =$	0	14	2
17 yds., at 3£ 5s. 10d., will cost		£55	10	2 *Ans.*

249. TABLE OF ALIQUOT PARTS.

Of a \$.		Of a £.		Of a Shilling.		Of a Ton.		Of a Cwt.		Of an Acre.	
Cts.	\$.	s. d.	£.	d.	s.	Cwt. qr.	ton.	Qr. lb.	cwt.	R. rd.	A.
50 =	$\frac{1}{2}$	10 =	$\frac{1}{2}$	6 =	$\frac{1}{2}$	10 =	$\frac{1}{2}$	2 =	$\frac{1}{2}$	2 =	$\frac{1}{2}$
$33\frac{1}{3}$ =	$\frac{1}{3}$	6 8 =	$\frac{1}{3}$	4 =	$\frac{1}{3}$	5 =	$\frac{1}{4}$	1 =	$\frac{1}{4}$	1 =	$\frac{1}{4}$
25 =	$\frac{1}{4}$	5 =	$\frac{1}{4}$	3 =	$\frac{1}{4}$	4 =	$\frac{1}{5}$	20 =	$\frac{1}{5}$	32 =	$\frac{1}{5}$
20 =	$\frac{1}{5}$	4 =	$\frac{1}{5}$	2 =	$\frac{1}{6}$	2 2 =	$\frac{1}{8}$	$12\frac{1}{2}$ =	$\frac{1}{8}$	20 =	$\frac{1}{8}$
$16\frac{2}{3}$ =	$\frac{1}{6}$	3 4 =	$\frac{1}{6}$	$1\frac{1}{2}$ =	$\frac{1}{8}$	2 =	$\frac{1}{10}$	10 =	$\frac{1}{10}$	16 =	$\frac{1}{10}$
$12\frac{1}{2}$ =	$\frac{1}{8}$	2 6 =	$\frac{1}{8}$	1 =	$\frac{1}{12}$	1 1 =	$\frac{1}{16}$	$6\frac{1}{4}$ =	$\frac{1}{16}$	8 =	$\frac{1}{20}$
10 =	$\frac{1}{10}$	2 =	$\frac{1}{10}$			1 =	$\frac{1}{20}$	5 =	$\frac{1}{20}$		
$8\frac{1}{3}$ =	$\frac{1}{12}$	1 =	$\frac{1}{20}$								
$6\frac{1}{4}$ =	$\frac{1}{16}$										
5 =	$\frac{1}{20}$										
2 =	$\frac{1}{50}$										

NOTE.—This table can be profitably extended by the pupil to other denominations, as Time, Length, etc.

250. EXAMPLES.

What is the cost of

1. 3 T. 13 cwt. of hay, at \$$12\frac{1}{2}$ per ton? *Ans.* \$45.625.
2. 13 cwt. 3 qr. 15 lb. of cotton, at \$$8.06\frac{1}{4}$ per cwt.? *Ans.* \$$112.068\frac{3}{4}$.
3. 200 yd. 3 qr. of sarcenet, at \$$.12\frac{1}{2}$ a yd.? *Ans.* \$$25.09\frac{3}{8}$.
4. 7 lb. 10 oz. of tea, at \$$.56\frac{1}{4}$ per pound? *Ans.* \$$4.289\frac{1}{16}$.
5. A farm of 40 A. 3 R. 31 rd., at \$82.50 per acre? *Ans.* \$$3377.859\frac{3}{8}$.
6. 1872 lbs. of butter, at $16\frac{2}{3}$ cents per pound? *Ans.* \$312.
7. 25350 ft. of gas, at $3\frac{1}{3}$ mills per foot? *Ans.* \$84.50.
8. 5 T. 12 cwt. 2 qr. $6\frac{1}{4}$ lb. of hemp, at \$180 a ton? \$$1013.06\frac{1}{4}$
9. Fencing 180 rd. 3 yd. 2 ft. of road, both sides, at \$1.25 per rd.? \$$451.66\frac{2}{3}$ \$650.40
10. Fencing a square lot of the above length, at \$.90 per rd.?
11. 3 pk. 7 qt. of berries, at \$2 a bushel? \$$1.937\frac{1}{2}$
12. Insuring a house 5 y. 7 m. 20 d., at \$$6.66\frac{2}{3}$ a year? (30 d. = 1 m.) \$37.592+

13. 12 boxes of shoes, each containing 48 pairs, at $.93\frac{3}{4}$ per pair? $450.

14. 17 cd. 7 cd. ft. 8 cu. ft. of wood, at $5.52 per cord? 99.015

15. Board for 9 w. $4\frac{1}{2}$ d., at $4.75 per week? 45.803+

16. $7\frac{1}{2}$ doz. knives, at $3.75 a dozen? $28.12½

17. $3\frac{1}{4}$ yd. of silk, at $\$1.12\frac{1}{2}$ per yard? $3.656¼

18. Making 7 m. 7 f. 35 rd. of railroad, at $650,000 per mile? $5,189

19. 6 hhd. 42 gall. of wine, at $110.15 per hogshead? $734.33⅓

20. 75 cwt. 75 lb. of tobacco, at $\$18\frac{3}{4}$ per cwt.? 1420.312½

21. 7 lb. 7 oz. 2 pwt. 16 gr. of silver, at $\$12.87\frac{1}{2}$ per pound? $97.77

22. If the interest of $1000 for 1 year is $105, what would be the interest of the same sum for 3 y. 8 m. 24 d.? $392.00

23. What is the amount of my salary for 3 y. 4 m. 25 d., at $600 per annum? $2041.66⅔

24. The rent of a musical instrument for a certain time was $60, the rent being $6 per quarter; what would have been the rent for the same time at $7 per quarter? at $5? at $8? at $3? at $9? at $10? at $\$7\frac{1}{2}$? at $\$4\frac{1}{2}$? at $\$12\frac{1}{2}$? at $1 per month? at $\$1\frac{1}{2}$ per month? *Sum of answers*, $740.

25. What cost 587 lb. of soap, at 1£ 8s. 9d. per cwt.? £.8.9.15

26. What cost 17 T. 9 cwt. 3 qr., at 5£ 3s. 8d. per ton? 90£.12.0.10

NOTE. — For further examples that may be performed by Practice, the pupil is referred to Art. 247.

251. GENERAL REVIEW, NO. 5.

PART I.

1. Multiply 675 by $\frac{1}{10}$, the product by 100; divide the last product by 1000, this quotient by $\frac{\frac{1}{2}}{5}$; multiply the last quotient by $\frac{1}{33\frac{1}{3}}$, and add the five results. 6893.775

2. From .1 lb. take .0678 lb. .0322

3. Multiply $3.05\frac{1}{2}$ by $.056\frac{1}{4}$. .17184375

4. What is the cost of 84 ft. of boards, at $20 per M.? $1.68

5. $.007644 \div 36 = ?$ .0002123

6. $8.052 \div .0044 = ?$ 1830

7. $10. \div 1000 = ?$ .01

8. $.065455 \div .065 = ?$ 1.007

9. Reduce $\frac{3}{80}$ *to a decimal.* .0375

10. $.3\frac{1}{3} + .92\frac{1}{7} = ?$ $1.25\frac{10}{21}$

11. What is the cost of 3 pk. 7 qt. of peas, at $3.75 per bushel? \$$3.63\frac{9}{32}$

12. Reduce $.8765\frac{2}{5}$ deg. to whole numbers of lower denominations. 60 m., 6 fur., 0 rd., 4 yd., 0 ft., 4.896 in.

13. Reduce .015 and .0096 to common fractions. $.015 = \frac{3}{200}$ $.0096 = \frac{6}{625}$

14.* Reduce $.\dot{3}\dot{9}$ and $.3\dot{4}3\dot{2}$ to common fractions. $.\dot{3}\dot{9} = \frac{13}{33}$

PART II.

1. What is the largest number that will exactly divide 475200 and 216000?

2. In 324 sheets of lead, each 12 in. by 8 in., $\frac{1}{12}$ in. thick, how many solid inches?

3. In 20692 sq. rd. how many acres?

4. Reduce 19 h. 12 m. to the fraction of a day.

5. What is the 5th power of .09?

6. $50.76\frac{2}{3} + .834\frac{1}{5} + .0\frac{1}{8} = ?$

7. If 6 yd. of cloth cost $$5\frac{3}{5}$, what will $14\frac{3}{4}$ yd. cost?

8. Add $\frac{1}{9}$ £, $\frac{3}{7}$ s., and $\frac{4}{5}$ d.

9. Reduce .21 pt. to the decimal of a peck.

10. How many inches in length of that which is $8\frac{5}{7}$ in. in breadth will make a square foot?

11. Reduce $\frac{6}{192}$ to a decimal fraction.

12. Carry out the following bill: —

FRANKLIN, *Dec.* 14, 1864.

B. FRANK WATSON,

Bought of B. COOLIDGE,

10850 ft.	Boards,	at $11.	per M.,	. . . .	
3000 "	"	" 19.375	"	. . . .	
2500 "	Lathing,	" 4.75	"	. . . .	
1500 "	Shingles,	" 6.	"	. . . .	
250 "	Plank,	" 13.	"	. . . .	
1250 "	Timber,	" 12.80	"	. . . .	
4220	Bricks,	" 12.50	"	. . . .	

Received payment,

B. COOLIDGE.

* *For Changes, see Key.*

PERCENTAGE.

252. The subject of **Percentage** comprises operations in hundredths. *Per cent.*, from the Latin *per*, by, and *centum*, hundred, signifies *by the hundred;* thus, 4 per cent. of any number or quantity is $\frac{4}{100}$ of that number or quantity.

Any per cent. may be expressed as a decimal fraction, as a common fraction, or by the use of the following sign, %; thus,

4 per cent. is written	.04	or $\frac{4}{100}$	or 4%.
$3\frac{1}{2}$ " " " "	$.03\frac{1}{2}$	" $\frac{3\frac{1}{2}}{100}$	" $3\frac{1}{2}$%.
$1\frac{1}{11}$ " " " "	$.01\frac{1}{11}$	" $\frac{1\frac{1}{11}}{100}$	" $1\frac{1}{11}$%.
$\frac{1}{4}$ " " " "	$.00\frac{1}{4}$	" $\frac{\frac{1}{4}}{100}$	" $\frac{1}{4}$%.

Examples.

Represent the following rates decimally:—

1. 8%. *Ans.* .08.
2. 20%. *Ans.* .20.
3. $12\frac{1}{2}$%.
4. $1\frac{3}{8}$%.
5. $6\frac{1}{4}$%.
6. $\frac{1}{2}$%.
7. $16\frac{2}{3}$%.
8. 106%.

253. Ill. Ex. Reduce 5 per cent. to its lowest terms.

$$5\% = \frac{5}{100} = \frac{1}{20}, \textit{ Ans.}$$

Examples.

Reduce the following rates to their lowest terms:—

1. 90%. *Ans.* $\frac{9}{10}$.
2. 50%. *Ans.* $\frac{1}{2}$.
3. 75%.
4. 40%.
5. $12\frac{1}{2}$%.
6. $8\frac{1}{3}$%.
7. $16\frac{2}{3}$%.
8. $33\frac{1}{3}$%.
9. 125%.
10. $62\frac{1}{2}$%.
11. $87\frac{1}{2}$%.
12. $31\frac{1}{4}$%.

254. Ill. Ex. Reduce $\frac{3}{8}$ to a per cent. (Art. 238.)

$$8\,)\,3.00$$
$$.37\frac{1}{2} = 37\frac{1}{2}\%, \textit{ Ans.}$$

EXAMPLES.

Reduce the following fractions to a per cent.:—

1. $\frac{3}{10}$. *Ans.* 30%.
2. $\frac{5}{8}$. *Ans.* $62\frac{1}{2}$%.
3. $\frac{7}{12}$.
4. $\frac{5}{6}$.
5. $\frac{1}{30}$.
6. $\frac{8\frac{1}{2}}{10}$.
7. $\frac{7}{5}$.
8. $1\frac{1}{16}$.

255. ILL. EX. Find the complement of 15 per cent., *i. e.*, what it wants of being 100 per cent.

$$100\% - 15\% = 85\%, \textit{Ans.}$$

EXAMPLES

Find the complement of the following rates:—

1. 92%. *Ans.* 8%.
2. 51%. *Ans.* 49%.
3. 11%.
4. $83\frac{1}{3}$%.
5. $87\frac{1}{2}$%.
6. $33\frac{1}{3}$%.
7. $18\frac{3}{4}$%.
8. $56\frac{1}{4}$%.

256. TO FIND ANY PER CENT. OF A NUMBER.

ILL. EX. What is 25% of 76 bushels of grain?

$$76 \text{ bu.} \times .25 = 19 \text{ bu.}, \textit{Ans.}$$

$$\text{or } 25\% = \tfrac{1}{4};\ \tfrac{1}{4} \text{ of } 76 \text{ bu.} = 19 \text{ bu.}, \textit{Ans.}$$

Hence the

RULE. To find any per cent. of a number: *Multiply by the rate per cent., expressed decimally.*

EXAMPLES.

What is

1. 8% of $800? *Ans.* $64.
2. 5% of 324.40? *Ans.* 16.22.
3. 20% of 375 men? *Ans.* 75 men.
4. $7\frac{1}{2}$% of 800 trees? *Ans.* 60 trees.
5. 12% of 78 bu.? *Ans.* $9\frac{9}{25}$ bu.
6. $\frac{3}{4}$% of $14.40? *Ans.* $.108.
7. $4\frac{1}{6}$% of 12£. 6s.? *Ans.* $10\frac{1}{4}$s.
8. 235% of $85? *Ans.* $$199\frac{3}{4}$.
9. 5% of $\frac{4}{5}$? *Ans.* $\frac{1}{25}$
10. 4% of 110% of $750? *Ans.* $33
11. $\frac{1}{2}$% of $200.75? $1.00 $3\frac{3}{4}$
12. $4\frac{1}{3}$% of 1000 gall.? $43\frac{1}{3}$ ga
13. 10% of 49£. 7s. 6d.? 4£, 18s.
14. $66\frac{2}{3}$% of 8 d. 5 h. 36 m.? 5 d.
15. $\frac{7}{8}$% of 3 cwt. 2 qr. (in lbs) 3
16. 75% of $8\frac{1}{3}$? $6\frac{1}{4}$

17. What is 25% of 125% of 75% of 50% of 384 inches? 45 in

18. Farmer F kept 75 sheep last year, and sold the wool for 40 cents per pound. He has 20% more sheep this year, and hopes to sell his wool 80% higher. How many sheep has he? What does he hope to get per pound for wool? 90 sheep $.72 p.

19. If $12\frac{1}{2}\%$ of \$97.50 be lost, what will remain? \$85.312+

20. The owner of a field of wheat allows $8\frac{1}{3}\%$ of the wheat for harvesting; what will be the owner's share if 80 bushels are harvested? $73\frac{1}{3}$ bu

21. I had \$125 in bills on the Cochituate Bank when it failed. I received 50% of this in good money from the bank, and afterwards 25% of the remainder; what did I lose? \$46.875

22. A and B had each \$2800 bequeathed to them. A gained 15% on his bequest, and B lost $12\frac{1}{2}\%$ of his bequest. How much had A and B then? A. \$3220. B \$2450.

23.* Carefully compiled statistics show that in 1860 in the United States $\frac{1}{5}\%$ of the population died from intemperance, that $\frac{2}{5}\%$ were sent to prisons and almshouses, that $\frac{2}{3}\%$ were made orphans, that .001% were murdered, and $.001\frac{1}{3}\%$ committed suicide, from the same cause. Considering the population to have been 30000000, what was the number of victims in each case? —in all? 60000 intem. 200000 orph. 300 m. 120000 prisoners & paupers. 400 sui. *Ans.* 380,700 in all.

24. A rope 12 yds. long shrank $2\frac{1}{2}\%$ on being wet. Required its length after shrinking. $11\frac{7}{10}$ yd

25. If a yard of cloth shrinks 6% in length in sponging, what part of a yard in length will it be after sponging? $\frac{47}{50}$ yd

26. If the cloth is a yard wide, and shrinks 6% in length and 6% in width, what will a yard contain after shrinking?

Ans. $\frac{2209}{2500}$ yd.

27. What would a yard have contained if it had been $1\frac{1}{4}$ yds. wide? $1\frac{209}{2000}$.

28.* How much cloth originally $1\frac{1}{3}$ yds. wide will be required to make a suit of clothes containing 9 full sq. yds., if the cloth purchased shrinks in sponging 10% each way? *Ans.* $8\frac{1}{3}$ yds.

☞ For Dictation Exercises, see Key.

257. To find 100% knowing a Certain Other %.

Ill. Ex. \$25 is $62\frac{1}{2}$% of what sum?

If \$25 is $62\frac{1}{2}$% of some number, 1% is $\frac{1}{62\frac{1}{2}}$ of \$25, and 100% is $100 \times \frac{1}{62\frac{1}{2}}$ of \$25 = \$40, *Ans.* Or

$62\frac{1}{2}\% = \frac{5}{8}$. If \$25 is $\frac{5}{8}$, $\frac{1}{8}$ is $\frac{1}{5}$ of \$25, and $\frac{8}{8}$ is $8 \times \frac{1}{5}$ of \$25, &c. Hence the

Rule. To find 100% knowing a certain other %: *Divide the given sum by the given number of %, and multiply by 100.*

Examples.

1. \$31.35 is 5% of what sum? *Ans.* \$627.
2. \$14.04 is 12% of what sum? *Ans.* \$117.
3. 153000 men is 9% of what number? *Ans.* 1700000.
4. $381\frac{7}{10}$ miles is 11% of what number? *Ans.* 3470 miles.
5. 250 is $4\frac{1}{6}$% of what? *Ans.* 6000.
6. $\frac{4}{9}$ is 15% of what? *Ans.* $2\frac{26}{27}$.
7. 84£. 14s. is $87\frac{1}{2}$% of what sum? *Ans.* 96£. 16s.
8. 75 is $37\frac{1}{2}$% of what sum?
9. \$700 is 140% of what sum? *Ans.* \$500.

10. The sum of the ages of a father and son is 44 years, the son's age being 10% of the father's; what is the age of each?

Note. — 44 years = the father's age, and 10% more, = $\frac{110}{100}$ of the father's age; ∴ 1% = $\frac{1}{110}$ of 44, and 100% = $100 \times \frac{1}{110}$ of 44 years = 40 years, father's age.

11. Having lost $12\frac{1}{2}$% of my money, I have \$84 remaining; what had I at first?

Note. — The complement of $12\frac{1}{2}$% is $87\frac{1}{2}\% = \frac{7}{8}$; ∴ $8 \times \frac{1}{7}$ of 84 = \$96, *Ans.*

12. 1865 bushels is 25% more than what number?

13. £67.76 is $12\frac{1}{2}$% less than what?

14. \$4.14 is $3\frac{1}{2}$% more than what?

15. A bankrupt is allowed to cancel all his debts by paying 40 cents on the dollar; what did he owe to a person to whom he paid \$2000.20?

16. An attorney receives \$1.26 for collecting a bill which is $\frac{7}{8}$% *of the* bill; what was the amount of the bill?

17. Drew out 25% of my deposit in a bank; of this I have spent \$500, which is 4% of what I drew out; what have I remaining in the bank? *Ans.* \$37,500.

18. A gain of $161\frac{7}{13}$% in the population of the United States from 1830 to 1860, shows an increase of 21000000; what was the population in 1830? in 1860?

Ans. 13000000 in 1830; 34000000 in 1860.

19. 8% allowance is given a debtor for making present payment of a debt due at a future time without interest; the amount paid is \$322.575; what was the sum due? *Ans.* \$350.625.

☞ For Dictation Exercises, see Key.

258. To find what Per Cent. one Number is of Another.

Ill. Ex. What per cent. of \$50 is \$15?

\$15 is $\frac{15}{50}$ of \$50; $\frac{15}{50}$, reduced, equals 30%, *Ans.* Hence the

Rule. To find what % one number is of another: *Divide the number expressing the part by the number with which it is compared, continuing the division to the hundredths' place.*

Examples.

1. \$23 is what % of \$92? *Ans.* 25%.
2. \$15 is what % of \$80? *Ans.* $18\frac{3}{4}$%.
3. What % of \$18 is 2 cts.? *Ans.* $\frac{1}{9}$%.
4. What % of 10 d. is 2 w. 1 d. *Ans.* 150%.
5. 5 oz. is what % of 4 oz. 7 pwt. 12 gr.? *Ans.* $114\frac{2}{7}$%
6. $4\frac{1}{2}$ is what % of 15? *Ans.* 30%.
7. What % of 48 doz. is 6 doz.?
8. What % of 1600 men are 1000 men?
9. What % of 1 dr. is $1\frac{1}{4}$ oz. indigo?
10. What % of 1 doz. is one score?
11. What % of 1£ is 1 s.
12. From a cask containing 120 gal. of oil, 6 gal. leaked out; what % was lost?
13. A field which yielded 90 bu. of rye, last year, yields 126 bu. this year; what is the gain %?

Note. $126 - 90 = 36$, the gain; $\frac{36}{90} = 40\%$, Ans

14. If gold and silver coin have 9 parts pure metal to 1 part alloy, what % of the coin is alloy? *Ans.* 10%.

15. In a class of 60 pupils 5 errors were made in spelling 1 word each; required the % of errors. *Ans.* $8\frac{1}{3}$%.

16. Subsequently, in spelling 10 words each, upon the slate, the same class made 33 errors; required the % of correct spelling. *Ans.* $94\frac{1}{2}$%.

17. Of a certain farm 18 acres are pasture, 30 acres woodland, 12 A. 1 R. a corn field, 70 A. a wheat field, 6 A. 2 R. a potato field, and 3 A. 1 R. a garden; what % of the whole farm is each part?

18. If wood, which should be cut 4 ft. in length, falls short 2 in., what % should be deducted from the price?

☞ For Dictation Exercises, see Key.

259. Miscellaneous Examples involving Profit and Loss.

1. A lot of coal cost $7.50 a ton; for what must it be sold to gain $33\frac{1}{3}$%? *Ans.* $10.

2. Bought 40 reams of paper at $2 a ream; at what price per quire must I sell it to gain 20 % on the cost? *Ans.* 12 cents.

3. What must I ask apiece for lamps that cost $4 a doz., that I may make 25%? *Ans.* $41\frac{2}{3}$ cts.

4. Sold nutmegs at 40 cents a pound, and lost 20%; what did they cost per lb.? *Ans.* 50 cents.

5. Sold a carriage for $240, which was 40% less than it cost; required the cost. *Ans.* $400.

6. Lost $15 by selling a watch at 25% below cost; what was the cost? *Ans.* $60.

7. By selling a lot of goods for $27.60, I gain 15%; what did I give for them? *Ans.* $24.

8. If, by selling gloves at 60 cents a pair, 20% is gained, what was the cost per dozen pairs? *Ans.* $6.

9. What would have been the cost per doz. if, by selling them at 60 cents, $6\frac{1}{4}$% had been lost? *Ans.* $7.68.

10. What was my property worth 10 years ago, if it has since increased 100%, and it is now worth $7000? $3500

11. What must be the amount of my sales for a year, that I may clear $800 at a profit of 16%? 5000

12. If I pay 45 cents a pound for tea, and sell it at 56 cents, what % do I gain?

56 — 45 = 11, the gain on 45; $\frac{11}{45}$ reduced = $24\frac{4}{9}$%, *Ans.*

13. What % is gained by selling goods at 10 cents a yd. which cost 8 cents? *Ans.* 25%.

14. What % will be lost by selling a book for 75 cents which cost 80 cents? *Ans.* $6\frac{1}{4}$%.

15. If I buy a horse for $75, and sell him for $120, what is the % of gain? 60 %

16. If $1000 be paid for a lot of goods, $640 be received for one half of them, and $300 for the remainder, is there a gain or loss, and what %? 6 % loss

17. Bought paper at $1.75 per ream, and sold it at 20 cents per quire; what % did I gain? $128\frac{4}{7}$ %

18. What % should I gain by selling at 1 cent a sheet? $174\frac{3}{7}$

19. Bought 150 beeves at the rate of $42.50 each, and 300 sheep at the rate of $4.50. I sold the lot for $10300; what did I gain %? $33\frac{1}{3}$ %

20. By selling wood at $6.50 per cord, I gain 30%; what did I give per cord? $5.

21. Bought a cord of wood for $5, and sold 2 cd. ft. for $1.62½; what was the gain %? 30 %

22. If a grocer buy 15 cwt. 3 qr. 20 lb. of coffee at $9 per cwt., what would he gain or lose % by selling the lot for $207.35? $44\frac{4}{9}$ % gain

23. What % is lost by selling a lot of goods for $\frac{2}{3}$ of their cost? $33\frac{1}{3}$ %

24. What % is gained by selling a lot of goods for 2 times their cost? 100 %

25. What was the cost of a lot of land which, selling at 20% below cost, brings $240? $300.

26. What was the original value of a share in a bridge, which, selling at 35% more than it cost, brings \$780? \$577$\frac{7}{9}$.

27. Bought a cow for \$87.50, which was 16$\frac{2}{3}$% more than her real worth; what was her worth? \$75.

28. A grocer, after losing 11% of his apples, has 133.5 bbls. of apples left; if they cost him \$2.50 per bbl. for what must they be sold per bbl. that he may lose nothing upon his purchase?

29. Bought and sold 250 lbs. of fish and gained \$3.75, which was 42$\frac{6}{7}$% of the cost, what did the whole cost, and what did they bring per lb.? \$8.75; \$.05 per lb

NOTE. — Further examples in Profit and Loss will be found in Miscellaneous Examples in Percentage.

INTEREST.

260. **Interest** is a certain per cent. of a sum of money, paid by the borrower to the lender for its use.

The **Principal** is the money lent.

The **Amount** is the sum of the Principal and Interest.

The **Legal Rate** is the rate per cent. per annum established by law.

Usury consists in taking more than the legal rate.

NOTE. — The laws often impose heavy penalties for usury.

261. The legal rate is 5% *per annum* in England, France, and Louisiana; 7% in New York and several of the Western and Southern States; 8% in the Gulf States, excepting La.; 6% in a majority of the United States, including all of N. England, and in Ireland, Canada and Nova Scotia.

In many of the States higher rates may be received by agreement; in California, any rate.

NOTE I. — In this book, 6% will be understood where no % is named.

NOTE II. — Business men reject mills from the products when less than 5, and call 5 or more 1 cent. In this book mills are retained.

262. First Method of Computing Interest.

At 6%, the interest on \$1 for 1 year, or 12 months, is 6 cents. If for 12 months the interest is 6 cents, for every 2 months it is 1 cent; for every 6 days, which is $\frac{1}{10}$ of 2 months, the interest is $\frac{1}{10}$ of 1 cent = 1 mill. The interest will be in the same proportion for a longer or a shorter period of time, and for larger or smaller sums.

Ill. Ex. What is the interest of \$200 for 5 y. 7 m. 19 d.?

Operation.

The interest of \$1	for 5 y.	$=5 \times \$.06 =$	\$.30
" " " "	" 7 m.	$=3\frac{1}{2} \times .01 =$	.035
" " " "	" 19 d.	$=3\frac{1}{6} \times .001 =$	$.003\frac{1}{6}$
" " " "	" 5 y. 7 m. 19 d. =		$\$.338\frac{1}{6}$
" " " \$200 "	" "	$=200 \times .338\frac{1}{6} = \$67.633\frac{1}{3}$, *Ans.*	

Hence the

Rule. To find the interest on \$1 for any time at 6%: *Take 6 times as many cents as there are years, one half as many cents as there are months, and one sixth as many mills as there are days given, and find their sum.*

To find the interest on any number of dollars: *Multiply the principal by the interest of* \$1 *for the given time.*

Examples.

Find the interest of \$1 for the following times: —

1. 1 y. 3 m. 6 d. *Ans.* \$.076.
2. 4 y. 16 d. *Ans.* $\$.242\frac{2}{3}$.
3. 4 m. 5 d. *Ans.* $\$.020\frac{5}{6}$.
4. 1 m. 25 d. *Ans.* $\$.009\frac{1}{6}$.
5. 1 y. 1 m. 10 d. $\$.066\frac{2}{3}$
6. The amount of \$1 for 1 y. 8 m. *Ans.* \$1.10.
7. What is the interest of \$1 for 16 y. 8 m.? *Ans.* \$1.
8. What is the interest of \$300 for 2 y. 5 m.? *Ans.* \$43.50.
9. What is the interest of \$4.20 for 3 y. 6 m. 12 d.? \$.8904
10. What is the amount of \$1000 for 7 y. 10 m. 18 d.? *Ans.* \$1473.

263. SECOND METHOD OF COMPUTING INTEREST.

We see by Ex. 7, Art. 262, that the interest of \$1 for 16 y. 8 m., or 200 months, is the same as the principal; this is true of any sum at 6%. Upon this fact is based an ingenious and practical method of computing interest.

The following table shows the relation which the interest bears to the principal at 6% for various periods of time: —

TABLE.

Interest for	16 y. 8 m.	or 200 m.	=	Principal.
" "	8 y. 4 m.	or 100 m.	=	$\frac{1}{2}$ do.
" "	4 y. 2 m.	or 50 m.	=	$\frac{1}{4}$ do.
" "	2 y. 1 m.	or 25 m.	=	$\frac{1}{8}$ do.
" "	1 y. 15 d.	or $12\frac{1}{2}$ m.	=	$\frac{1}{16}$ do.
" "	5 y. 6 m. 20 d.	or $66\frac{2}{3}$ m.	=	$\frac{1}{3}$ do.
" "	2 y. 9 m. 10 d.	or $33\frac{1}{3}$ m.	=	$\frac{1}{6}$ do.
" "	1 y. 4 m. 20 d.	or $16\frac{2}{3}$ m.	=	$\frac{1}{12}$ do.
" "	8 m. 10 d.	or $8\frac{1}{3}$ m.	=	$\frac{1}{24}$ do.
" "	3 y. 4 m.	or 40 m.	=	$\frac{1}{5}$ do.
" "	1 y. 1 m. 10 d.	or $13\frac{1}{3}$ m.	=	$\frac{1}{3}$ of $\frac{1}{5}$ do.
" "	1 y. 8 m.	or 20 m.	=	$\frac{1}{10}$ do.
" "	6 m. 20 d.	or $6\frac{2}{3}$ m.	=	$\frac{1}{3}$ of $\frac{1}{10}$ do.
" "	3 m. 10 d.	or $3\frac{1}{3}$ m.	=	$\frac{1}{6}$ of $\frac{1}{10}$ do.
" "	10 m.		=	$\frac{1}{20}$ do.
" "	5 m.		=	$\frac{1}{4}$ of $\frac{1}{10}$ do.
" "	2 m.	or 60 d.	=	$\frac{1}{100}$ do.
" "	1 m.	or 30 d.	=	$\frac{1}{2}$ of $\frac{1}{100}$ do.
" "	$\frac{1}{2}$ m.	or 15 d.	=	$\frac{1}{4}$ of $\frac{1}{100}$ do.
" "	$\frac{2}{5}$ m.	or 12 d.	=	$\frac{1}{5}$ of $\frac{1}{100}$ do.
" "	$\frac{1}{3}$ m.	or 10 d.	=	$\frac{1}{6}$ of $\frac{1}{100}$ do.
" "	$\frac{1}{5}$ m.	or 6 d.	=	$\frac{1}{10}$ of $\frac{1}{100}$ or $\frac{1}{1000}$ do.
" "	$\frac{1}{10}$ m.	or 3 d.	=	$\frac{1}{2}$ of $\frac{1}{1000}$ do.
" "	$\frac{1}{15}$ m.	or 2 d.	=	$\frac{1}{3}$ of $\frac{1}{1000}$ do.
" "	$\frac{1}{30}$ m.	or 1 d.	=	$\frac{1}{6}$ of $\frac{1}{1000}$ do.

To obtain the interest at 6% by this method for 200 months, 20 months, 60 days, or 6 days, nothing is required but *removing the decimal point;* facility in computing for any other period of time, depends upon *subdividing the given time into convenient factors and multiples of* 200 *months*, 20 *months*, 60 *days, or* 6 *days.*

ILL. Ex., I. What is the interest of \$480 for 1 y. 3 m.?

1ST METHOD.

\$480 multiplied by
.075 int. of \$1 for 1 y. 3 m.

2400
3360

\$36.000, *Ans.*

2D METHOD.

Principal,	(1),	\$480
Int. for 10 m. $\frac{1}{20}$ of (1) = (2),		24
" " 5 m. $\frac{1}{2}$ of (2) = (3),		12
" " 1 y. 3 m.	= *Ans.*	\$36

ILL. Ex., II. Required the amount of \$872.32 for 6 y. 2 m. 6 d.

1ST METHOD.

\$872.32 mult'd by
1.371 am't of \$1 for
6 y. 2 m. 6 d.

87322
610624
261696
87232

\$1195.95072, *Ans.*

2D METHOD.

Principal	(1),	\$872.32
Int. for 50 m.	$\frac{1}{4}$ of (1) = (2),	218.08
" " 20 m.	$\frac{1}{10}$ of (1) = (3),	87.232
" " 4 m.	$\frac{1}{5}$ of (3) = (4),	17.446
" " 6 d.	$\frac{1}{1000}$ of (1) = (5),	.872
" " 6 y. 2 m. 6 d.	*Ans.*	\$1195.950.

ILL. Ex., III. Find the interest of \$762.75 for 3 y. 10 m. 29 d.

1ST METHOD.

762.75, multiplied by
.234⅚, int. of \$1 for 3 y. 10 m. 29 d.

63562½
305100
228825
152550

Ans \$179.119+ int. of \$762.75 for 3 y. 10 m. 29 d.

2D METHOD.

Principal	(1),	\$762.75
Int. for 40 m. $\frac{1}{5}$	(1) = (2),	152.55
" " 6⅔ m. $\frac{1}{3}$ of $\frac{1}{10}$ of	(1) = (2),	25.425
" " 6 d. $\frac{1}{1000}$	(1) = (4),	.762
" " 3 d. $\frac{1}{2}$	(4) = (5),	.381
" " 3 y. 10 m. 29 d.	*Ans.*	\$179.118+

EXAMPLES.

1. Find the interest of \$100 for 1 y. 4 m. *Ans.* \$8.
2. \$75.085 for 1 y. 8 m. 6 d. *Ans.* \$7.583.
3. \$987.35 for 4 y. 2 m. 28 d.
4. \$36.18 for 3 m. 7 d.
5. \$96.34 for 1 m. 10 d. 642+
6. \$130.50 for 2 y. 9 m. 13 d. 161.64
7. \$800.20 for 3 y. 4 m. 12 d. 21.815
8. \$16.82 for 9 m. 27 d.
9. \$1000 for 3 y. 10 m. 2 d.
10. \$25.50 for 1 y. 1 m. 1 d. 1.661+
11. Find the amount of \$14.98 for 2 y. 6 m. 29 d. *Ans.* \$17.299.
12. Find the amount of \$490.82 for 4 y. 7 m. 17 d.
13. Find the amount of \$97.65 for 5 y. 11 m. 14 d.

264. TO FIND INTEREST AT ANY OTHER RATE THAN 6%.

ILL. EX. What is the interest of \$490 for 1 y. 5 m. 24 d. at 7%?

Principal,	(1),	\$490.
Int. for 1 yr., $.07 \times (1)$	$= (2)$,	34.30
" " 4 m., $\frac{1}{3}$ of (2)	$= (3)$,	11.433+
" " 1 m., $\frac{1}{4}$ of (3)	$= (4)$,	2.858+
" " 24 d. $\frac{1}{5}$ of (3)*	$= (5)$,	2.286+
" " 1 y. 5 m. 24 d., *Ans.*,		\$ 50.877+

RULE I. To find interest at any %: *Find the interest for* 1 *year by multiplying the principal by the given rate; and from that interest compute the interest for the given time,* by Practice. (Art. 249.) Or,

RULE II. *Find the interest at* 6%, *and increase or diminish that interest as the given % is greater or less than* 6%.

Thus, for 7 % take 7 times $\frac{1}{6}$ of the interest at 6 %, or add $\frac{1}{6}$; for 5 % take 5 times $\frac{1}{6}$ of the interest at 6 %, or subtract $\frac{1}{6}$; for $7\frac{1}{2}$ % take $7\frac{1}{2}$ times $\frac{1}{6}$ of the interest at 6 %, or add $\frac{1}{4}$, &c.

* 24 d. $= \frac{4}{5}$ of 1 mo. $= \frac{1}{5}$ of 4 mo.

EXAMPLES.

Find the

1. Interest of \$837.36 for 3 y. 2 mo. at 7%. *Ans.* \$185.614+.
2. Interest of \$400.08 for 2 y. 4 m. 2 d. at 9%. *Ans.* \$84.216+.
3. Amount of \$640 for 1 y. 6 m. at 7%. *Ans.* \$707.20+.
4. Interest of \$75.85 for 10 m. 3 d. at 9%. *Ans.* \$5.745+.
5. Amount of \$416 for 3 y. 16 d. at 7%.
6. Interest of \$450 for 5 y. 4 m. 3 d. at 8%.
7. Interest of \$658 for 9 m. at $\frac{1}{2}$%. *Ans.* \$2.467+.
8. Amount of \$325 for 3 d. at $7\frac{1}{4}$%. *Ans.* \$325.196+.
9. Interest of \$896 for 2 y. $6\frac{1}{2}$ m. at $6\frac{2}{3}$%. *Ans.* \$151.822+.
10. Interest of \$187.50 for 2 m. 12 d. at 10%.

265. TO FIND INTEREST ON ENGLISH CURRENCY.

ILL. EX. Find the interest of 10 £. 15 s. 6 d. for 16 yr. 10 mo. at 6%.

OPERATION.

10 £. 15 s. 6 d. = 10.775 £.
10.775 £ multiplied by
1.01, interest of 1 £ for 16 y. 10 m.
10.88275 £ = 10 £. 17 s. 7 d. 3+ qr., *Ans.*

Hence the

RULE. *Reduce the shillings, pence, and farthings to the decimal of* 1 £ † (Art. 244), *compute interest as in Federal Money, and reduce the decimal of* 1 £ *to s. d., &c.,* (Art. 245.)

† Shillings, pence, and farthings may be reduced to the decimal of a pound by inspection as follows:

ILLUSTRATION.

5 s. 10 d. 3 far. = .295 £.
5 s. = .25
10 d. 3 f. = 43 f. = .045
∴ 5 s. 10 d. 3 f. = .295 £.

Call half the number of even shillings tenths, and the odd shilling, if any, 5 hundredths. Reduce pence and farthings to farthings, and call them, thousandths, adding one to the number when it exceeds 12, and 2 when it exceeds 36.

The decimal of a pound may be reduced to shillings, pence, and farthings, as follows:

EXAMPLES.

Find the

1. Interest of 10 £. 15 s. for 2 y. 9 m. 10 d.
Ans. 1 £. 15 s. 10 d.

2. Interest of 17 s. for 1 y. 1 m. 4 d.

3. Interest of 241 £. 10 s. 6 d. for 1 y.

4. Interest of 15 £. 7 s. 10 d. for 3 y. 11 m. 14 d. at 4%.

5. Interest of 27 £. 7 s. 11 d. for 1 y. 7 m. 18 d. at 5%.

6. Amount of 20 £. 8 s. for 4 y. 2 m. 27 d. at 8%.

7. Interest of 482 £. 10 s. for 3 y. 2 m. at a rate equal to 1 shilling on £1. *Ans.* 76 £. 7 s. 11 d.

266. In computing interest, it is often necessary to find the time between two dates. This may be done by subtraction, as in Art. 204, or mentally as follows:—

ILL. EX., I. What is the time from May 19, 1860, to Mar. 28, 1862?

From May 19, '60, to May 19, '61, =	1 y.
From May 19, '61, to Mar. 19, '62, =	10 m.
From Mar. 19, '62, to Mar. 28, '62, =	9 d.
From May 19, '60, to Mar. 28, '62, =	1 y. 10 m. 9 d., *Ans.*

ILL. EX., II. What is the time from May 19, '60, to Mar. 15, '62?

From May 19, '60, to Mar. 19, '62, as above, = 1 y. 10 m.; but to Mar. 15, '62, it being 4 days less, it is 1 y. 9 m. 26 d. Hence

RULE I. *Find the number of years and months between the first date and the same day of the month in the second date. If this falls short of the true time, add the difference of days; if it reaches beyond it, subtract the difference of days.*

RULE II. *Find the number of years and* ENTIRE *calendar months between the dates, and then the remaining days.* (p. 329.)

NOTE.—The answers given in the book are by Rule I. Answers obtained by *both* rules are given in the Key.

ILLUSTRATION.

.584 £ = 11 s. 8 d. 1 far.
.55 £ = 11 s.
.034 £ = 33 far. = 8 d. 1 far.
.584 £ = 11 s. 8 d. 1 far.

Call every tenth 2 shillings, and every 5 hundredths, 1 shilling. Call the remainder farthings, subtracting 1 if the number exceeds 12, and 2 if it exceeds 36.

It is just to compute interest by the exact number of days in each calendar month. This is the method employed by the United States Government and throughout Great Britain. For method of computing interest at $7\frac{3}{10}\%$, see Appendix, p. 335.

267. Miscellaneous Examples.

What is the interest of

1. \$270.87 from Oct. 17, 1860 to Dec. 28, 1863?
Ans. \$51.961+.

2. \$400.37 from Mar. 14, 1857, to Sept. 9, 1859?
Ans. \$59.722+.

3. \$1000 from Nov. 11, 1856, to Aug. 15, 1862, at 7%?
Ans. \$403.277+.

4. \$19.80 from Oct. 15, 1859, to Apr. 19, 1860, at 5%?
Ans. \$0.506.

5. \$130.16 from Feb. 7, 1866, to Dec. 1, 1870, at 8%?
Ans. \$50.154+.

6. \$99.99 from Jan. 15, 1860, to Mar. 10, 1863, at 3%?
Ans. \$9.457+.

7. \$62.50 from Aug. 3, 1862, to Apr. 11, 1863, at $7\frac{1}{2}\%$?
Ans. \$3.229+

8. \$175 from Dec. 4, 1861, to May 1, 1864? \$25.257+

9. \$2000 from Sept. 8, 1859, to Jan. 3, 1861? 158.933+

10. \$120.90 from July 10, 1865, to Feb. 2, 1868, at 9%? 27.867

11. \$456.82 from June 15, 1830, to June 6, 1865, at 1%? 159.772

Find the

12. Amount of \$365 from Dec. 12, 1860, to Mar. 7, 1861.
Ans. \$370.171—.

13. Amount of \$58.80 from Nov. 1, 1844, to Feb. 1, 1849, at $7\frac{1}{2}\%$. 77.5421

14. Interest of \$40.75 from Aug. 19, 1835, to June 17, 1838, at 4%. \$4.609+ or 4.615+

15. Interest of \$150 from July 5, 1860, to Mar. 17, 1862.

16. Interest of £1000 from July 8, 1858, to Mar. 5, 1860, at 7%. 116£ 1s 8d

17. Amount of 430 £. 7 s. $8\frac{3}{4}$ d. from July 15, 1870 to Oct. 5, 1874, at 12%.
Ans. 648 £. 8 s. 21 d.

18.* Interest of 15 £. 10 s. from Dec. 28 to Jan. 4, by the exact number of days. *Ans.* 4 d. 1 + f.

19. Interest of $1600 from Aug. 10, 1864, to Jan. 1, 1865, by the exact number of days, at 5 %. *Ans.* $32.

20. A man gave his note (Art. 268) May 7, 1830, for $1800, with interest; what sum would discharge the note June 21, 1834? *Ans.* $2245.20.

21. In settling with a person, Jan. 1, 1859, I found I owed him $387.20; for this sum I gave my note on interest at 7 %; what should I pay to discharge this note Oct. 20, 1859?

22. There are two notes, one dated Jan. 19, 1850, for $375.83, the other dated May 19, 1851, for $76.19; what is the amount of both notes Jan. 1, 1852, each bearing interest from its date?

23. Reed and Prescott bought goods to the following amounts, agreeing to pay 7 % interest from the date of purchase: July 8, 1864, $470; July 28, $235; Oct. 2, $206. What will be the amount due Jan. 1, 1865? *Ans.* $937.366+.

24.* What is the balance of the following account Jan. 1, 1862, and due to whom, reckoning interest on each item from its date?

LEONARD HARRIS

DR. In a/c with MARTIN LINCOLN.† CR.

1861.					1861.				
May	5	To Wool,	$400	00	Feb.	16	By Oats,	$200	00
June	7	" Goods,	420	00	May	5	" Corn,	174	00
Aug.	28	" Goods,	225	00	Sept.	14	" Hay,	380	00

Ans. Due M. Lincoln, $301.505.

25. Find the balance due Cabot in the following account, Oct. 1, 1865, interest at 6 %, from the date of the items: —

ARTHUR LEE

DR. In a/c with GEO. D. CABOT. CR.

1865.					1865.				
Mar.	29	To Mdse.,	$476	93	Apr.	24	By Mdse.,	$389	51
Apr.	22	" Cash,	869	82	May	15	" Mdse.,	379	84

† The Dr. side of this account shows what goods Harris has bought of Lincoln; the Cr. side what he has sold to Lincoln.

PARTIAL PAYMENTS.

NOTES.

268. A **Promissory Note,** usually called a *Note*, is a written promise to pay money or merchandise for value received.

269. The sum promised is called the **Principal** or **Face of the Note,** and should be written in words.

270. In order that a note shall draw interest from date, "*with interest*" must be inserted in it; but notes *on demand*, without interest specified, draw interest from the time payment is demanded; and notes *on time*, from the time when due, if not then paid, though interest is not specified.

271. To be negotiable, *i. e.*, transferable or salable, a note must be made payable "to order" or "bearer." If "to order," the holder cannot transfer it without endorsing it, *i. e.*, *writing his name on the back of it.*

The endorser of a note becomes liable for its payment under certain circumstances. He may endorse, without becoming liable, by writing above his name "without recourse."

272. According to custom, and, in many States, by law, a note is not considered due till *three days* after the time specified for its payment. These are called **days of grace,** and interest is taken for them.

NOTE. — A note is said to *mature* when it becomes due.

273. **Partial Payments** are payments in part of notes or other obligations.

274. To compute the interest on notes, when partial payments have been made, observe the following, called

THE UNITED STATES RULE.

Find the amount of the sum due from the time interest com- to the time of the first payment ; subtract the payment, if cancel the interest, and consider the remainder a new prin- ; find the amount of this new principal from the time of the

first payment to the time of the second; subtract the second payment as before, and so proceed to the time of settling the note.

2. *By the decisions of the United States Court, when a payment will not cancel the interest due, interest is computed on the principal till sufficient sums have been paid to cancel all the interest due, when all the payments are subtracted from the amount due as if paid in one sum.*

NOTE. — When partial payments are made, the account is kept by entering the same, with their dates, on the back of the note. The entries are called *endorsements*.

EXAMPLES.

1. Suppose a note for \$1908.42, dated Aug. 9, 1851, to be on interest till Feb. 15, 1852, when a payment of \$1732.59 is made; what sum will remain due? *Ans.* \$234.991.

2. Suppose the above balance (\$234.991) to remain on interest till April 3, 1853, when another payment of \$50 is made; what will then be due? *Ans.* \$200.97.

3. Suppose the balance (\$200.97) to continue on interest to Jan. 9, 1860, what will be due at that time? \$282.363

4. A note for \$75.83, with interest, is dated Jan. 19, 1850; suppose \$15 to be paid July 15, 1850, what will remain due? 61.

5. Suppose \$40 of the above balance to be paid April 13, 1852, what will then be due? 63.04 29.65

6. If this balance remains on interest till Feb. 7, 1853, what sum will then be due? *Ans.* \$31.105+.

7. A note for \$50, dated Jan. 1, 1862, is on interest at 6% till April 15, 1862, when a payment of \$25.87 is made. What sum will remain due? 24.996

8. If the balance of the above should remain on interest at 7% till Jan. 13, 1863, what will then have to be paid to discharge the note? 26.298

PROMISSORY NOTE. (Art. 268.)

9. \$300. PHILADELPHIA, *April* 5, 1857.

On demand, I promise to pay E. VARNUM, or bearer, three hundred dollars, with interest, value received. C. J. POTTER.

On the above note were the following endorsements: —

Received, May 29, 1860, $217.49.

" Apr. 23, 1862, $50.

What will be the balance due on the above note December 15 1869? *Ans.* $153.275

OPERATION.

Principal, ...	$300.	, on interest from Apr. 5, '57.
Interest on principal, .	56.70	, to May 29, '60, (3 y. 1 m. 24 d.)
Amount, ...	$356.70	
1st payment, ...	217.49	
2d principal, ...	$139.21	, on interest from May 29, '60.
Interest on 2d principal,	15.869,	to Apr. 23, '62, (1 y. 10 m. 24 d.)
Amount, ...	$155.079	
2d payment, ...	50.	
3d principal, ...	$105.079,	on interest from Apr. 23, '62.
Interest on 3d principal,	48.196,	to Dec. 15, '69, (7 y. 7 m. 22 d.)
Amount, ...	$153.275, *Ans.*	

10. $1000. BURLINGTON, *Oct.* 5, 1854.

For value received, I promise to pay to the order of JOSEPH P. BATTLES, one thousand dollars, with interest, on demand.

J. BUSNELL.

ENDORSEMENTS.

Received of within Dec. 8, 1854, $125.

" " " May 12, 1855, 316.

" " " Sept. 2, 1855, 417.

" " " Mar. 9, 1856, 100.

What balance remained due June 15, 1856? *Ans.* $93.353+.

11. $700. LANCASTER, *April* 5, 1847.

On demand, with interest at 7%, we promise to pay H. K. OLIVER, or order, seven hundred dollars, value received.

WARREN BURTON & Co.

ENDORSEMENTS.

Received of within, Oct. 29, 1850, $217.49.

" " " July 23, 1852, 200.00.

What remained due Dec. 12, 1859? *Ans.* $814.681.

Note. — In the following examples in Partial Payments, consider each note to be *on demand with interest* from its date, unless otherwise specified.

12. A note for $960 is dated Nov. 16, 1855, on which was paid $140, Nov. 11, 1856; $80, July 30, 1857; $70, Jan. 2, 1858; and $100, Dec. 1, 1858. What balance is due Oct. 30, 1859, reckoning interest at 7%? *Ans.* $806.077.

13. $350. BRISTOL, *April* 5, 1850.

For value received, we jointly and severally promise JOHN INGALLS to pay him, or order, three hundred fifty dollars, on demand, with interest at 5% per annum, after three months from date. HOOD & BISHOP.

On this note were the following endorsements: Nov. 1, 1852, received $87; March 7, 1855, received $150; Feb. 19, 1858, received $115. What was due Sept. 15, 1862? *Ans.* $125.61+.

14. A note for $935 is dated Sept. 1, 1855, on which was paid $125.75, Jan. 15, 1856; $250, March 25, 1861; $300, May 10, 1861. What was the balance due July 1, 1861? *Ans.* $549.713.

OPERATION.

Principal, ...	$935.	, on interest from Sept. 1, 1855.
Interest on principal, .	20.881,	to Jan. 15, '56 (4 m. 14 d.).
Amount, ...	$955.881	
1st payment, ...	125.75	
2d principal, ...	$830.131,	on interest from Jan. 15, '56.
Interest on 2d principal,	258.724,	to Mar. 25, '61 (5 y. 2 m. 10 d.).
Interest on 2d principal,	6.225,	to May 10, '61 (1 m. 15 d.*).
Amount, ...	$1095.08	
2d and 3d payments, .	550.00	, both required to cancel interest.
3d principal, ...	545.08	, on interest from May 10, '61.
Interest on 3d principal,	4.633,	to July 1, '61 (1 m. 21 d.).
Amount, ...	$549.713,	*Ans.*

15. $500. SALEM, *April* 1, 1855.

For value received, I promise to pay W. J. ROLFE, or order, five hundred dollars, on demand, with interest from Oct. 1, 1855.

IRENAS EDWARDS.

* See Art. 274, Rule, 2d Clause.

ENDORSEMENTS.

Received of the within, Apr. 1, 1856, \$12.
" " " " Apr. 1, 1857, \$100.
" " " " Apr. 1, 1858, \$100.

What is due June 19, 1858? *Ans.* \$363.646+.

16. A note for \$1000 is dated June 1, 1860. The endorsements are: \$75, paid Aug. 1, 1860; \$125.75, paid Dec. 15, 1860; \$250, paid Feb. 25, 1866; and \$300, paid Apr. 10, 1866. What will be due on this note June 1, 1866? *Ans.* \$549.713+.

17. A note for \$790, dated Oct. 9, 1862, is endorsed Sept. 6, 1863, with \$320; Jan. 30, 1864, with \$10; Oct. 9, 1864, with \$190. What balance is due Feb. 3, 1865, interest at 5%?

Ans. \$338.77+.

18.* A note for \$800, dated Jan. 15, 1860, is on interest after 6 months, and is endorsed Apr. 18, 1861, \$100; Jan. 1, 1863, \$70; and June 15, 1864, \$62.50, — interest being at 7%. What was due July 15, 1865? *Ans.* \$830.415+.

19.* Upon a note of \$425, dated July 13, 1859, there are the following endorsements: August 10, 1861, \$50; November 18, 1862, \$150. What will be due, if the note is settled July 13, 1863? \$ 322.508

20.* A note for \$250, dated May 15, 1838, is endorsed Feb. 25, 1841, \$111.66⅔; Oct. 19, 1842, \$15; May 9, 1848, \$62; and Oct. 15, 1849, \$100.30. Required the balance due July 22, 1851.

21.* A note for \$489 is dated Jan. 20, 1850, and endorsed as follows: June 26, 1850, received \$50; Feb. 26, 1852, received \$40; July 8, 1855, received \$90; Jan. 26, 1856, received \$200; June 20, 1856, received \$200. If this note was on interest from three months after date, what was due Nov. 20, 1856?

275. The following rule for Partial Payments is in general use, when the whole period of time is less than one year:—

RULE. *Find the amount of the principal for the whole time the note is on interest; find, also, the amount of each payment from the time it is made to the time of settling the note; and deduct the sum of the payments, with their interest, from the amount of the principal.*

ILL. EX. $800. BURLINGTON, *July* 7, 1860.

Three months after date, I promise to pay JOHN THETFORD, or bearer, eight hundred dollars, with interest.

BENJAMIN STOKES.

On the back of the above note were recorded the following payments:—

Received, Aug. 16, 1860, $200.
" Oct. 8, 1860, $480.
" Feb. 20, 1861, $49.92.

What balance was due at the time of settlement, July 1, 1861? *Ans.* $84.65.

OPERATION.

Principal, $800, from July 7, '60, to July 1, '61 (11 m. 24 d.), amounts to		$847.20
1st pay't, $200, from Aug. 16, '60, to July 1, '61 (10 m. 15 d.), amounts to	$210.50	
2d pay't, $480, from Oct. 8, '60, to July 1, '61 (8 m. 23 d.), amounts to	501.04	
3d pay't, $49.92, from Feb. 20, '61, to July 1, '61 (4 m. 11 d.), amounts to	51.01 —	762.55
Balance due, ...	*Ans.*	$84.65

1. 10000\frac{75}{100}$. CONCORD, *Oct.* 4, 1863.

In two months from date, I promise to pay to the order of BENJAMIN TYLER, at Suffolk Bank, Boston, ten thousand $\frac{75}{100}$ dollars, with interest, value received.

THOMAS BEEMAN.

ENDORSEMENTS.

Received of within, $672.41, Nov. 5, 1863.
" " " $7682.42, Nov. 15, 1863.
" " " $437.98, Nov. 16, 1863.
" " " $833.42, Nov. 19, 1863.

What was the balance due on the above note, when the note became due? *Ans.* $443.555.

2. $1200 ALBANY, *April* 1, 1862.

One year from date, for value received, I promise to pay to J. V. SMILEY or order, twelve hundred dollars, with interest, at 7%. ORRIN JONES.

The above was indorsed as follows:—

April 11, 1862, $161.08; July 18, 1862, $224.14;
July 27, 1862, $17.90; Jan. 28, 1863, $100.25.

What was still due April 1, 1863?

Ans. $756.565.

276. The following is the

CONNECTICUT RULE.

1. *When a year's interest or more has accrued at the time of a payment, and always in case of the last payment, follow the Government Rule.* (Art. 274.)

2. *When* LESS *than a year's interest has accrued at the time of a payment, except it be the last payment, find the difference between the amount of the principal for an* ENTIRE *year, and the amount of the payment for the balance of a year after it is made; this difference will form the new principal.*

3. *If the interest which has arisen at the time of a payment exceeds the payment, no interest will be computed upon the payment, but only upon the principal.*

1. $1000. HARTFORD, *March* 9, 1855.

In one year from date, for value received, I promise to pay GEO. YATES or order, one thousand dollars, with interest, at 6%. JOSEPH W. BOOMER, Jr.

ENDORSEMENTS.

Received Nov. 19, 1855, $204; Mar. 3, 1857, $50;
June 15, 1858, $600; Nov. 1, 1858, $85.

What balance was due Jan. 1, 1859? *Ans.* $241.798.

13

OPERATION.

$1060	Amount of principal from Mar. 9, '55 to Mar. 9, 56, (1 yr.).
207.74	" 1st payment from Nov. 19, '55 to " " (3⅔ m.).
852.26	Balance, forming 2d principal.
51.135	Interest from Mar. 9, '56 to Mar. 9, '57, (1 yr.).
903.395	Amount.
50.	2d payment, being less than interest, has no interest.
853.395	Balance, forming 3d principal.
64.857	Interest from Mar. 9, '57 to June 15, '58, (1 yr. 3 m. 6 d.).
918.252	Amount.
600.	3d payment, time being more than 1 year, has no interest.
318.252	Balance, forming 3d principal.
10.396	Interest from June 15, '58 to Jan. 1, '59, (6 m. 16 d.).
328.648	Amount of 3d principal to time of last payment.
85.85	Amount of $85 from Nov. 1, '58 to Jan. 1, '59, (2 m.).
$242.798, *Ans.*	Balance due Jan. 1, '59.

277. Annual Interest.

Ill. Ex. What is the amount due on a note for $1000, interest payable annually, if no payment should be made till the expiration of 4 y. 6 m. 12 d.?

The holder of this note should be allowed interest on the *interest* from the time it is payable to the time of settlement, in addition to the interest upon the note.

The int. on $1000 for 4 y. 6 m. 12 d. =		$272.00
" " $60 for 3 y. 6 m. 12 d.	= the int. on $60 for 8 y. 1 m. 18 d.	= 29.28
" " $60 " 2 y. 6 m. 12 d.		
" " $60 " 1 y. 6 m. 12 d.		
" " $60 " 6 m. 12 d.	Principal,	1000.00
	Amount due,	$1301.28, *Ans.*

The interest is first taken upon the face of the note for the full time; then upon the $60 due at the end of the first year for the balance of the time for which the note has to run; and so on for the other payments. Hence the

Rule for Annual Interest. *Compute interest on the principal for the entire time it is on interest; compute interest*

also upon one year's interest for the sum of all the periods of time for which each yearly interest remains unpaid. The sum of the interests thus found will be the annual interest.

EXAMPLES.

1. What is the annual interest of $200 for 4 y. 6 m. 3 d.?
Ans. $59.884.

2. What is the annual interest of $334 for 3 y. 8 m. 10 d.?
Ans. $80.148+.

3. What is the annual interest of $118.50 for 5 y. 3 m. 18 d.?
Ans. $42.588+.

4. What is the amount at annual interest of $175 for 6 y. 2 m. 25 d.?
Ans. $250.821+.

NOTE. — For New Hampshire rule for annual interest with partial payments, see Appendix, page 334.

COMPOUND INTEREST.

278. **Compound Interest** is interest on both principal and interest, the sum of the two forming a new principal at specified intervals of time.

NOTE. — Interest may be compounded, or added to the principal, annually, semi-annually, or for any period of time agreed upon.

279. ILL. EX., I. What is the compound interest of $212 for 2 y. 5 mo. 6 d., at 6% ?

OPERATION.

Principal,	$212
Amount of $1 for 1 year,	1.06
" $212 for 1 year,	224.72
" $1 " "	1.06
" $212 " 2 years,	238.2032
" $1 " 5 m. 6 d.,	1.026
" $212 " 2 y. 5 m. 6 d.,	244.3964832
Principal, subtracted,	212.
Compound Interest,	$32.396+, *Ans.*

ILL. EX., II. What is the compound interest of \$520, at 7%, for 4 y. 1 m. 24 d.?

OPERATION.

Amount of \$1 at 7% for 4 y. by the table, . .	\$1.310796
Multiplied by the principal, ...	520
Amount of \$520 for 4 yrs., ...	681.613+
Multiplied by am't of \$1 for 1 m. 24 d., ...	1.0105
Amount of \$520 for 4 y. 1 m. 24 d., ...	688.769+
Principal subtracted, ...	520.
Compound interest, ...	*Ans.* \$168.769+

13. What is the compound interest of \$480 for 7 y. 10 m.? *Ans.* \$277.829+.

14. What is the amount of \$100 for 2 y. 4 m., at 7%?

15. What is the compound interest of \$200 for 3 y. 2 m. 6 d.?

16. What is the amount of \$221.075 for 3 y. 5 m., at 7%?

17. What is the amount of \$280 for 1 y. 10 m. 22 d., interest payable semi-annually?

18.* What is the amount of \$50 for 3 y. 13 d., at 5%?

19.* What is the compound interest of \$896 for 2 y. 6 m. 15 d., at 5%?

20.* Find the compound interest of \$300 for 3 y. 4 m. 12 d., at 7%.

21.* Find the amount of £58 for 3 y. 5 m.

Ans. 70 £. 16 s. 1+d.

22.* Find the compound interest of 75 £. 9 s. 9 d. for 4 y. 8 m. 27 d., at 5%.

23.* What is the difference between the compound and simple interest of \$678.25 for 3 y. 6 m. 6 d.? *Ans.* \$11.488—.

24.* What is the difference between the compound and simple interest of \$100 for 1 y. 4 m., the compound interest payable semi-annually?

25.* What is the difference between the amount of \$175.08, at compound and at simple interest, from May 7, 1861 to Sept. 25, 1863, at 7%?

26.* Find the difference between the simple and compound interest of 94 £. 12 s. 6 d. for 2 y. 6 m. 12 d., at 8% ?

☞ For Dictation Exercises, see Key.

PROBLEMS IN INTEREST.

281. Since interest is always the product of the three factors, *principal*, *rate*, and *time*, it follows that to find the time, rate, or principal, when the interest and two of the other terms are given, it is only necessary to divide the interest by the product of the two given terms.

282. TO FIND THE TIME, WHEN THE INTEREST, PRINCIPAL, AND RATE ARE GIVEN.

ILL. EX. In what time will \$300 gain \$63 interest at 6% ?

OPERATION.

Int. of \$300 for 1 y. = \$18

18) 63.0

3.5 yrs. *Ans.*

The interest of \$300 for 1 *year* at 6% is \$18; it will require as many years for \$300 to gain \$63 as \$18 is contained times in \$63, which is $3\frac{1}{2}$ times. *Ans.* $3\frac{1}{2}$ y. Hence the

RULE. To find the time, when the interest, principal, and rate are given: *Divide the given interest by the interest of the principal at the given rate for* 1 *year.*

EXAMPLES.

What time will be required

1. For \$400 to gain \$20, at 6% ? *Ans.* 10 m.
2. For \$500 to gain \$60, at 4% ? *Ans.* 3 y.
3. For \$68.25 to gain \$3.003, at 6% ?
4. For \$640 to gain \$67.20, at 7% ?
5. For \$3000 to gain \$205, at 5% ?
6. For \$408 to gain \$170, at $7\frac{1}{2}$% ?
7. For \$450 to gain \$192.30, at 8% ?
8. For \$280 to amount to \$301, at 5% ?

NOTE. — Subtract \$280 from \$301 to find the interest.

9. In what time will \$200 amount to \$400, at 6% ?
10. In what time will \$500 amount to $\$658.33\frac{1}{3}$, at 6% ?

283. TO FIND THE RATE, WHEN THE INTEREST, TIME, AND PRINCIPAL ARE KNOWN.

ILL. EX. At what rate per cent. will \$250 gain \$25 in 2 years?

OPERATION.

	\$
Int. of \$250 for 2 y. at 1%, . . \$5)	25
	5

If the interest of \$250 for 2 y. at 1 *per cent.* is \$5, it will require as many times 1% to gain \$25 as \$5 is contained times in \$25, which is 5 times. *Ans.* 5%. Hence the

RULE. To find the rate, when the interest, time, and principal are given: *Divide the given interest by the interest of the principal for the given time at* 1 *per cent.*

EXAMPLES.

At what % will

1. \$360 gain \$40.80 in 1 y. 5 m.? *Ans.* 8%.
2. \$100 gain \$33⅓ in 12 y. 6 m.? *Ans.* 2⅔%.
3. \$250 gain \$3.75 in 4 m.?
4. \$25 gain \$7.87½ in 3 y. 6 m.?
5. \$100 gain \$25 in 7½ y.?
6. \$48.24 gain \$8.71 in 2 y. 9 m. 10 d.?
7. \$75 amount to \$78.75 in 2 y. 6 m.?

NOTE. — \$78.75 — \$75 = \$3.75 interest.

8. At what rate will \$50 amount to \$55.25 in 2 y.?
9. At what rate will \$1000 amount to \$1058.33⅓ in 10 m.?

284. TO FIND THE PRINCIPAL, WHEN THE INTEREST, TIME, AND RATE ARE KNOWN.

ILL. EX. What principal will yield \$42.50 interest in 8 m. 15 d. at 6%?

OPERATION. \$

Int. of \$1 for 8 m. 15 d. at 6%, . . . \$.0425) 42.50 (1000.

The interest of \$1 for 8 m. 15 d. at 6% is \$.0425; it will require as many dollars of principal to gain \$42.50 as \$.0425 is contained times in is 1000 times. *Ans.* \$1000. Hence the

RULE. To find the principal, when the interest, time, and rate are known: *Divide the given interest by the interest of* 1 *dollar at the given rate for the given time.*

EXAMPLES.

What principal will gain

1. \$15 in 2 y. at 6%? *Ans.* \$125.
2. \$20 in 4 y. at 5%? *Ans.* \$100.
3. \$76.50 in 2 y. 6 m. at 3%?
4. \$1.705 in 7 m. 15 d. at 4%?
5. \$68.999 in 1 y. 4 m. 24 d. at 5%?
6. \$4.128 in 11 m. 14 d. at 6%?

NOTE.—($4.128 \div .057\frac{1}{3}$). Reduce dividend and divisor to thirds before dividing. *Ans.* \$72.

7. What principal will be required to gain \$24 in 60 days at 2% a month?

8. What principal must be on interest 2 y. 5 m. 29 d. at 6% to gain \$89.40?

9. What is the principal which being on interest at 7% per annum, gains \$62.50 semi-annually?

285. TO FIND THE PRINCIPAL, WHEN THE AMOUNT, TIME, AND RATE ARE KNOWN.

ILL. EX. What principal will amount to \$17.238 in 2 m. 12 d. at 7%?

OPERATION. $\$17.238 \div \$1.014 = 17.$ The amount of 1 dollar for 2 m. 12 d. is \$1.014; it will require as many dollars to amount to \$17.238 as \$1.014 is contained times in \$17.238, which is 17 times. *Ans.* \$17. Hence the

RULE. To find the principal, when the amount, time, and rate are known: *Divide the given amount by the amount of* 1 *dollar at the given rate for the given time.*

EXAMPLES.

What principal will amount

1. To \$870 in 7 y. 6 m. at 6%? *Ans.* \$600.
2. To \$537.50 in 2 y. 6 m. at 6%? *Ans.* $\$467.39\frac{3}{23}$.

3. To \$2072.25 in 30 da. at 5% ? *Ans.* \$2063.651+.
4. To \$412 in 90 da. at 1% a month? *Ans.* \$400.
5. To \$100 in 3 y. 6 m. at $5\frac{1}{2}$% ?
6. To \$343.75 in 2 y. 1 m. at 7% ?
7. To \$206.25 in 7 m. 15 da. at 5% ?

☞ For Dictation Exercises, see Key.

PRESENT WORTH AND DISCOUNT.

286. This subject is a practical application of Art. 285. It embraces all examples in which it is required to know what sum will equitably discharge a note or debt at a given time before it is due.

287. The **Present Worth** of any sum of money due at a future time without interest, is such a sum as put at interest at the given rate will amount to the debt when it becomes due.

It is evident that where money is worth 6% a year, \$106 due in one year is the same in value as \$100 paid now; for \$100 put at interest for 1 year will amount to \$106.

288. **Discount** is that part of an obligation which is abated or given up when the payment is made before it becomes due, and should in justice equal the interest upon the present worth for the given time.

289. Ill. Ex. What is the present worth of \$210 due 1 year hence, money being worth 5% ?

Here \$210 is the amount of some principal for 1 year at 5%; \$1 amounts to \$1.05 in a year; hence it will require as many dollars to amount to \$210 as \$1.05 is contained times in \$210, which is 200 times. *Ans.* \$200. Hence the

Rule. To find the present worth: *Divide the given sum by the amount of* 1 *dollar at the given rate for the given time.*

The discount of the above (\$210) is found by subtracting \$200 from \$210; this leaves \$10, which is precisely the same as the interest of \$200 for 1 year at 5%. Hence the

Rule. To find the discount: *Subtract the present worth from the given sum.*

EXAMPLES.

1. Find the present worth of \$27.50 for 1 y. 8 m. at 6%

$27.50 \div 1.10 = 25.$ *Ans.* \$25.

2. Find the present worth of \$100.96 for 8 mo. at 6%. *Ans.* \$97.076+.

3. Find the present worth and discount of \$200 due in 3 mo. at 6%. *Ans.* \$197.044+ pres. worth; \$2.956— discount.

4. What is the discount of \$100 for 9 mo. at 4%? *Ans.* \$2.912+.

5. What is the present worth of \$1609.30 for 10 m. 24 d. at 5%? *Ans.* \$1540.

6. What is the present worth of \$175.80 for 9 m. 20 d. at 6%?

$175.80 \div 1.048\frac{1}{3} = 527.40 \div 3.145 = 167.694+.$ *Ans.* \$167.694+.

7. What is the discount of \$661.375 for 3 m. 15 d. at 6%? *Ans.* \$11.375.

8. What is the present worth of \$96.347 for 8 m. 3 d. at 7%? *Ans.* \$92.

9. Find the present worth and discount of \$75.50 for 8 m. 10 d. *Ans.* \$72.48 pr. w.; \$3.02 disc.

10. Find the present worth of \$800.75 for 1 y. 1 m. 10 d. *Ans.* \$750.703+.

11. What is the present worth of \$75.85 due in 4 m. at 5%?

12. What is the present worth of \$221.075 due in 3 y. 5 m. at 7%?

13. If a note for \$500 be due in 2 years without interest, what is its value at the present time, money being worth 7%?

14. What is the present value of \$50 due in 3 y. 13 d., interest being 5%?

15. A note for \$240 is dated June 1, 1860, due in 8 m. 25 d.; what money will discharge it at date? *Ans.* \$230.215+.

16. A note for \$500 is dated April 6, due in 90 days; what money will discharge it at date?

17. What would discharge the above June 23? *Ans.* \$499+.

18. A note for \$2000, dated July 15, was given for 1 year, without interest; what will discharge it at date?

19. What will discharge it Oct. 15 of the same year?
Ans. \$1913.875+.

20.* What sum paid down will discharge a note of \$500, due in $2\frac{1}{2}$ years, the rate being 5%?

21.* What is the cash value of a note for \$927.60 on 7 days' credit?

22.* What is the value of a note for \$139.50 Dec. 11, 1863, which is dated Sept. 9, 1863, and given for 1 year?

\$251.90. TRENTON, April 1, 1862.

In nine months from date, I promise to pay J. ADAMS, or bearer, two hundred fifty-one $\frac{90}{100}$ dollars, value received.

C. QUINT.

23.* What will discharge the above at its date, the rate of discount being 6%?

24.* What will discharge the above April 16, 1862, the rate being 7%?

☞ For Dictation Exercises, see Key.

BANK DISCOUNT.

290. **Bank Discount** is an allowance made to a bank for advancing money on a note before it is due.

291. Bank discount is the interest on the face of the note or its amount at maturity for the time it is discounted (Art. 272).

292. The holder of a note discounted at a bank receives the face of the note *minus* the discount. This is called the **present worth,** the **proceeds,** or **avails** of the note.

ILL. EX. What is the bank discount on a note of \$300 for 4 months, at 6%? What are the avails?

OPERATION.

The interest on \$300 for 4 mo. is \$6.00
" " " \$300 for 3 d. is .15
" " " \$300 for 4 m. 3 d. is \$6.15, discount.
\$300 — \$6.15 = \$293.85, avails of note.

Hence the

RULE. To find the Bank Discount: *Compute simple interest on the given sum for the time it is to remain on interest, plus three days of grace.*

To find the avails: *Subtract the discount from the given sum.*

NOTE. Suppose the above (Ill. Ex.) was a 6 months note dated Jan. 5, which was to be discounted at a bank March 5, the operation would be precisely the same; the note would mature† July 5, with grace, July 8, and would be discounted for the time to elapse between March 5 and July 8, which is 4 months and 3 days.

EXAMPLES.

Find the bank discount

1. On \$75 for 30 days. *Ans.* \$.412+.

2. On a 90 days note for \$500, dated May 10, and discounted June 9. (Discount for days.) *Ans.* \$5.25.

3. On a 60 days note for \$256.84, dated Oct. 28, and discounted Nov. 12. *Ans.* \$2.054+.

4. On \$1000 for 3 mo. at 7%. *Ans.* \$18.083+.

5. What are the avails of a note of \$700, discounted at a bank for 69 days? *Ans.* \$691.60.

6.* A trader buys 900 pairs of shoes at \$.75 a pair cash, and immediately sells them at \$.90 on a note payable in 4 months without interest; suppose he gets his note discounted at a bank for the 4 months, what will he have made? *Ans.* \$118.395.

☞ For Dictation Exercises, see Key.

293. TO FIND FOR WHAT A NOTE MUST BE GIVEN, WHICH, DISCOUNTED AT A BANK, WILL YIELD A CERTAIN SUM.

ILL. EX. What must be the face of that note which, being discounted at a bank for 60 days, will yield \$148.425?

The bank discount of \$1 for 63 d. = \$.0105; \$1 − \$.0105 = \$.9895, avails of \$1. \$148.425 ÷ \$.9895 = \$150, face of note.

If \$1 were discounted at a bank, it would yield \$.9895; to yield \$148.42, the note must be given for as many dollars as \$.9895 is contained times in \$148.425, which is 150 times. *Ans.* \$150.

† See Art. 272, also Appendix, p. 330.

Hence the

RULE. To find the face of a note which, discounted at a bank, will yield a certain sum: *Divide the required sum by* \$1, *minus the bank discount of* \$1 *for the given time at the given rate, and the quotient will be the face of the note.*

EXAMPLES.

1. What must be the face of a note that it may yield \$80 when discounted at a bank for 30 days? *Ans.* \$80.442+.

2. For what must a note on 4 months, without interest, be given, that, when discounted at a bank, it may yield \$489.75? *Ans.* \$500.

3. For what must a note be given, which is to run 90 days, that it may yield \$400?

4. What must be the face of a note having 60 days to run, that it may yield \$989.50?

5. For what must a note, dated Sept. 1, on 4 months, be given to yield at its date \$400, when interest is 7%?

6.* For what must a note, dated Jan. 1, payable in 3 months, discount being $5\frac{1}{2}$%, be given, to yield \$150? *Ans.* \$152.162—.

7.* For what must a note on 6 months be written, to yield \$495.85, when the discount is $7\frac{1}{2}$%?

☞ For Dictation Exercises, see Key.

294. MISCELLANEOUS EXAMPLES IN BANKING, &c.

NOTE. — All examples in Present Worth or Discount should be considered in True Present Worth or Discount (page 202), unless Bank Present Worth or Bank Discount is definitely stated.

1. A note for \$500, dated July 1, is given for 20 days without interest; what is its true value July 15? *Ans.* \$499.50+

2. What will discharge the above, Aug. 14? (Exact days.) *Ans.* \$502

\$200. BOSTON, *April* 1, 1862.

Four months from date, I promise to pay JOHN BILLS, or order *two hundred* dollars, value received. JOHN ORNE, JR.

3. Suppose the above to be a good and true note, what is it really worth to the holder in cash at its date, money being 6%? *Ans.* \$196.078+.

4. What could he get from a bank for it at its date? *Ans.* \$195.90.

5. What would he get for it May 1, by true discount? (Art. 287.) *Ans.* \$197.044+.

6. What ought he to get for it April 1, 1863? *Ans.* \$208.

7. What is the difference between the avails of a note for \$200, payable without interest in 18 months, whether it be paid by true or by bank discount? *Ans.* \$1.586+.

8. What will be the difference between the true and the bank discount of a note for \$90.50, due Feb. 9, 1862, and discounted June 15, 1861? *Ans.* \$.177+.

\$300. HARTFORD, *July* 15, 1860.

Six months after date, I promise to pay to the order of Cyrus Ingraham three hundred dollars. Value received.

JOHN A. ANDREW.

9. What sum will the holder of the above receive, if it be discounted at a bank Sept. 15, 1860?

10. What sum would the holder of the above receive at its date by true discount?

11. What would discharge the above note May 8, 1861?

12. What would be the bank discount of the above at its date?

\$500. NEW BEDFORD, *Oct.* 5, 1860.

For value received, I promise to pay Alvin Dow, or order, five hundred dollars in three months. ALLEN JONES.

13. What cash must be paid to discharge the above note at its date by true present worth?

14. What would be the avails of it at a bank Dec. 5, 1860?

15. What would be its real cash value March 17, 1861?

16. What would be the true discount on it Nov. 5, 1860?

17. What would be the bank discount of it Nov. 5, 1860?

18. What would be due on the above note Feb. 15, 1862, if \$50 had been paid on it at the termination of each six months from its date, interest being 5%?

295. General Review, No. 6.

1. Reduce 75%, $16\frac{2}{3}$%, $37\frac{1}{2}$%, 95%, and $83\frac{1}{3}$% to their lowest terms, and give their sum in a common fraction.

2. If you buy socks at \$4.80 per dozen pairs, and sell at \$.50 per pair, what % do you gain?

3. For what must apples which cost \$1.25 per bbl. be sold to gain 20%?

4. If 25% is lost by selling a pair of boots at \$$4\frac{1}{2}$, what was the cost?

5. What is the simple interest of \$300 from May 5, 1860, to Feb. 2, 1862, at $1\frac{1}{2}$% a month?

6. What is the amount at compound interest of \$271.36 for 2 y. 6 m. at 6%?

7. What is the present worth of \$4508.25 for 11 days, at 6%?

8. What is the bank discount of \$450 for 30 days, at 5%?

9. What are the avails of a note of \$100 discounted at a bank for 27 days?

10. What is the amount at simple interest of 5£. 4s. 6d. for 2 years, at 5%?

11. The interest of \$400 for a certain time at 6% was \$60; what was the time?

12. What principal at 5% will gain \$4.50 in 10 months?

13. At what % will \$462 gain \$103.95 in 2 y. 3 m.?

14. For what must a note be given, which, discounted at a bank at 6 % for 60 days, will yield \$1295?

15. Given a note for \$2500, dated Sept. 5, 1862, on which were paid \$50 Jan. 29, 1863, \$500 July 1, 1864. The note being on interest at 6 % from its date, what was due Sept. 5, 1864?

☞ For changes, see Key.

COMMISSION, BROKERAGE, AND STOCKS.

296. **Commission** is a certain percentage received by a commission merchant for transacting business as factor, or agent for another.

297. **Brokerage** is the percentage received by a broker. A **Broker** is one who exchanges money and deals in stocks and bills of credit.

298. **Stocks** are Government Bonds of all kinds, and shares of the *capital* invested in Banks, Insurance Companies, &c.

299. When stocks and money sell for their original or nominal value, they are said to be *at par;* when they sell for more than their nominal value, they are said to be *at an advance, above par*, or *at a premium;* when they sell for less than their nominal value, they are said to be *at a discount*, or *below par.*

300. In Commission, the % is estimated *upon the sum actually expended;* in Brokerage, upon the par value, or an assumed value.

Ill. Ex. My agent buys a quantity of goods for \$220; what is his commission at 5 %?

$$\$220 \times .05 = \$11, \textit{Ans.} \text{ Or,}$$
$$5\% = \tfrac{1}{20};\ \tfrac{1}{20} \text{ of } \$220 = \$11, \textit{Ans.}$$

Examples.

1. What should a commission merchant receive for selling 4750 pounds of sugar at $12\frac{1}{2}$ cents a pound, his commission being 1%? *Ans.* 5.937+.

2. A stock broker purchases for a person 8 shares of stock in a manufacturing company at \$72 a share; what is his commission at $\frac{7}{8}$%?

3. What is a broker's commission for negotiating a loan of \$4500 at $\frac{1}{2}$%?

4. Dupee & Sayles bought on account of T. Winship, 4 shares

of Essex Company's stock, at \$27 each, their commission being $\frac{1}{4}$%: what is Winship's bill? *Ans.* \$108.27.

5. What amount of current money will be given in exchange for \$450 of that which is at 5% discount? *Ans.* \$427.50.

6. When gold is at a premium of 25%, what must be paid for \$275 of gold? *Ans.* \$343.75.

7. If an auctioneer sells, on a commission of 8%, 14 chairs at \$1.25, 1 bedstead for \$10, and a miscellaneous lot for \$53.79, what sum will be due the person for whom he makes the sale, his commission being deducted?

8. What is the commission on the sale of 200 yards of broadcloth at \$4.80 per yard, 6% being paid for selling, and $2\frac{1}{2}$% for guaranteeing the sales? *Ans.* \$81.60.

NOTE. — The seller sometimes guarantees the payment for the goods sold; for this he is paid a premium.

9. What are the net proceeds on the sale of a lot of crockery amounting to \$10650, commission being $4\frac{1}{2}$%, and $1\frac{1}{2}$% being allowed for guaranteeing payment? *Ans.* \$10011.

NOTE. — To obtain net proceeds, deduct commission.

10. What are the net proceeds from the sale of 1260 barrels flour, at \$3.50 per bbl., charges for freight and storage being 40c. per bbl., commission for selling being 2%, and for guaranteeing sales $1\frac{1}{4}$%?

11. What are 50 shares of railroad stock worth, at 4% advance, the par value being \$100?

$\$100 \times 1.04 \times 50 = \$5200.$ *Ans.* \$5200.

12. What would be the value of 15 shares of the above stock at 7% premium?

13. What would be the cost of 8 shares of the above, at a discount of 35%?

14. What would be the value of 4 shares in the stock of a gas company, originally worth \$200, at 3% above par?

15. What would be the value of 12 shares of above stock, at 17% below par?

16. A certain corporation, wishing to increase their capital

stock without multiplying their shares, assessed the stockholders 40% on the par value of their stock, which was $500 per share; what was assessed on a person holding 3 shares?

17. What was the par value of the stock per share in the corporation after the assessment was made?

18. If I buy 10 shares of stock, originally worth $100, at 18% above par, and sell it at 7% below par, what do I lose?

Ans. $250.

19. What would have been my gain if I had bought the above at 10% discount, and sold it at a premium of 8%?

20. Bought 75 shares in a savings-bank, par value being $50, at $6\frac{1}{2}$% advance, and sold at $3\frac{1}{2}$% above par; what did I lose on the lot?

21. The amount of the deposits in the savings-banks of Massachusetts for 1863, was $44,785,438.56; the ordinary dividends were at the rate of $4\frac{1}{2}$% of the deposits; what was the total of the dividends?

301. To find the Commission or Brokerage and the Sum invested, when both are contained in a certain Sum sent to a Factor or Broker.

Ill. Ex. I send to my agent at St. Salvador $1224; what part of this sum will remain to invest in sugars, after deducting his commission of 2% on what he lays out?

Operation.

$1224 ÷ $1.02 = $1200, sum to invest.
$1224 — $1200 = $24, commission.

Since the commission is 2% of the sum laid out, the agent receives $1.02 for every dollar which he is to lay out. If he receives $1224, he will have as many dollars to lay out as $1.02 is contained times in $1224, which is 1200 times. *Ans.* $1200.

Hence the

Rule. To find the sum invested: *Divide the amount named by* $1 *plus the commission on* $1; *the quotient will be the sum invested.*

To find the commission or brokerage: *Subtract the sum invested from the amount.*

Examples.

1. I have sent to a commission merchant in New York $450, of which he is to lay out what he can in butter, after reserving his commission of 2% on the purchase; what is the purchase?

Ans. $441.176+.

2. What part of a remittance of $328.25 will remain to be invested after 1% of the investment has been deducted?

Ans. $325.

3. How many barrels of flour at $5 each can a factor purchase with a remittance of $2575, after deducting his commission of 3%? *Ans.* 500 bbl.

4. How many shares of stock at $100 each can a broker purchase with a remittance of $520, allowing himself a brokerage of 4%?

5. A real estate broker receives $2593.75; what number of acres of land at $1.25 per acre can he purchase with the balance, after allowing himself $3\frac{3}{4}$% brokerage on the purchase?

6. Having remitted to my agent in New Orleans $891.75, to be expended for sugars, after reserving his commission of $2\frac{1}{2}$%, I received from him 29000 pounds of sugar; what was the cost per pound?

7. I have authorized a broker to employ $292.32 in the purchase of a certain stock for me, allowing him $1\frac{1}{2}$% commission; what number of shares originally worth $100 can he purchase, if they are now 72% below par?

8.* Wm. H. Ladd sells for Chas. Smith 2500 pounds of butter at 14 cts., and 100 pelts at $1.50; from the proceeds he deducts his commission of 3% and $4 for cartage, &c., and with the balance purchases for Smith, after deducting his commission of $1\frac{1}{2}$% on the purchase, a lot of sheeting at 10 cts. per yard; how many yards can he purchase?

☞ For Dictation Exercises, see Key.

INSURANCE.

302. **Insurance** is security to indemnify for loss.

Property Insurance indemnifies for loss by fire, shipwreck, &c.

Life and Health Insurance indemnify for loss of life or health.

303. The persons or company that insure are called *underwriters.*

304. A **Policy** is the written contract between the insurer and the insured.

305. **Premium** is a certain per cent. of the sum insured paid to the underwriters for the insurance.

306. Examples.

1. Required the premium for insuring a house for $1600 at $\frac{1}{2}\%$. 1% of $1600 = $16; $\frac{1}{2}\%$ of $1600 = $8, *Ans.*

2. What is the insurance on $1000 worth of furniture at $\frac{1}{2}\%$, including $1 for policy? *Ans.* $6.

3. Insured $\frac{2}{3}$ of a store valued at $15000 at $\frac{5}{8}\%$, and paid $1 for policy. What amount is paid?

4. Effected insurance on the ship Susan to Cadiz and back for $10000 at 2%, and on her return cargo, worth $7500, at $1\frac{1}{2}\%$; what is the amount of insurance, including $1 for policy? *Ans.* $313.50.

5. A lot of clothing worth $4000 is insured for $\frac{2}{3}$ of its value at $\frac{3}{4}\%$; if the goods are damaged by fire to the amount of $500, what will be the net loss to the underwriters, making no account of interest? *Ans.* $480.

Note. — The underwriters will make good to the insured his *actual* loss. Their net loss will be $500 minus the premium.

6. What will be the premium for insuring $15500 on a school-house for 10 years at $2\frac{2}{5}\%$?

7. What would be the loss to the insurance company if the

above building should be destroyed by fire, and the insurance be paid in 6 months from the date of the policy? *Ans.* $15116.84.

Note. — Reckon interest on the premium for 6 months.

8. Insurance was effected upon $\frac{3}{4}$ of a ship and cargo, valued at $50000, at $1\frac{7}{8}$%; what would be the actual loss to the underwriters if the ship and cargo should be totally lost at sea, making no allowance for interest?

9. What would be the actual loss to the owners?

Ans. $13203.125.

307. To find for what Sum an Insurance Policy must be taken out, to secure a certain Sum and cover the Premium.

Ill. Ex. For what must a policy be taken out to insure $500 on a ship's freight, and cover the premium of 2%?

Operation.

$1 - .02 = .98$

$\$500 \div .98 = \$510.204.$

Since the premium is 2% of the policy, the property ($500) must be 98% of the policy; if $500 is 98%, 1% will be $\frac{1}{98}$ of $500, and 100% will be $100 \times \frac{1}{98}$ of $500, which is $510.204. *Ans.* $510.204. Hence the

Rule. To find for what sum an insurance policy must be taken out, to secure a certain sum and cover the premium: *Divide the sum to be secured by* 1 *minus the rate per cent. of insurance; the quotient will be the amount of the policy.*

Examples.

1. What policy will cover $2000 of property and a premium of 3%? *Ans* $2061.855+.

2. I have loaned $1140 to a friend, to be secured by a policy on his life; for what must a policy be taken to secure the sum loaned and cover the premium of 5% also? *Ans.* $1200.

3. Having adventured $1800 to Calcutta, what policy should I take out to secure both the adventure and the premium of 6%?

4. For what must a policy be taken out to cover a loan of $588 and a premium of $12\frac{1}{2}$% upon it?

☞ For Dictation Exercises, see Key.

AVERAGE, OR EQUATION OF PAYMENTS.

308. **Equation of Payments** is the process of finding an average time for the equitable payment of several sums due at different times.

309. The **Equated Time** is the date at which all the items may be paid without loss to either party.

310. The **Term of Credit** is the time from the contracting of a debt to the date of its becoming due.

311. To find the Equated Time, when the Terms of Credit begin at the same Date.

Ill. Ex. I owe P. Benjamin two notes dated March 1, — one for \$80, to be paid in 12 months, the other for \$40, to be paid in 3 months. When, without loss to either Benjamin or myself, can I pay both notes at once?

Interest Method.

Operation.

12 months' interest on \$80 = \$4.80
3 " " " 40 = .60
120 5.40

$\frac{1}{2}$ of $\frac{1}{100}$ of 120 = .60 ; .60) 5.40
9

Mar. 1 + 9 mo. = Dec. 1, *Ans.*

I am entitled to keep these two notes till their interest equals \$5.40; if I pay them both at once, it should be at such time after Mar. 1 as will be required for \$120 to gain \$5.40; \$120 gains 60 cts. a month; ∴ to gain \$5.40, it will require as many months as 60 cts. is contained times in \$5.40 = 9 months, which added to Mar. 1, is Dec. 1. Hence

Rule I. To find the equated time when all the terms of credit begin at the same date: *Find the interest on each item for its time of credit; divide the sum of the interests by the interest of the sum of the items for one month. The quotient will be the equated time in months.*

Add the equated time to the date.

Note I. — To obtain the interest for 1 month, remove the decimal point two places to the left, and divide by 2.

NOTE II.— If any item contains cents, reject them if they are less than 50, and increase the dollars by *one* if they equal or exceed 50.

PRODUCT METHOD.

OPERATION.

$$\begin{array}{rrl} 80 \times 12 & = & 960 \\ 40 \times \ 3 & = & 120 \\ \hline 120 \quad) & & 1080 \\ \hline & & 9 \text{ m.} \end{array}$$

The use of \$80 for 12 m. = the use of \$1 for 960 m.; the use of \$40 for 3 m. = the use of \$1 for 120 m. \$1 for 960 m. + \$1 for 120 m. = \$1 for 1080 m., which = \$120 for $\frac{1}{120}$ of 1080 m. or 9 m., which added to Mar. 1. = Dec. 1, *Ans.* Hence

RULE. *Multiply each payment by the number of days or months to elapse before it becomes due; divide the sum of the products by the sum of the payments, and add the quotient to the date.*

NOTE.— The examples in this book are performed by the *Interest method.*

EXAMPLES.

1. What is the equated time for paying \$50 due in 5 m. from May 14, 1863, \$35 due in 4 m., and \$25 due in 2 m. from the same date? *Ans.* Sept. 14, 1863.

2. B. Frank holds five notes against me, dated June 7, 1864; one for \$500 on 4 m., one for \$750 on 5 m., one for \$200 on 12 m., one for \$400 on 2 m., and one for \$400 on 17 months' credit; what is the time at which all should be paid if paid in one sum? *Ans.* Jan. 7, 1865.

3. Having sold Samuel Bond real estate to the amount of \$2000, he gave me four equal notes for it, dated Oct. 4, and payable in 5, 6, 9, and 12 months; what is the average time for the payment of all the notes? *Ans.* 8 m.

4. What is the average time for paying \$20 due in 20 days, \$20 due in 100 days, \$70 due in 30 days, \$20 due in 60 days, and \$40 in 70 days? *Ans.* 1 m. 20 d.

5. April 1, C. A. Brown purchased coal to the amount of \$5000, $\frac{1}{4}$ of which was to be paid in 6 months, $\frac{1}{2}$ in 12 months,

and the remainder in 9 months; for what time should a note without interest, dated April 1, 1865, in payment of all the sums, be allowed to run, and when should the note be paid?

Ans. Jan. 28, 1866.

6. A owes B $360.25 (Note 2, Art. 311), $\frac{1}{3}$ of which is to be paid in 7 months, $\frac{1}{4}$ of the remainder in 8 months, $\frac{1}{3}$ of what then remains in 10 months, and the balance in $4\frac{1}{5}$ months; in how many months and days should the whole be paid? *Ans.* 6 m. 22 d.

7. Sept. 25, bought 3 parcels of goods, as follows: a bill amounting to $225.25 on 12 months' credit, a bill amounting to $125.25 on 8 months' credit, and a bill amounting to $40 on 5 months' credit; what was the mean time for paying all?

Ans. 10 m.

NOTE.—When a sum is paid immediately, the term of credit expires instantly, and it will have no corresponding interest or product in time.

8. A person promised to pay $7000, $\frac{1}{4}$ of it immediately, $\frac{1}{5}$ of the remainder in 8 months, $\frac{1}{3}$ of what then remained in 22 months, and the balance in 16 months; what is the equated time for paying the whole? *Ans.* 12 m.

9. A merchant tailor finds, on examining his account with Jones & Co., May 5, that he owes them for 150 yds. of silk, at $.50 per yd., which is due that day; for 2339 yds. of cambric, at $.10, which will be due in 6 days; for $12\frac{1}{2}$ yds. broadcloth, at $5.00 per yd., which will be due in 16 days; for 50 yds. of doeskin at $3.75 per yd., which will be due in 20 days; what is the average time for paying the whole? If the tailor settles the account by giving his note, when should the note begin to bear interest?

312. TO FIND THE EQUATED TIME, WHEN THE TERMS OF CREDIT BEGIN WITH DIFFERENT DATES.

ILL. EX. J. Rives bought of A. Ainger the following bills of goods:—

Sept. 15, a bill amounting to $100,
" 30, " " " $400,

Oct. 8, a bill amounting to $250,
" 10, " " " $250,

What is the equated time for paying the whole?

To equate the above bills, it is necessary to assume a date from which to compute the interest on the several items; any date may be assumed, but the most convenient date for examples generally, on account of reckoning the time, will be found to be *the last day of the month before the earliest date at which any item becomes due*; this in the above example is Aug. 31.

OPERATION BY INTEREST METHOD.

15 days' interest on $100	=	.25		
30 " " " 400	=	2.00		
38 " " " 250	=	$1.58\frac{1}{3}$		
40 " " " 250	=	$1.66\frac{2}{3}$		
1000		5.50		
$\frac{1}{2}$ of $\frac{1}{100}$ of 1000 = 5;	5)	5.50		
		1.1 m.		

1.1 m. = 1 m. 3 d.
Aug. 31 + 1 m. 3 d. = Oct. 3.

By paying at the assumed date, Aug. 31, Rives would lose interest on the several bills from Aug. 31 to their respective dates, amounting in all to $5.50; ∴ payment should be made at such time after Aug. 31 as will be required for $1000 to gain $5.50, = 1 m. 3 d., which added to Aug. 31 is Oct. 3, *Ans.* Hence the

RULE. To find the equated time when the terms of credit begin with different dates: *Assume that the time for paying all the items is on the last day of the month previous to the earliest day at which any item is due; find the interest on each item from the assumed date to the date when it is due, and divide the sum of the interests by $\frac{1}{2}$ of $\frac{1}{100}$ of the sum of the items; the quotient will be the time after the assumed date, in months, when all should be paid.*

NOTE I. — .1 month = 3 d.; $.03\frac{1}{3}$ month = 1 day nearly, etc.

NOTE II. — Reject the fraction of a day when it is less than $\frac{1}{2}$; reckon it 1 day when it is $\frac{1}{2}$ or more.

1. Find the equitable time for the payment of the following $300, due April 1, 1869; $450, due Dec. 1, 1869; and $600, due March 10, 1870. (Assumed date, March 31, 1869.)

Ans. Nov. 25, 1869

2. Find the mean or average time for paying the following: \$12.45, due Feb. 10, 1860; \$24.17, due Mar. 1, 1860; \$15, due Mar. 14, 1860; \$30, due Mar. 16, 1860; and \$12.70, due Mar. 7, 1860. *Ans.* Mar. 5, 1860.

Edwin Foote's ledger contains the following account.

	THOMAS WING,	*Dr.*		*Cr.*	
1861.					
July 1.	To Merchandise.	250	00		
1862.					
Apr. 1.	" "	400	00		
Oct. 2.	" "	600	00		

NOTE.—This account shows that Wing bought of Foote merchandise at the times and to the amount indicated.

3. Allowing interest on each item from its date, what is the time from which a note should draw interest in payment of all of the above items? *Ans.* May 9, 1862.

4. What is the equated time for paying the following bill?

NEW YORK, *Jan.* 1, 1861.

E. TRAIN & Co.

1860. Bought of F. FOGG & Co.

Jan. 20.	M'd'se on 3 mo.,	\$100.
Mar. 15.	" " 2 "	100.
Apr. 12.	" " 2 "	100.

The above items will be due as follows: April 20, May 15, and June 12. Equate from these dates. Assumed date, Mar. 31. *Ans.* May 16, 1860.

5. Equate the following:—

ROXBURY, *Jan* 1, 1865.

MR. J. STOW

1864. Bo't of Z. CHURCHILL,

Jan. 5.	M'd'se on 5 mo.,	\$400.
May 5.	" " 4 "	600.
" 15.	" " 4 "	200.

Ans. Aug. 8.

6. If one note should be given for the following three, when should interest commence upon it?

A note for	$200,	dated	May 15, 1864,	on	90 days.
" "	" 250,	"	June 1,	"	" 60 "
" "	" 700,	"	July 8,	"	" 30 "

7. What is the mean time for the payment of the following bills of goods purchased by Calrow & Co. of Armstrong & Co.?

1856.	June 1,	a bill of	$200	on 90 days.
"	Feb 1,	" "	800	" 75 "
"	Apr. 1,	" "	300	" 60 "
"	July 1,	" "	650	" 40 "
1857.	Feb. 1,	" "	1000	" 20 "

8.* What is the equated time for paying the following?

BOSTON, *July* 1, 1864.

J. P. PUTNAM,

To WEYMOUTH IRON CO., *Dr.*

1864.				
Jan.	10.	To Merchandise on 3 mo., ...	$437	90
"	28.	" " " 3 mo., ...	254	25
Feb.	29.	" " " 60 d., ...	144	63
Mar.	12.	" " " 4 mo., ...	159	00
Apr.	8.	" " " 90 d., ...	300	00

9.* Find the equated time for the payment of the following notes held by Page & Son against Washington Manufacturing Co.

A note for	$560.	dated	Jan. 1, 1856,	on 5 months.
" "	846.15	"	Feb. 11, "	" 6 "
" "	728.50	"	Mar. 20, "	" 6 "
" "	400.	"	July 30, "	" 6 "
" "	560.	"	Sept. 12, "	" 8 "
" "	600.	"	Dec. 18, "	" 6 "
" "	500.	"	May 10, 1857,	" " "
" "	350.75	"	" 7, "	" " "
" "	820.20	"	Apr. 17, "	" " "

☞ For Dictation Exercises, see Key.

AVERAGE OF ACCOUNTS.

313. To Average an Account.

Ill. Ex. I have in my ledger an account with F. E. Clarke, both the debt and credit sides of which consist of sundry items. The footing is as follows: —

Dr. side \$250, averaging due Feb. 9, 1863; the Cr. side \$300, averaging due Apr. 4, 1863; at what time should I pay Clarke the balance?

Assuming, as in Art. 312, Jan. 31, 1863, as the time for settling this account, we compute interest on each item from this time till it is due.

By settling Jan. 31, 1863,

I should lose 63 days' int. on	\$300 = \$3.15	
Clarke would lose 9 days' int. on	\$250 = \$.375	
The balance due Clarke is	\$50.	\$2.775 int., my net loss.

$\frac{1}{2}$ of $\frac{1}{100}$ of \$50 = \$.25.
2.775 ÷ .25 = 11.1 m. = 11 m. 3 d.
Jan. 31, 1863 + 11 m. 3 d. = Jan. 3, 1864.

If, by settling at the assumed date, my net loss of interest would be \$2.775, I shall be entitled to keep the balance, \$50, till it has gained \$2.775, which, found by dividing it by the interest of \$50 for 1 month, is 11 m. 3 d.; this added to Jan. 31, 1863, is Jan. 3, 1864, *Ans.*

If, however, the dates were transposed, making Clarke's \$250 due to me Apr. 4, and my \$300 due to Clarke Feb. 9, by settling at the assumed date

Clarke would lose 63 days' interest on	\$250 = \$2.62$\frac{1}{2}$,	
I should lose 9 days' interest on	300 = .45	
The balance due Clarke is	\$50.	\$2.17$\frac{1}{2}$ int., C.'s net loss.

If, by settling at the assumed date, Clarke's net loss of interest is \$2.17$\frac{1}{2}$, he may justly require me to pay the balance, \$50, at such time prior to Jan. 31, 1860, as will be required for \$50 to gain \$2.17$\frac{1}{2}$ of interest, which is 8 m. 21 d.; this, reckoned back from Jan. 31, 1863, is May 10, 1862.

From the above example we deduce the following

RULE. To equate an account: *Assume that all the items are to be paid on the last day of the month previous to the earliest day at which any item becomes due; find the interest on each item from the assumed date to the date at which it becomes due; find the difference between the interest on the Dr. and Cr. sides of the account; divide this difference by the interest on the balance of the account for* 1 *month; add the quotient to the assumed date when the larger side has the more interest, and subtract it from the assumed date when the larger side has the less interest.*

314. Settlement can be effected earlier than the equated time, by deducting interest from the balance of the account for the time between the equated time and the desired time of settlement. It can be effected later by adding interest. The latter will be necessary when the equated time is already past. Or,

315. If it be desired to settle an account at a specified time, add interest to the items due before the specified time, and subtract interest from those due after the specified time; the difference between the total of the Dr. and Cr. items *plus* or *minus* their interest, will be the balance due.

316. EXAMPLES.

1. When can the balance of the following ledger account be paid without loss to either party?

DR. EDWARD C. DAMON. CR.

1863.				1863.		
Apr. 1	To Cash,	$1000	00	Apr. 14	By Mdse.,	$1392 59
July 8	" Mdse.,	118	98	Aug. 10	" Real Estate,	94 33

This account shows that Damon receives,
Apr. 1, $1000.
July 8, 118.98.

And that he is credited,
April 14, with $1392.59.
Aug. 10, " 94.33.

We assume March 31 for settling the account.

[illegible]

Date.	Dr.			Cr.			
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Apr. 1,	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
July 6,	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]		[illegible]		[illegible]		[illegible]
					[illegible]		[illegible]

[illegible]

[illegible]

[illegible]

2. Add 3 [illegible] credit [illegible] each [illegible] in the following, and equate the [illegible].

Dr.			A. [illegible] in a/c with C. [illegible]			Cr.
1863.			[illegible]			
Jan. 1	To Bal. Ledger [illegible]	[illegible]	[illegible]	[illegible]	By Mdse.,	[illegible]
Mar. 31	" Real Estate,	[illegible]	[illegible]	[illegible]	"	[illegible]
May 30	" do. do.	[illegible]				

Assumed date, Mar. 31, 1863. *Ans.* May 12, 1863.

3. When is the balance of the following a/c due?

Dr.			Smith, Dove, & Co.			Cr.
1862.			1862.			
Jan. 6	Mdse., 30 d. cr.,	$600 00	Jan. 1	By R. Estate, 90 d. cr.,		$300 00
Feb. 7	do. 60 d. cr.,	540	Mar. 16	" Cash,		300 00

Assumed date Jan. 31, 1862. *Ans.* Feb. 25, 1862.

4.* When is the balance of the following account due?

Dr.				Day, Wilcox, & Co.			Cr.
1864.				1864.			
July 21	To Mdse., 90 da.,	$173	15	June 25	By Mdse., 30 da.,	$300	00
Aug. 15	" do. 60 da.,	13	68	June 30	" do. 60 da.,	32	71
Aug. 31	" do. 4 mo.,	81	55	Aug. 20	" do. 4 mo.,	16	48
Oct. 17	" Cash,	230	00	Sept. 12	" do. 30 da.,	102	10

Ans. Dec. 26, 1863.

317. To find the equitable time for the payment of the balance of a debt, when partial payments are made before the debt

is due: *Make the whole debt the Dr. side of an account, and the partial payments the Cr. side.*

5. A holds a note against B, dated Nov. 14, 1864, for $620, due 7 months hence, without interest. On this note B paid A $220 Jan. 14, 1865, and $100 Feb. 14, 1865; what is the equated time for paying the balance? *Ans.* Nov. 16, 1865.

6. T. Ropes owes R. Treat $250, due May 29. If he should pay $50 on the 29th of April previous, when should he pay the balance? *Ans.* June 7.

7. A farmer purchased, on the 1st day of April, 1864, 3 acres of land at $183 per acre, agreeing to pay for it in 7 months; if he should pay $50.75 at the date of the purchase, $148.25 in 4 months, and $150 in 3 months, in what time should he pay the balance?

8. A owes B $2000 Oct. 5; if he should pay $1200 of it Sept. 8, at what time should the balance be paid?

9.* J. Edwards owes J. Adams $1200 on a note dated Oct. 9, 1863, payable in 4 months without interest; if Edwards should pay Adams $250 on this note Jan. 16, 1864, and $400 Feb. 9, 1864, when should the balance of the note be paid?

10.* Bought a lot of land for $800, for which I gave my note, dated May 7, 1864, payable in 6 months; June 28, I paid $158; Aug. 7, I paid $320.60, and Sept. 7, $179.40; when should the balance be paid?

11.* Find the equated time for the settlement of the following account:—

ROBERTSON & REYNOLDS in a/c with JAMES LORING & Co.

DR. CR.

1864.				1864.			
July 12.	To Balance,	$562	17	July 18.	By Cash,	$480	00
" 20.	" Mdse. on 4 mo.	1524	82	" 27.	" Note on 90 d.	1218	65
Aug. 8.	" " " 2 mo.	210	00	Aug. 20.	" Real Estate,	600	00
Sep. 30.	" " " 4 mo.	783	25	Sep. 30.	" Cash,	459	50
Nov. 25.	" Bill due,	286	58	Oct. 28.	" Draft at 60d.†	425	00
Dec. 1.	" " "	424	60	Dec. 1.	" Cash,	185	20

☞ For Dictation Exercises, see Key.

† Allow 3 days of grace.

TAXES.

318. **A Tax** is a sum of money assessed upon a person or upon property for public purposes.

319. **A Poll Tax** is a sum assessed upon each male citizen liable to be taxed, without regard to his property. The persons thus taxed are called the *polls*.

320. **Real Estate** consists of immovable property, such as houses, lands, &c.

321. **Personal Property** consists of movable property, such as money, stocks, cattle, ships, &c.

322. **Assessors** are officers appointed to levy taxes. It is their duty to ascertain the value of the taxable property and the number of polls, and to apportion the tax to each person.

323. ILL. EX. The whole state, county, and town tax of Oxford for the year 1862, was $5300; the value of the real estate and personal property is $1250000; there are 200 polls in the town, each of which is taxed $1.50. What is the tax on $1, and what is J. Swan's tax, who has $3000 of real estate and $1000 of personal property, and who pays 1 poll tax?

OPERATION.

$1.50 × 200 = $300, amount of poll taxes.
$5300 — $300 = $5000, property tax.
$5000 ÷ $1250000 = 4 mills, tax on $1 of property.
$3000 + $1000 = $4000, Swan's taxable property.
$4000 × $.004 = $16.00, Swan's property tax.
$16.00 + $1.50 = $17.50, Swan's whole tax.

Hence the

RULE FOR APPORTIONING TAXES. *Multiply the tax on one poll by the number of polls, and subtract the product from the whole tax; divide the balance by the taxable property; the quotient is the tax on* $1. *Multiply each person's taxable property by the tax on* $1, *and add his poll tax, or taxes, if he have any.*

EXAMPLES.

1. The whole tax of the town of H. is \$70352; the valuation of the town being \$9329000, the number of polls being 3366, each taxed \$1.50, what is the tax upon \$1 and what is the tax on the following named tax-payers?

A has property amounting to \$8500, and pays 1 poll.
B has property amounting to \$3570, and pays 1 poll.
C has property amounting to \$5800, and pays 0 polls.
D has property amounting to \$1000, and pays 2 polls.
E has property amounting to \$2800, and pays 3 polls.

Ans. 7 mills on \$1; A's tax, \$61; B's tax, \$26.49.

2. The town of L. votes to raise a tax of \$8343.20; the valuation of the town is \$2000000; there are 1679 polls, each taxed \$.80; what is the tax on a dollar, and what is the tax of J. L. Partridge, who has \$1500 of real estate and \$3000 of personal property, and pays two poll taxes?

Ans. $3\frac{1}{2}$ mills on \$1; Partridge's tax, \$17.35.

3. What would be the tax upon a non-resident who had property in the above-named town of L. to the amount of \$15225?

Ans. \$53.287+.

4. The state tax of a certain town is \$3093; the county tax is \$5110; the town tax, \$33860; the valuation of the town is \$6700000; there are 2542 polls, each taxed \$1.50. What is the tax on \$1, and what is the tax on a person having \$4500 in real estate, \$2750 in personal property, and who pays one poll tax? *Ans.* \$.005$\frac{95}{134}$ on \$1; \$42.889$\frac{63}{67}$.

5. There is a town whose valuation is \$1100000, in which there are 300 polls, each taxed \$1.20; the tax to be raised is \$9600. What is the tax on \$1, and what is the tax of a person having \$4000 in real estate, an annual income of \$3000, on all above \$800 of which he is taxed as for personal property, and who pays three poll taxes? *Ans.* $8\frac{2}{5}$ m. on \$1; \$55.68.

6. The valuation of a certain town in real estate is \$3200000; in personal property \$1186000; the tax to be raised is \$31579.20,

one sixth of which is to be levied upon the polls, of which there are 3096. What is the tax on $1, and what on each poll?

Ans. 6 m. on $1; poll tax, $1.70.

Assessors commonly construct a table showing the tax on $1, $2, $3, &c., from which they compute the individual taxes.

7. The valuation of a certain town is $11522400; the tax to be raised is $108391.60; there are 3350 polls, each taxed $1.40. Find the tax on $1, and perform the remaining examples by the following

TABLE,

Showing the tax on various sums at the rate of $.009 on $1.

$1	pays	$.009	$10	pays	$.09	$ 100	pays	$.90
2	"	.018	20	"	.18	200	"	1.80
3	"	.027	30	"	.27	300	"	2.70
4	"	.036	40	"	.36	400	"	3.60
5	"	.045	50	"	.45	500	"	4.50
6	"	.054	60	"	.54	600	"	5.40
7	"	.063	70	"	.63	700	"	6.30
8	"	.072	80	"	.72	800	"	7.20
9	"	.081	90	"	.81	900	"	8.10
10	"	.09	100	"	.90	1000	"	9.00

8. At the above rate, what is A's tax, he being assessed for $4250, and paying 2 polls?

OPERATION.

$4000	pays	$36.
200	"	1.80
50	"	.45
2 polls	"	2.80

Ans. $41.05, A's tax.

Find the tax of the following at the above rate: —

9. Of B, who is assessed for $2800 and 1 poll.

10. Of C, who is assessed for $7850 and 3 polls.

11. Of D, who is assessed for $1565 and 1 poll.

12. Of E, who is assessed for $9068 and 2 polls.

13. Of F, who is assessed for \$6555 and 1 poll.
14. Of G, who is assessed for \$5687 and 1 poll.
15. Of H, who is assessed for \$10793 and 3 polls.
16. Of I, who is assessed for \$3384 and 1 poll.
17. Of J, who is assessed for \$4597 and 1 poll.
18. Of K, who is assessed for \$8979 and 2 polls.
19. Of L, who is assessed for \$2972 and 1 poll.
20. Of M, who is assessed for \$1000 and 1 poll
21. Of N, who is assessed for \$6587 and 2 polls.
22. Of O, who is assessed for \$7572 and 2 polls.
23. Of P, who is assessed for \$2956 and 1 poll.

☞ For Dictation Exercises, see Key.

CUSTOM HOUSE BUSINESS.

324. **Custom Houses** are places where Government Officers collect *duties*.

325. **Duties** are taxes upon imports and upon the tonnage or weight which a vessel may carry. They are of two kinds, *specific* and *ad valorem*. They furnish a revenue for the government.

326. An **Invoice** is a list of imported goods, showing their quantity and price.

327. A **specific** duty is a tax upon each article of a certain kind, without regard to its value.

328. An **ad valorem** duty is a certain per cent. of the cost of goods, estimated upon the invoice.

329. **Leakage** and **Breakage** are allowances for loss from the leaking and breaking of bottles, boxes, &c.

330. **Tare** is an allowance for the weight of boxes, &c.

331. **Gross Weight** is the weight of goods including whatever is used for packing.

332. **Net Weight** is the weight of the goods alone.

333. EXAMPLES.

1. What is the net weight of 120 boxes of raisins, gross weight being 31 lbs. each, the tare being $6\frac{1}{2}$ lbs. per box?

Ans. 2940 lbs.

OPERATION.

$31 - 6\frac{1}{2} = 24\frac{1}{2}$, net weight of 1 box.
$24\frac{1}{2} \times 120 = 2940$, " " " 120 boxes.

2. What is the duty, at 5 cents per lb., on the net weight of the above? *Ans.* $147.

3. What is the specific duty, at 15 cents per gallon, on 25 barrels spirits turpentine, containing 32 gallons each, 5% being deducted for leakage? *Ans.* $114.

4. What is the duty, at 25 %, on 75 boxes of tin, 112 lbs. per box, invoiced at 7 cents per pound, tare being 6 lbs. per box? *Ans.* $139.125.

5. What is the duty on 100 dozen watch crystals, at 35%, invoiced at $1 per dozen, 3% being allowed for breakage?

Ans. $33.95.

6. What is the duty, at 20% ad valorem, on an invoice of 24 boxes of tea, gross weight being 1284 lbs., 8 lbs. for tare being allowed on each box, the cost of the tea being 38 cents per pound? *Ans.* $82.992.

7. I have imported 3 tons, 3 cwt. 3 qrs. 7 lbs. of steel invoiced at 20 cents per pound; 8% being allowed for damage, what is the duty at 20%? †

8.* What is the cost at the store of 5 hhds. of sugar, weighing gross 2556 lbs., which was bought in Havana for $178.92, and on which is paid $35.75 for freight and carting, and $2\frac{1}{2}$ cents per pound for duty after deducting 15% for tare?

9.* Find the cost of two cases of gum arabic, at 21£. 5s. per cwt., duty 30% ad valorem; the gross weight of the first being 1 cwt. 3 qrs. 20 lbs., of the second 1 cwt. 1 qr. 10 lbs., 35 lbs. being allowed for the weight of each case. *Ans.* 73£ 0s. $2\frac{1}{4}$d.

☞ For Dictation Exercises, see Key.

* See Art. 164, Note.

DRAFTS AND BILLS OF EXCHANGE.

334. A **Draft** is a written order, directing one person to pay money to another. The following is a simple form of a

Draft.

$100. BALTIMORE, April 4, 1864.

Thirty days after sight, pay to Samuel Price, or order, One Hundred Dollars, and charge the same to my account.

CHARLES SMITH.

To BREWER & TILESTON,
Publishers, Boston.

335. The **Drawer** is the person who signs the draft.

336. The **Drawee** is the person to whom the draft is addressed.

337. The **Payee** is the person in whose favor the draft is drawn.

NOTE. — In the above, Charles Smith is the Drawer, Brewer & Tileston are the Drawees, and Samuel Price is the Payee.

338. The **Holder** of a draft is the person who has legal possession of it.

339. The **Endorsement** of a draft is the writing upon its back, by which the payee transfers his right in it to another person.

340. If the drawee does not pay the money when the draft is presented, he may signify his acceptance of it by writing his name on its face, after the word "Accepted," by which act he becomes responsible for its payment.

Bills of Exchange.

341. A **Bill of Exchange** is a draft used by merchants for the discharge of debts payable at a distance.

342. Bills of Exchange are **Inland**, or **Domestic**, when they are drawn and payable in the same country.

343. Bills of Exchange are **Foreign** when they are drawn in one country and payable in another.

ILLUSTRATION. — Suppose A, in New York, ships butter to B, in Liverpool, to the amount of $1000. He makes a draft on B to pay to himself, or "bearer," or "order," the $1000 due. But C, in Boston, wishes cutlery from D, in Sheffield, to the same amount. So he buys A's draft, paying its value in United States money, and sends it to D. D receives it and presents it to B, in Liverpool. B, having received his butter in good condition, *accepts* the draft, and when the time comes for payment of the money, pays it to D, of Sheffield, in English currency. Thus A receives payment for his butter, and D for his cutlery, without the risk or inconvenience of sending the money from one country to another.

344. Bills of Exchange thus bought and sold are said to be **negotiable**, or **marketable.**

345. When the value of the goods sent from the United States to another country— England for example—is greater than the value of those received from England, more money is due to us from the English merchants than is due to them from our merchants, and we hold more bills against England than are needed to pay our debts; consequently, bills become cheap, and exchange is at a **discount.** When the value of the goods imported from England exceeds the value of those sent to England, our merchants hold fewer bills against England than are needed to pay their debts, and bills thus become dear, and exchange is at a **premium.** The current price of exchange is called the **Course of Exchange.**

346. A **Set of Exchange** consists of two or more drafts of the same tenor and date, the payment of either one of which cancels the other one or two.

To provide against accident in the transmission of a draft, it is customary to send two, at least, of a set by different modes of conveyance, or at different times.

INLAND OR DOMESTIC EXCHANGE.

347. To find the Value of an Inland Draft.

Ill. Ex., I. What must be paid in St. Louis for a draft on Philadelphia, for \$2500, payable at sight, exchange being $1\frac{1}{2}\%$ premium in favor of Philadelphia?

Operation.

\$2500 × 1.015 = \$2537.50, *Ans.*

If exchange is $1\frac{1}{2}\%$ premium, the exchange value of \$1 is \$1.015 and the price of \$2500 will be \$2500 × 1.015 = \$2537.50, *Ans.*

Ill. Ex., II. What must be paid in New Orleans for a draft on New York for \$1500, payable in 60 days after sight, exchange being 1% premium?

Operation.

\$1500 × 1.01 = \$1515., Exchange value of \$1500 at sight.
\$1500 × .0105 = 15.75, bank discount of \$1500 for 63 days.
Ans. \$1499.25, cost of draft.

Hence the

Rule. To find the value of an inland draft: *Multiply the face of the draft by the exchange value of \$1. If the draft is payable after sight, deduct from the product the bank discount of the face of the draft for the given time and grace.*

Examples.

1. What is the value of a draft on Boston for \$2500, when exchange is at a premium of $\frac{1}{4}\%$? *Ans.* \$2506.25.

2. What must I pay for a draft on New York for \$700 at 12 days without grace, exchange being at $\frac{1}{8}\%$ premium? *Ans.* \$699.475.

3. Bought a bill on New Orleans for \$400, at a discount of $\frac{1}{2}\%$; what did I pay? *Ans.* \$398.

4. Messrs. B. & T., of Boston, sold a draft for \$206.59 on Billings & Son, of Baltimore, at 30 days' sight, discount being $\frac{1}{2}\%$: what did they receive for it?

348. To find the Face of a Draft.

Ill. Ex. What is the face of a 30 days' draft on Cincinnati at 1% discount, which can be purchased at New York for $200?

Operation.

$\$1 - \$.01 = \$.99$, exchange value of $1 at sight.
.0055, bank discount of $1 for 33 days.
.9845, exchange value of $1 at 30 days.
$\$200 \div .9845 = \$203.148+$, *Ans.*

The exchange value of $1 of the draft will be $.9845; if $1 can be purchased for $.9845, as many dollars can be purchased for $200 as $.9845 is contained times in $200, which is 203.148+ times. *Ans.* $203.148+. Hence the

Rule. To find the face of a draft which may be purchased for a given sum: *Divide the given sum by the exchange value of* $1.

Examples.

1. Wishing to remit to my correspondent at St. Louis the net proceeds of a lot of wheat, amounting to $1275, I purchase with that sum a draft at $1\frac{1}{8}\%$ discount; required the face of the draft. *Ans.* $1289.506+.

2. What is the face of a draft for 15 days, which may be purchased for $1050, at $1\frac{1}{2}\%$ premium? *Ans.* $1037.549+.

3. What is the face of a bill on Boston for 60 days, at $\frac{1}{8}\%$ premium, which may be bought for $3000?

FOREIGN EXCHANGE.

349. The method of computing foreign exchange is similar to that of computing inland exchange, except that the currency of one country must be reduced to that of another.

350. The value of 1£ sterling, which is the English sovereign, compared with the old United States coin, equals $\$4.44\frac{4}{9}$. But Congress has from time to time reduced the weight and purity of United States coins, making their value as metals less than their value as coins, that they might not be used for transportation or

the arts, and has established the legal value of the pound sterling at $4.84. The intrinsic value of the pound is $4.861. The commercial value varies from $4.83 to $4.87, as it is in greater or less demand.

351. Exchange, however, is reckoned upon the old or nominal value of the pound ($$4.44\frac{4}{9}$), and the present value is said to be at about 9% premium. Thus, when exchange on England is quoted at 10 or 11% premium, it is really only at about 1 or 2% premium upon the real value.

352. Examples in Reduction of Currency and Exchange.

1. What is the nominal value of £250 sterling expressed in United States money?

$$\$4.44\tfrac{4}{9} = \$\tfrac{40}{9}. \quad \tfrac{250 \times 40}{9} = \$1111.11\tfrac{1}{9},\ Ans.$$

2. What is the United States legal value of the above? *Ans.* $1210.

3. Reduce 1£ sterling to Federal money at $9\frac{1}{2}$% advance upon the nominal value. *Ans.* $$4.86\frac{2}{3}$.

4. Reduce 40£. 10s. to Federal money, at $9\frac{1}{2}$% advance. (See Art. 265, Note.)

5. What will be the value in Federal money of 84£. 19s. $11\frac{1}{4}$d. at 10% advance?

6. What is the cost in New York of a bill on Liverpool for 470£. 13s. 9d., at $9\frac{1}{4}$% premium?

7. A gas company purchased Newcastle coal in England to the amount of 1000£. 15s. $7\frac{3}{4}$d.; exchange being $8\frac{1}{2}$% premium, what will this be worth in United States money?

8. A dealer in flour shipped to London 3000 barrels of flour, which cost $4.20 a barrel in Baltimore; the flour was sold in London at 1£. 6s. 6d. per barrel, exchange being 10% advance; what was the gain, without regard to expenses?

9.* How much Federal money will pay for 3 T. 15 cwt. 2 qr. 1 lb. of iron, at 7£. 10s. 9d. sterling per ton, when the premium is $9\frac{1}{2}$%?

10.* A cotton broker sent to Manchester, England, 50 bales of

cotton averaging 460 pounds each; the cotton was sold at 11d. per pound. What was the amount of sales in United States money, premium being $10\frac{3}{4}\%$, and what was the broker's commission at $1\frac{1}{4}\%$ upon the sales?

11. Reduce \$200 to sterling money when exchange on England is 10% advance. $\frac{200 \times 9 \times 100}{40 \times 110} = 40£.\ 18s.\ 2\frac{2}{11}d.$, *Ans.*

12. Reduce \$575 to sterling money at $9\frac{3}{4}\ \%$ advance.

13. When exchange is 9% premium, what is the value of \$6874.40?

14.* What will a merchant gain by buying 4000 bushels of corn in Albany at 65 cents per bushel, paying for shipping and other expenses 30 cents per bushel, and selling it in Liverpool, England, at 4s. 9d. per bushel, when exchange is 10% premium?

15.* A merchant in 1864 shipped to Liverpool 5000 pounds of butter, which cost him in New York 35 cents per pound; he paid 4% for freight and other expenses, and sold it in Liverpool for 11d. per pound. Exchange being 120% premium, did he gain or lose, and how much?

16. What is the cost of a set of exchange on Paris, for 6000 francs, exchange being $6\frac{2}{3}$ francs per dollar?

17. What is the cost of a set of exchange on Paris for 4500 francs at 5% premium, the value of 1 franc being $18\frac{2}{3}$ cents?

18. Mr. James Payne, an American gentleman, while travelling in England received the following draft; what was the cost of the draft in America, the premium being 41% in favor of England?

£127 BOSTON, *Aug.* 23, 1863.

At sight of this, our first Bill of Exchange (second and third of same tenor and date unpaid), pay to the order of James Payne, in Manchester, one hundred and twenty-seven pounds sterling, value received, and charge the same to our account.

J. E. THAYER & BRO.

To Messrs. CALMONT BROS. & Co, Bond Street, London.

☞ For Dictation Exercises, see Key.

353. Questions in Review.

What is PERCENTAGE. From what is per cent. derived? In what four ways represented? Represent $\frac{3}{8}\%$ decimally.

How is a per cent. reduced to lowest terms?

How is a common fraction reduced to a per cent.?

What is the complement of any rate per cent.?

Reduce $\frac{3}{8}$ to a per cent.; represent it decimally; find its complement, and reduce that to its lowest terms.

How do you find any required per cent. of a number?

Name the applications of percentage in this book. (See Contents.)

Upon what is the percentage of PROFIT or LOSS reckoned?

If goods cost 24 cents, for what must they sell to gain $8\frac{1}{3}\%$? to lose $16\frac{2}{3}\%$? What per cent. would be gained or lost by selling the above at 30 cents? at 21 cents? If 24 cents is 20% less than the value of goods, what is the value? if 24 cents is $33\frac{1}{3}\%$ more than value? If 18 cents is 10% less than cost, for what would you sell to gain 10%? to lose 25%? If 10% of what you receive for goods is gain, what is your gain per cent.?

What is INTEREST? principal? amount? legal rate? usury?

Usual rate in United States? Rule for finding interest on $1 at 6%? on any sum?

At 6%, in what time will a sum *double*? will the interest equal $\frac{1}{2}$ of the principal? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{1}{12}$? $\frac{1}{24}$? $\frac{1}{25}$? $\frac{1}{50}$? $\frac{1}{100}$? $\frac{1}{1000}$?

At 6%, what part of the principal is the interest for 1 mo.? for 3 mo.? 5 mo.? $6\frac{2}{3}$ mo.? 11 mo.? 13 mo. 10 d.? 1 y. 4 mo. 20 d.? 1 y. 8 mo.? 5 y. 6 mo. 20 d.?

What is the interest on $1 for 3 y. 4 m.? 6 d.? 3 d.? 1 d.? 20 d.? $33\frac{1}{3}$ mo.?

How obtain interest at any rate besides 6%?

How obtain interest on pounds, shillings, &c.?

What is the rule for reducing shillings, pence, and farthings, to decimal of 1£ by inspection? What is the value of 1s. in decimal of £1? 3s.? 5s.? 1 qr.? 12 qr.? 6d.? 9d.?

Name the first month in the year; the fourth; the seventh; the tenth; third; twelfth; ninth; fifth; eighth.

How many months and days forward from June 7th to the 2d of each of the above? from November 15th? from March 28th?

What are PARTIAL PAYMENTS? What is the legal rule for partial payments? How is the record of payments kept? What are the pay-

ments called? Suppose an endorsement will not cancel the interest? Rule in common use when the note is paid within one year? Rule for annual interest?

What is COMPOUND INTEREST? How often may interest be compounded? For how many periods of time will interest be compounded in 2 y. 9 mo., if it is compounded semi-annually? quarterly? monthly?

What three factors are employed to produce interest? The interest, principal, and rate being known, give the rule to find time. The interest, principal, and time being known, give the rule to find rate. The interest, rate, and time being known, give the rule to find principal. What is the dividend in each case? To find time, for what time do you take interest for a divisor? to find rate, at what rate? to find principal, on what principal? Amount, rate, and time being known, give rule to find principal.

What does PRESENT WORTH embrace? What is discount? Give rule for present worth. How find discount? How prove the work?

What is a PROMISSORY NOTE? What the face of a note? What is bank discount? What are days of grace? avails of a note?

Which is the larger, true or bank present worth? true or bank discount?

Describe the process of getting a note discounted at a bank.

What is endorsing a note?

Rule for finding the face of a note, which, discounted at a bank, will yield a certain sum?

What is COMMISSION? BROKERAGE? What are STOCKS? When are stocks above par? below par? at an advance? discount? premium?

Upon what is the per cent. of commission or brokerage estimated?

What is the rule for finding what sum is to be laid out when a remittance is made which contains that sum together with the commission? How obtain commission?

What is INSURANCE? policy? premium? What are underwriters?

What is the rule for finding the policy which will cover a certain sum and the premium?

What is EQUATION of PAYMENTS? the interest rule? Rule for equating an account?

What are TAXES? polls? real estate? personal property? assessors? How find tax on $1?

What are CUSTOM-HOUSES? DUTIES? What is a specific duty? an ad valorum duty? leakage and breakage? tare? tonnage of vessels? gross weight? net weight?

What is a DRAFT? Who is the drawer? the drawee? the payee? the holder? What is the endorsement of a draft? the acceptance?

What is a BILL OF EXCHANGE? What is Inland Exchange? Foreign Exchange? When a bill negotiable? when at a premium? when at a discount? Define course of exchange; a set of exchange. Give the rule for finding the value of a draft; for finding the face of a draft.

What is the nominal value of 1£. sterling? the legal value? the intrinsic value?

Upon what is Sterling Exchange reckoned?

354. MISCELLANEOUS EXAMPLES.

1. Reduce $\frac{1}{13}$ to a per cent.
2. Represent $1\frac{1}{20}\%$ decimally.
3. Reduce $106\frac{1}{4}\%$ to its lowest terms.
4. Find the complement of $84\frac{3}{7}\%$.
5. What is 124% of 5 T. 3 cwt.?
6. What is the amount at 6%, simple interest, of $38.75, from Aug. 5 to Nov. 10?
7. What is the amount of $380.25, at 6% compound interest, for 2 yrs. 5 mo.?
8. What is the simple interest, at 5%, of 10£. 8s. 6d., for 4 yrs. 2 mo.?
9. What is the compound interest of the above at the same rate and for the same time?
10. $2500. CHICAGO, *April* 4, 1862.

In three months from date, I promise to pay JOHN PEIRCE, or bearer, twenty-five hundred dollars, with interest after at 6%, value received. P. T. IVISON.

The above was endorsed as follows: $1010 March 28, 1863; $100 Aug. 10, 1864; $1000 Jan. 1, 1865. What was still due?

11. At what per cent. must $450 be on interest 9 months to gain $13.50?
12. I received $7.35 for the use of $150 a certain time at 7%. Required the time.

13. Lent a certain sum for 1 y. 6 mo. at 5% ; the interest being \$9.40, what was the sum?

14. What principal will amount to \$63.25 in 1 y. 3 mo. at 8%?

15. \$150.25. WINTHROP, *Jan.* 5, 1860.

On the fifteenth of May, 1860, I promise to pay to the order of B. F. TWEED one hundred fifty $\frac{25}{100}$ dollars, with interest after, at 6%, value received. D. D. DANIELS.

If the holder of the above have it discounted at a bank Feb. 15, 1860, what will he receive?

16. What will be the true present worth of the above at its date?

17. What would settle the above Oct. 27, 1860?

18. What would settle the above at compound interest Oct. 27, 1865?

19. What must be the face of a note, which, discounted at a bank for 30 days and grace, would yield \$500?

20. In what time will a sum of money double at 2% simple interest?

21. Find the commission for selling the following lot of hogs at $2\frac{1}{2}$%:

Sales on a/c of Messrs. BISHOP & TROWBRIDGE,
By BONNEY & WAITE.

7 Hogs, as follows: 293 lb. 317 lb. 300 lb.
219 lb. 314 lb. 323 lb. 184 lb.

Amount, ______________ at $7\frac{1}{2}$ cts. per lb.

22. What will a broker's bill be for 5 shares of stock purchased for me at 7% advance, shares having originally been \$500, his brokerage at $\frac{1}{2}$% included?

23. Required the value of 1£ sterling at 9% advance upon the nominal value.

24. What will be the contents of a piece of cloth originally

1 yard long by $2\frac{1}{4}$ yards wide, after sponging, if in that operation it shrinks 4% in length and 6% in width?

25. A commission merchant receives $544; of this he is to invest such a portion as remains after deducting his commission of $2\frac{1}{2}$%. What is his commission, and what will remain?

26. What is the cost of insuring $2500 at $17.50 on $1000?

27. What will be the net loss to an insurance company in case of the loss by fire of a property insured for $4500, on which the company had received 3% premium, no allowance for interest?

28. For what sum must a policy be taken out to cover $2575 and the premium of $1\frac{1}{4}$%?

29. What per cent. of 10£ is 15s.?

30. Of how many rods is 84 rods $87\frac{1}{2}$%?

31. Take 9% of 7.5 acres.

32. Paid $18.77 for insuring my schooner at a premium of $\frac{1}{2}$%; what was the sum covered?

33. What is the par value of stock, which, selling at 25% above par, brings $500?

34. In the year 1862 the town of B voted to raise, by taxes, $97290; $\frac{1}{8}$ of this was levied upon the polls; the valuation of the town was $10134375; what was the tax on $1, and what was the tax of a non-resident who owned a house in town valued at $2000?

35. Reduce 750£. 10s. 4d. sterling to United States currency, exchange at 17% advance upon the nominal value.

36. A debtor owes $200, $\frac{1}{2}$ due in 2 months, $\frac{1}{4}$ in 3 months, and the remainder in 5 months; what is the equated time for paying the whole?

37. If $\frac{1}{3}$ of a sum of money be due in 2 months, $\frac{1}{4}$ in 4 months, $\frac{1}{6}$ in 3 months, and the remainder in 4 months, in what time should the whole be paid?

38. What is the average time for paying for $200 worth of goods purchased May 17, 1859, on 4 months' credit; $500 worth, purchased June 18, 1859, on 60 days' credit; and $300 worth, purchased June 19, 1859, on 90 days' credit?

39. There is a note, dated July 1, on 60 days' credit, for $200.

July 20 there is paid \$50; Aug. 19, \$60; and Aug. 21, \$10. When should the balance be paid?

40.* Thomas Swain owes Wm. C. Chapin \$500.18, due Jan. 7, 1863, and \$207.06, due April 4, 1863; Mr. Chapin owes Mr. Swain \$800, due Feb. 8, 1863. When should the balance be paid?

41.* What is the cash value of the above March 25, 1863?

42.* What is the date of a note which must be given to settle the following, allowing 4 months' credit on each of the *merchandise* items?

JOHN F. STONE in a/c
with THOMAS EMERSON'S SONS. *Dr.* *Cr.*

		Dr.		Cr.	
1863.					
Sept. 15.	To 200 bbls. Apples, @ \$3.25, ...	650	00		
Dec. 1.	" 100 " Flour, " 3.48, ...				
1864.					
Feb. 10.	" 35,000 Bricks, " 6.00, per M. . .				
Mar. 1.	" Cash, ...	500	00		
Jan. 9.	By Merchandise, ...			510	00
" 25.	" do., ...			476	00
Apr. 1.	" Cash, ...			265	00

355. MISCELLANEOUS EXAMPLES IN PROFIT AND LOSS.

NOTE. — Younger pupils can omit this article till they review.

1. By selling goods at 6 cents 3 mills per yard, I lose 30%; what did they cost?

2. If I lose 10% by selling goods at 18 cents per yard, for what should they have been sold to gain 20%?

$$\frac{18 \times 100}{90} = 20 \text{ cts., the cost; } \quad \frac{20 \times 120}{100} = 24 \text{ cts.} \quad \textit{Ans.}$$

3.* By selling a lot of iron at $12\frac{1}{2}\%$ below cost, I received \$14.75 less than I should have received if I had sold it at $12\frac{1}{2}\%$ above cost; what did it cost? what should it have sold for to gain $12\frac{1}{2}\%$?

4. A merchant bought 5 cwt. 1 qr. of coffee for \$63; for what must he sell it per lb. to gain $16\frac{2}{3}\%$?

5. For what must hay be sold per ton to gain $13\frac{1}{3}\%$, if by selling at \$16, $33\frac{1}{3}\%$ be gained?

NOTE. — $\$16 \div 1.33\frac{1}{3} = \$12$; $1.13\frac{1}{3}$ of $\$12 = \$13.62$, Ans.

6. If $12\frac{1}{2}\%$ be gained by selling ladies' slippers, at \$9 per dozen pairs, for what should they have been sold per pair to gain 35%? *Ans.* 90 cents.

7. 13% is lost by selling a lot of land for \$783; what would it have brought if it had been sold at a loss of $8\frac{1}{3}\%$?

Ans. \$825.

8. 50% of a certain number exceeds 35% of it by \$13.70; what is the number? *Ans.* $\$91.33\frac{1}{3}$.

9. A person takes a note on 2 months' credit for \$110 in payment for a watch; on getting the note discounted at a bank, he finds that he has lost 40% on the first cost of the watch. Required the cost? *Ans.* $\$181.40\frac{5}{8}$.

10. A broker purchases a lot of stocks at an average of 9% below par, and sells them at an average of $7\frac{2}{3}\%$ above par, and makes \$300; what was the par value of the stocks?

11. If, by selling goods at 60 cents per lb., 20% is gained, what % would have been gained by selling them at 75 cents per lb.?

Ans. 50%.

12. If 10% is lost by selling boards at \$7.20 per M., what % would be gained by selling them at 90 cents per C.?

13. Sold boots at \$3.55 per pair, and thereby lost 29%; what % would have been lost by selling them at \$57.50 per dozen pairs?

14. A dealer has 18 barrels of sound apples remaining in a lot of which 10% have decayed; if his lot cost him \$1.50 per bbl. would he gain or lose on the lot, and what % by selling the remainder at \$1.75 per bbl.? *Ans.* 5% gain.

15. Sold 4 ploughs at \$24 each; on 2 of them I made 20%, and on 2 I lost 20%; what did I gain or lose on the whole?

Ans. Lost \$4.00.

16. 20% of a lot of barley, originally 5000 bushels, was destroyed by fire, the cost having been $\$1\frac{1}{4}$ per bushel; what per cent. will be gained on the lot by selling the remainder at \$2 per bushel?

17. By losing 3 cents on a pound, I get $87\frac{1}{2}\%$ of the cost of butter; if I had lost 4 cents a pound, what % should I have received? *Ans.* $83\frac{1}{3}\%$.

18. By having 5 pupils absent from school my attendance is $93\frac{3}{4}\%$; if my attendance had been 95%, how many pupils must have been absent? *Ans.* 4 pupils.

19. What will be the % of gain on the cost of 174 shares of Roxbury Gas Co.'s stock, nominal value of shares being \$87.50, if it be bought at 15% below par, and sold at $19\frac{1}{2}\%$ above par?

20. If I buy coal at \$4.12 per ton on 6 months' credit, for what must I sell it immediately to gain 10%? *Ans.* \$4.40.

21. If I pay \$3.90 cash, for what must I sell it on 4 months to gain 20%?

22. If, by selling wood at 75 cts. per cd. ft., $6\frac{1}{4}\%$ be lost, for what should it have been sold per cord to gain $3\frac{1}{2}\%$? *Ans.* \$6.624.

23. I sell $\frac{1}{3}$ of a lot of goods for \$9, and thereby lose 25%, for what must I sell the remainder to make $8\frac{1}{3}\%$ on the whole? *Ans.* \$30.

24. A grocer bought 100 gallons of oil, at \$1 per gallon; he mixed with it 50 gallons that cost \$1.60 per gallon, then sold the whole at the rate of \$1.44 per gallon; did he gain or lose, and what %?

25. Suppose the above was sold on a credit of 6 months, what was the % of gain?

26. Suppose the oil to have been bought on 4 months, and sold for cash; what % was gained?

27. For what should he sell the above per gallon to make 25 %, if he bought for cash and waited 10 months for his pay?

28. How much water must be mixed with a barrel of ink (31 galls.), which cost \$34.10, that it may be sold at \$1.10 a gallon, and a profit of 25% be realized by it? *Ans.* $7\frac{3}{4}$ gall.

29. What water would be required in the above, if the cost had been \$25, the profit 20%, and the selling price \$.75 per gallon?

30. If 20% of what I receive for an article is gain, what is the gain %?

NOTE. — If 20% is gain, 80% is cost; the gain then is $\frac{1}{4}$ of the cost, which equals 25%, *Ans.*

31. If 25% of what I receive is gain, what is the gain %?

RATIO.

357. **Ratio** is the relation which one number bears to another number of the same kind.

Ratios are of two kinds, **Arithmetical** and **Geometrical.**

358. **Arithmetical Ratio** is ratio of numbers with respect to their difference; as $6 - 4 = 2$.

GEOMETRICAL RATIO.

359. **Geometrical Ratio** is ratio of numbers with respect to their quotient; as $2 : 4 = \frac{1}{2}$, read 2 is to 4, or the ratio of 2 to $4 = \frac{1}{2}$; $6 : 3 = 2$, read 6 is to 3, or the ratio of 6 to $3 = 2$.

360. The first term of a ratio is called the **Antecedent,** the second, the **Consequent;** both together are called a **Couplet.**

What is the antecedent in the first illustration in Article 359? the consequent in the second? the ratio in the first? the consequent in the first? the ratio in the second?

361. When the terms of a ratio are equal, the ratio is one of **equality;** when the antecedent is greater than the consequent, it is a ratio of **greater inequality;** when the antecedent is less than the consequent, it is a ratio of **less inequality.**

362. It will be readily seen that ratios, being expressions for division, are similar to fractions. They can at any time be written in a fractional form, the antecedent taking the place of the numerator, and the consequent that of the denominator. The principles applicable to fractions apply also to ratio. Hence,

Multiplying the antecedent, or dividing the consequent, } *multiplies the ratio.*

Dividing the antecedent, or multiplying the consequent, } *divides the ratio.*

Multiplying or dividing both terms of a ratio by the same number, } *does not alter its value.*

363. Ratios, like fractions, may be **simple, complex,** or **compound.** A ratio is **simple** when each term is a simple number; it is **complex** when either term contains a fraction; it is **compound** when it is the indicated product of two or more ratios.

Simple Ratio.	Complex Ratio.	Compound Ratio.
$2:8.$	$\frac{2}{3}:\frac{2\frac{1}{2}}{7}.$	$2\times3:5\times5.$

364. Exercises.

1. Write the ratio of 2 to 3; of 7 to 10; of $\frac{2}{3}$ to $\frac{4}{5}$; of 2×7 to 5×4.
2. Multiply the ratio $3:4$ by 2.
3. Divide the same by 2.
4. Reduce the ratio $6:8$ to lower terms.
5. Write any ratio of equality; of greater inequality; of less inequality.

365. Ill. Ex. Reduce $\frac{2}{3}:\dfrac{5}{2\frac{1}{3}}$ to a simple ratio.

Operation.

$\dfrac{5}{2\frac{1}{3}}=\frac{15}{7}$. Multiplying each term of the ratio $\frac{2}{3}:\frac{15}{7}$ by 3×7, we have $\dfrac{2\times\cancel{3}\times7}{\cancel{3}}:\dfrac{15\times3\times\cancel{7}}{\cancel{7}}=14:45$, *Ans.* Hence,

To reduce a complex ratio to a simple one: *Reduce each term to its simplest form, then multiply each by the least common multiple of the denominators, and cancel.*

Reduce to simple ratios,

1. $2\frac{1}{2}:\dfrac{6}{7\frac{1}{3}}.$
2. $\dfrac{8\frac{1}{5}}{\frac{5}{2}}:8.$
3. $3\times5:4\times8.$
4. $5\times4:3\times\frac{1}{2}.$
5. $\frac{2}{3}\times5:\frac{2}{5}\times1\frac{1}{2}.$

PROPORTION.

366. **Proportion** is an expression of equality between two ratios; thus, $2:3 = 4:6$, read 2 is to three as 4 is to 6; that is, 2 is the same part of 3 that 4 is of 6. 2 is $\frac{2}{3}$ of 3, and 4 is $\frac{2}{3}$ of 6.

367. The first and fourth terms of a proportion are called the **extremes,** and the second and third are called the **means.** The first ratio is called the **first couplet,** and the second ratio the **second couplet.** Read the following proportions: —

1.	$5:10 = 3:6.$	3.	$8:2 = 12:3.$
2.	$\frac{1}{3}:1 = 5:15.$	4.	$56:7 = 8:1.$

Name the extremes of the first proportion; the means of the second; the antecedents of the third; the consequents of the fourth; the second couplet of the first proportion.

368. Inverse Proportion. Four terms are **directly proportional** when the first is to the second as the third is to the fourth. They are **inversely proportional** when the first is to the second as the fourth is to the third, or when one ratio is direct and the other inverse. Thus, the *amount of work* done in any given time is *directly* proportional to the men employed; *i. e.*, the more men, the more work; but the *time* occupied in doing a certain work is *inversely* proportional to the men employed; *i. e.*, the more men, the less time.

369. A **compound proportion** is an equality between a compound ratio and a simple ratio, or between two compound ratios.

370. Three terms are in proportion when the first is to the second as the second is to the third. The second term is called a **mean proportional** between the other two; thus, in the proportion, $3:6 = 6:12$, 6 is a mean proportional between 3 and 12.

371. The performance of arithmetical examples by proportion depends upon the following important principle: —

In every proportion the product of the means equals the product of the extremes.

ILLUSTRATION.

$$2:3=4:6$$
$$\tfrac{2}{3}=\tfrac{4}{6}$$
$$\frac{2\times \cancel{3}\times 6}{\cancel{3}}=\frac{4\times 3\times \cancel{6}}{\cancel{6}}$$
$$2\times 6=4\times 3$$

Writing the given proportion in a fractional form, we have $\frac{2}{3}=\frac{4}{6}$. Multiplying each fraction by the product of the denominators, and cancelling, we have $2\times 6=4\times 3$. But 2 and 6 are the extremes, and 4 and 3 the means; hence the product of the extremes equals that of the means.

372. From the above, it follows that whenever an extreme in a proportion is wanting, it can be found *by dividing the product of the means by the given extreme; and whenever a mean is wanting, it may be found by dividing the product of the extremes by the given mean.*

Supply the terms wanting in the following proportions:—

1. $5:1=50:?$
2. $8:?=3:10.$
3. $9:7=?:56.$
4. $?:2=15:5.$

373. In the proportion $2:4=4:8$, $4^2=2\times 8$, $\therefore 4=\sqrt{2\times 8}$ (Arts. 91, 92); hence *a mean proportional between two numbers equals the square root of their product.*

Supply the mean proportionals between the following numbers and write the proportions:—

5. 2 and 8.
6. 2 and 18.
7. 2 and $24\frac{1}{2}$.
8. 20 and 5.
9. 3 and 27.
10. 16 and 4.

ANALYSIS AND PROPORTION.

374. ILL. EX., I. If 15 boxes of oranges cost \$60, what will 17 boxes cost?

OPERATION BY ANALYSIS.

$$\frac{\overset{4}{\cancel{\$60}}\times 17}{\cancel{15}}=\$68,\ \textit{Ans.}$$

If 15 boxes cost \$60, 1 box will cost $\frac{1}{15}$ of \$60, and 17 boxes will cost $17\times\frac{1}{15}$ of \$60. Cancelling and multiplying, the result is \$68.

OPERATION BY PROPORTION.

$$15:17=\$60:\text{Ans.}$$
$$\frac{17\times \overset{4}{\cancel{\$60}}}{\cancel{15}}=\$68,\ \textit{Ans.}$$

\$60, the price of 15 boxes, must bear the same relation to the price of 17 boxes that 15 bears to 17. We have then three terms of a proportion ($15:17=\$60:$), and can find the fourth by multiplying the second and third together, and dividing the product by the first. Hence we *derive the following*

RULE FOR SIMPLE PROPORTION. *Make the number that is of the same kind as the required answer the third term. If the answer should be greater than the third term, make the larger of the other two numbers, upon which the answer depends, the second term, and the smaller the first. If it should be less, make the smaller number the second term, and the larger the first. Multiply the means together, and divide their product by the first extreme.*

NOTE. — Analysis is the more natural and philosophical method of solving arithmetical questions; but the principles of Proportion are applicable to certain classes of examples. It is recommended to the pupil to perform the following examples in both ways. He should, at least, perform a sufficient number of them by Proportion to fix the method in his mind.

EXAMPLES.

1. If 2 men build 17 rods of wall in a week, how many rods will 100 men build in the same time?

We make 17 rods, which is of the same denomination as the required answer, the third term. As 100 men will build more wall than 2 men, we make 100 the second term, and 2 the first term, and the statement is, $2 : 100 = 17 : 850$. *Ans.* 850 rods.

2. If 9 lbs. of lead make 150 bullets, how many bullets can be made from 105 lbs.? *Ans.* 1,750 bullets.

3. If 65 pairs of boots can be made from 75 lbs. of calfskin, how many pairs can be made from 850 lbs.? *Ans.* $736\frac{2}{3}$ prs.

4. How many tons of hay can be made from 750 acres of land, if 13 tons can be made from 3 acres? *Ans.* 3250 tons.

5. If $2000000 will support an army of 500000 men a day, how many men can be kept for $400000? *Ans.* 100,000 men.

6. If $500 purchase 200 hats, how many hats can be purchased for $$87\frac{1}{2}$? *Ans.* 35 hats.

7. If $800 yield $56 of interest in a certain time, what will $390 yield at the same rate? *Ans.* $27.30.

8. If 16 horses eat a certain quantity of hay in 13 weeks, how long would the same quantity last 24 horses? *Ans.* $8\frac{2}{3}$ weeks.

9. What time would be required for 5 men to mow an acre of land, if 2 men can mow it in $1\frac{1}{2}$ days of 10 hours in length? Ans. 6 hours

10. If 500 bushels of plaster were sufficient for the dressing of $3\frac{1}{2}$ acres of land, what would be required for $17\frac{1}{2}$ acres?

Ans. 2500 bu.

11. If 95 acres of land were mowed by 20 men in 2 days of 12 hours each, how much time would be required for 3 mowing machines to do the same work (1 machine being equal to 4 men)? *Ans.* 3 d. 4 h.

12. If a Graham loaf weighs 1 lb. 2 oz. when flour is worth $\$7\frac{1}{2}$ a bbl., what should it weigh, selling at the same price, when flour is \$6 per bbl.? *Ans.* 1 lb. $6\frac{1}{2}$ oz.

13. If 400 lbs. of coal are required to run an engine 12 hours, what number of tons will be required to run three similar engines for 30 days constantly?

14. If it requires 13 days of 9 hours each to do a piece of work, how many days of 14 hours each will be required to do the same work? *Ans.* $8\frac{5}{14}$.

15. If soda-crackers can be sold at 10 cents a pound when flour is worth \$6.50 per bbl., for what can they be sold when flour is worth \$9.75 per bbl.?

16. If it requires 30 men to do a piece of work when they work 11 hours a day, what number will be required when they work 15 hours a day?

17. If 30 men, working 11 hours a day, can do a piece of work in a certain time, how many more men must be employed, when it is half done, to finish it in the same number of days, working 10 hours a day? *Ans.* 3 more.

18. A pole 10 ft. long casts a shadow of 7 ft. 1 in. at 8 o'clock; what is the height of a flagstaff that casts a shadow 58 feet at the same time of day? *Ans.* $81\frac{1}{17}$ ft.

19. If my friend lends me \$7000 for 15 days, for what time should I lend him \$4500 to requite the favor? *Ans.* $23\frac{1}{3}$ d.

20. If my friend lend me money for 4 months when interest is 10 per cent., for what time should I lend him the same sum when interest is 7 per cent.?

21. A sail vessel has 10 hours the start of a steamer, and sails at the rate of 7 miles an hour, while the steamer sails 16 miles an hour; when will the steamer overtake the sail vessel? *Ans.* $7\frac{7}{9}$ h.

22. A deer, 150 rods before a hound, runs 30 rods a minute; the hound follows at the rate of 42 rods a minute; in what time will the deer be overtaken?

23. Two armies are on opposite sides of a river, one being 300 miles east and the other 250 miles west of it, and marching towards each other, the first at the rate of 15 and the other of 18 miles in a day; in what number of days will they meet, and where? *Ans.* $16\frac{2}{3}$ d.; 50 m. east.

24. If 2 lbs. 5 oz. of wool make 1 yd. of cloth 32 inches wide, how much will make a yard $1\frac{1}{4}$ yards wide of the same quality? *Ans.* 3 lb. $4\frac{1}{32}$ oz.

25. How much cloth that is $\frac{3}{4}$ yd. wide will cover 24 tables 6 ft. long and 3 ft. wide? *Ans.* 64 yds.

26. If it requires 10 yards of cloth that is $1\frac{7}{8}$ yds. wide to make a garment, how much will be required of that which is $1\frac{3}{8}$ yds. wide?

27. How many yards of cambric 24 inches wide will be required to line $14\frac{1}{2}$ yards of silk which is 22 inches wide?

28. How long must a piece of land be to contain 3 acres, if it is 4 rods wide?

29. If 400 bushels of potatoes were bought for \$350.90, and sold for \$425.50, what would be gained on 25 bushels?

30. A man failing in trade owes \$7,865; his property is sold for \$4385.70; what will he be able to pay to a creditor to whom he owes \$1500?

31. If it costs \$30 to paint the outside of a house 40 ft. by 30 ft. and 25 ft. high, what will it cost to paint one 50 ft. by 40, of the same height? *Ans.* \$38$\frac{1}{4}$.

32. If a building 13 ft. high casts a shadow of 4 ft., what is the length of a shadow cast by a tree $346\frac{3}{4}$ ft. high at the same time?

33. If \$110 be paid for 8 T. 12 cwt. 20 lb. of hay, what will be the cost of 6 T. 3 cwt. 3 lbs.? *Ans.* \$78.59$\frac{11}{287}$.

34. If a hind wheel, which is $8\frac{3}{4}$ feet in circumference, turns 800 times in a journey, how many times will the fore wheel which is $6\frac{1}{2}$ feet in circumference, turn in the same journey?

35. The weight of a cubic foot of water being $62\frac{1}{2}$ lbs., how many pounds' weight will a tank contain which is $2\frac{1}{2}$ ft. square at the bottom, and 4 ft. high?

36. If 15 A. 2 R. 20 r. pasture 2 cows a certain time, what will be required to pasture 25 cows the same time?

37. How many yds. of cloth can be bought for 17£. 5 s., if 10 s. were paid for 5 quarters, or 1 Ell English?

38. What is the value of 7 cords 3 cord feet of wood at the rate of \$18 for 2 cords 5 cord feet?

39. If $5\frac{5}{8}$ cwt. of leather pay for $\frac{3}{7}$ of an acre of land, how many pounds would be required to pay for $1\frac{1}{2}$ sq. rods?

Ans. $12\frac{39}{128}$ lbs.

40. If 9 oz. 10 pwt. 3 gr. of gold be worth \$150, what will be the value of 7 lb. 5 oz. 5 pwt.?

41. If $15\frac{3}{8}$ lbs. of tea pay for 48 lbs. 12 oz. of butter, how many pounds of butter can be purchased with a box of tea which contains $42\frac{3}{4}$ lbs.?

42. If $.80\frac{1}{11}$ acres of land be worth \$75.20, what is the value of $.373\frac{1}{3}$ acres? *Ans.* \$35.053+.

Note.—Perform the following examples by analysis.

43. If a cow and a calf sell for \$27, and the value of the calf is $\frac{1}{8}$ that of the cow, what is the value of each?

44. The sum of the ages of a father and son is 48 years, that of the son being $\frac{1}{5}$ of the age of the father; what is the age of each?

45. By one pipe a cistern can be emptied in 2 hours; by another it can be emptied in 3 hours; in what time can it be emptied if both are running?

46. A certain cistern has 3 pipes; the first will empty it in 5 hours, the second in 4 hours, and the third in 10 hours; in what time will all empty it?

47. A certain piece of work can be performed by A in 8 days, by B in 10 days, and by C in 16 days; in what time can all do it working together?

48. In what time can A and B do it together?

49. In what time can A and C do it together?

50. In what time can B and C do it together?

51. A cistern has one pipe which will fill it in $5\frac{1}{2}$ hours, and another which will empty it in 33 h.; in what time will it be filled with both open? *Ans.* $6\frac{3}{5}$ h.

52. If A and B can do a piece of work in 3 mo. 10 d., and A alone can do it in 5 mo., in what time will B do it alone? *Ans.* 10 mo.

53. If A and B can dig a trench in $6\frac{2}{3}$ days, and B can do it alone in 10 days, in what time can A do it alone?

54. If A and B can do it in $5\frac{1}{4}$ days, and A, B, and C, can do it in $3\frac{1}{2}$ days, in what time can C do it alone?

55. If A and B can do a piece of work in $4\frac{4}{9}$ days, and A and C can do it in $5\frac{1}{3}$ days, and B and C in $6\frac{2}{13}$ days, in what time will all do the work together, and in what time will each do it alone? *Ans.* A, B, C, $3\frac{11}{23}$d.; A, 8 d.; B, 10 d.; C, 16 d.

56. A man sold a load of coal, containing $\frac{7}{8}$ T., at \$.75 for a hundred lbs., and received pay in corn at \$.87$\frac{1}{2}$ per bushel; how many bushels did he receive?

57. Purchased a number of pieces of goods, each containing 22 yards, at \$4 for 3 yards, and sold them at \$5 for 2 yards, and gained \$154 on the lot; how many pieces were purchased? *Ans.* 6 pieces.

☞ For Dictation Exercises, see Key.

ANALYSIS AND COMPOUND PROPORTION.

375. Ill. Ex. If 10 gas-burners consume 800 feet of gas in 3 hours, how many feet will 12 burners consume in 15 hours?

Operation by Analysis.

$$\frac{800 \times \overset{6}{\cancel{12}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times \cancel{3}} = 4800 \text{ ft., } \textit{Ans.}$$

If 10 burners consume 800 feet of gas in 3 hours, 1 burner will consume $\frac{1}{10}$ of 800 feet, and 12 burners will consume $12 \times \frac{1}{10}$ of 800 feet in the same time; if this is consumed in 3 hours, in 1 hour there will be consumed $\frac{1}{3}$ of $12 \times \frac{1}{10}$ of 800 feet, and in 15 hours, $15 \times \frac{1}{3}$ of $12 \times \frac{1}{10}$ of 800 feet.

OPERATION BY PROPORTION.

$$\left.\begin{array}{r}10:12\\3:15\end{array}\right\} = 800 : \text{Ans.}$$

$$\frac{800 \times \overset{6}{\cancel{12}} \times \overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}} \times \cancel{3}} = 4800 \text{ ft., } Ans.$$

800 feet being of the same kind as the answer, we make it the third term. But the number of feet required depends upon two conditions: 1st, the number of gas-burners, and, 2d, the number of hours during which they are burning. We shall deal with each condition separately. As 12 burners will consume more gas than 10 burners, we make 12 the second term and 10 the first term; and as they will burn more in 15 hours than in 3 hours, we make 15 the second term and 3 the first term. We find the fourth term, as in simple proportion, *by dividing the product of the means*, $12 \times 15 \times 800$, *by the product of the first extremes*, 10×3.

The process may sometimes be shortened by cancelling the terms as they stand in the proportion, remembering that the numbers which constitute the first terms are divisors, and those which constitute the second and third are multipliers. Thus,

$$\left.\begin{array}{r}\overset{2}{\cancel{10}}:\overset{\overset{2}{\cancel{4}}}{\cancel{12}}\\ \cancel{3}:\underset{3}{\cancel{15}}\end{array}\right\} = 800 : 4800 \text{ ft., } \quad Ans.$$

From the above we derive the following

RULE FOR PERFORMING EXAMPLES BY COMPOUND PROPORTION. *Make the number that is of the same kind as the answer the third term. Take each two numbers that are of the same kind, and consider whether, depending upon them alone, the answer will be greater or less than the third term. Arrange them as in simple proportion. Divide the continued product of the second and third terms by the continued product of the first terms.*

EXAMPLES.

1. If \$90 is paid for the labor of 20 men 6 days, what should be paid for 5 men 8 days? *Ans.* \$30.

2. If 85 tons of coal were required to run 6 engines 17 hours a day, what number would be required to run 25 engines 12 hours a day? *Ans.* 250 tons.

3. If 120 rods of wall were laid by 72 men in 83 days of 14

hours each, how many rods would be laid by 88 men, working 12 hours a day for $3\frac{1}{2}$ days? *Ans.* $13\frac{1}{3}$ rds.

4. If the wages of 75 men for 84 days were $68.75, for how many days could 90 men be employed for $41.25? *Ans.* 42 d.

5. If the freight on 450 lbs. of merchandise is 30 cents for 26 miles, how many miles can 3 tons be carried for $4? *Ans.* 26 m.

6. If, when flour is $7.50 per bbl., a 3-cent loaf weighs 2 oz., what should a 12-cent loaf weigh when flour is $16? *Ans.* $3\frac{3}{4}$ oz.

7. If a loaf, which sells for 10 cents when wheat is $2 a bushel, weighs $1\frac{1}{2}$ lbs., what is the price of wheat when a 6-cent loaf weighs $1\frac{1}{4}$ lbs.? *Ans.* $1.44.

8. If 500 lbs. of wool worth 42 cents a lb. is given for 75 yds. of cloth $1\frac{5}{8}$ yds. wide, how much wool worth 36 cents a lb. should be given for 27 yds. that is $1\frac{1}{2}$ yds. wide? *Ans.* $193\frac{11}{13}$ lb.

9. If it costs $135.02 to carry 7 cwt. 2 qr. 15 lb. a distance of 64 miles, what will be the cost of carrying 11 cwt. 2 qr. a distance of $15\frac{1}{2}$ miles? *Ans.* $49.157+.

10. If 25 men, in $9\frac{1}{2}$ days of 10 hours each, build 200 rods of wall, how many rods would be built in 1 day of 12 hours by 12 men? *Ans.* $12\frac{12}{95}$ rds.

11. If 11 men, in 20 days of 12 hours each, can build a wall 48 feet long, 8 feet high, and 3 feet thick, in how many days can 15 men, working 10 hours a day, build 440 ft. of wall 12 ft. high, and 4 ft. thick? *Ans.* $322\frac{2}{3}$ d.

12. How many men will be required, working 12 hours a day for 250 days, to dig a ditch 750 ft. long, 4 ft. wide, and 3 ft. deep, if it requires 27 men, working 13 hours a day for 62 days, to dig a ditch 403 ft. long, 3 ft. wide, and 3 ft. deep? *Ans.* 18 men.

13. If 5 men, working 12 hours a day for 8 days, cut 44 loads of wood, each 8 ft. long, 4 ft. wide, and 4 ft. 6 in. high, in how many hours would 16 men cut $49\frac{1}{2}$ loads 8 ft. long, 4 ft. wide, and 5 ft. 6 in. high? *Ans.* $41\frac{1}{4}$ hours.

14. If 7 cases of boots can be made by 9 men laboring 12 hours a day for 7 days, what length of time will be required for 3 men and 4 boys (2 boys being equal to a man and a half), laboring 11 hours a day, to make 33 cases? *Ans.* 54 d.

15.* If $421\frac{7}{8}$ rods of fence which is 4 feet high be built by 5 men in 10 days of 9 hours each, how many rods of a fence 5 ft high will be built by 8 men in 6 days of 8 hours each?

16.* What length of canal 27 ft. wide, 24 ft. deep, can be dug by 250 men in 100 days of 12 hours each, if 750 men in 3 months of 25 days each, working 11 hours a day, could dig 4 miles of a canal 30 ft. wide and 10 ft. deep?

Ans. 7 fur., 7 rods, 5 ft., $2\frac{8}{9}$ in.

17.* If the type for a book of 84 pages, 50 lines to a page, lines averaging 8 words, $1\frac{1}{2}$ syllables to a word, $2\frac{1}{2}$ letters to a syllable, was set by 2 men in 5 days of 12 hours each, how many pages of a book, each page containing 75 lines, averaging $5\frac{1}{4}$ words each, 2 syllables to a word, 3 letters to a syllable, would be set by 7 men, laboring 8 hours a day for the working days of a week? *Ans.* $142\frac{6}{11}$ pages.

18.* If 12 men, in 2 months of 4 weeks each, working 6 days per week, 12 hours a day, can set the type for 12 books, of 600 pages each, 120 lines to a page, 20 words to a line, 10 letters to a word, in how many months of $4\frac{1}{4}$ weeks each will 7 boys, working 4 days per week, 16 hours a day, set the type for 6 books, of 500 pages each, 150 lines to a page, lines averaging 24 words, $4\frac{1}{2}$ letters to a word, each boy doing $\frac{5}{7}$ of the work of a man? *Ans.* $1\frac{73}{175}$ months.

19.* If it requires 15 yds. of silk, $\frac{5}{8}$ yd. wide, to line a cloak made of 12.25 yds. of cloth, 1 yd. $1\frac{1}{5}$ qrs. wide, how many yards of silk, $\frac{5}{4}$ yd. wide, will be required to line a cloak made of $8.33\frac{1}{3}$ yds., $4\frac{1}{3}$ qrs. wide? *Ans.* $4\frac{37}{147}$ yds.

20.* If 4 men dig a cellar 33.75 ft. long, 18 ft. wide, and 9.6 ft. deep, in 4.5 days of 11.25 hours each, in how many days of 11.7 hours will 15 men dig a cellar 15.2 yds. long, 7.8 yds. wide, and 10.8 ft. deep, the latter cellar being twice as hard to dig as the former?

☞ For Dictation Exercises, see Key.

\$150, which was $\frac{2}{3}$ of the whole gain; what was M's gain, and what the sum each invested?

14.* Adams & Brown built a schooner. A. furnished \$8000, and B. \$1700 and 15000 ft. of lumber. Her freights for the first year were \$1125, of which B.'s share was \$225; what was the price of his lumber per thousand feet? *Ans.* \$20 per M.

15.* Jones, Styles & Carpenter enter into partnership. J. puts in \$750, S. \$420, and C. 60 tons of coal. They gain \$624, of which C. is to have $\frac{1}{2}$ for conducting the business, the balance to be shared among the partners in proportion to their stock in trade. C. receives \$390; what is his coal per ton, and what are the shares of the other partners?

☞ For Dictation Exercises, see Key.

COMPOUND PARTNERSHIP.

377. When stock in trade is employed for different periods of time, the partnership is called **Compound Partnership.**

Ill. Ex. Three persons formed a partnership. A put in \$170 for 9 mo., B \$130 for 12 mo., and C \$150 for 8 mo. They gained \$286; what was the share of each?

OPERATION.

$170 \times 9 = 1530$ $\frac{1530}{4290} \times \$286 = \$102$, A's share.
$130 \times 12 = 1560$ $\frac{1560}{4290} \times \$286 = \$104$, B's share.
$150 \times 8 = 1200$ $\frac{1200}{4290} \times \$286 = \$80$, C's share.
4290

The use of \$170 for 9 mo. = the use of \$1530 for 1 mo.; the use of \$130 for 12 mo. = \$1560 for 1 mo.; the use of \$150 for 8 mo. = \$1200 for 1 mo. The amount in trade was, therefore, equal to \$1530 + \$1560 + \$1200, = \$4290, for 1 mo.; hence the gains should be as follows: A's, $\frac{1530}{4290}$ of \$286 = \$102; B's, $\frac{1560}{4290}$ of \$286 = \$104; C's, $\frac{1200}{4290}$ of \$286 = \$80. Hence the

Rule for Compound Partnership. *Multiply each partner's stock by the time it is in trade, and apportion the gain or loss according to the products.*

EXAMPLES.

1. A and B engaged in business, and gained \$2008.25. A put in \$4500 for 9 months, and B \$5690 for 7 months. What was the gain of each? *Ans.* A, \$1012.50; B, \$995.75.

2. A, B, C, and D work a mine in company. A furnishes \$1400 for 3 years, B \$500 for 5 years, C \$1800 for 2 years, and D \$700 for 4 years. At the end of 5 years they divide \$2620 of profits; what is the share of each?

Ans. A, \$840; B, \$500; C, \$720; D, \$560.

3. Webb, Clapp, and Calhoun form a partnership. Webb puts in \$8500 for 7 months, Clapp \$10000 for 4 months, and Calhoun \$6750 for 9 months. They lose \$2499.90. What is each partner's loss?

Ans. Webb, \$928.20; Clapp, \$624; Calhoun, \$947.70.

4. Hooker, Brown, and Lear traded in company. H. put \$2500 for 10 months, B. \$2300 for 11 months, and Lear conducts the business, which is considered equal to \$2000 in trade, for 12 months. They gain \$1486. What should each receive?

Ans. Hooker, \$500; Brown, \$506; Lear, \$480.

5. Four persons, J, K, L, and M, loaned money as follows: J \$1500 for 5 years, K \$750 for 3 years, L \$1700 for $2\frac{1}{2}$ years, and M \$950 for 4 years. They received of interest money \$1246. What was the share of each, and what the rate per cent.?

Ans. J, \$525; K, \$$157\frac{1}{2}$; L, \$$297\frac{1}{2}$; M, \$266; rate, 7%.

6. A, B, and C formed a copartnership. A furnished $\frac{4}{9}$ of the capital for 6 months, B $\frac{1}{3}$ of the capital for 10 months, and C the balance for 12 months. The whole gain was \$1560. What was the share of each? *Ans.* A, \$480; B, \$600; C, \$480.

7. Hooker & Brown were in business together for 3 years, and gained \$5750. Hooker put in \$2000 for the first year, and \$1500 for the other two; Brown put in \$2500 for the first two years, and \$1500 for the last year. What was the gain of each?

Ans. Hooker, \$2500; Brown, \$3250.

8. A and B received \$857.50 for grading a road. A furnished 5 hands for 20 days, and 6 others for 15 days; B furnished 10 hands for 12 days, and 9 others for 20 days. What was the share of each contractor? *Ans.* A, \$332.50; B, \$525.

9. Lincoln and Hurd hired a pasture, for which they paid \$117. Lincoln put in 217 head of cattle for 20 days, 150 for 5 days, 189 for 10 days, and 500 for 7 days; Hurd put in 650 head for 6 days, 48 for 15 days, and 400 for 11 days. What should each pay? *Ans.* Lincoln, \$62.88; Hurd, \$54.12.

10. Jones and Avery engaged in business as brokers for the year 1862. Jan. 1, Jones advanced \$3600 and Avery \$1250; April 1, Tyler was admitted to the firm with \$1500; June 1, Childs was admitted with \$1200; Sept. 1, Hewins with \$1800; and, Nov. 1, Jenkins with \$2550. The losses for the year were \$7560; what was the loss of each partner?

Ans. Jones, \$3534$\frac{6}{11}$; Avery, \$1227$\frac{3}{11}$; Tyler, \$1104$\frac{6}{11}$; Childs, \$687$\frac{3}{11}$; Hewins, \$589$\frac{1}{11}$; Jenkins, \$417$\frac{3}{11}$.

11.* Wallis and Winn engaged in trade. The former had in \$900 from January 1 till April 1, when he withdrew \$450; July 1 he added \$600. The latter had in \$2000 from Feb. 1 to Oct. 1, when he added \$200; Nov. 1 he withdrew \$800. The whole gain during the year was \$2500; what was the share of each. *Ans.* Wallis, \$825$\frac{75}{209}$; Winn, \$1674$\frac{134}{209}$.

12.* Ames & Rice ran a steamer for 3 years. Ames furnished \$3000 for the first 10 months, when he added \$1000 more, and at the end of the second year \$500 more. Rice put in \$2500 for the first 18 months, when he put in \$3500 more. At the end of the third year they found their loss to be \$5565; what should each sustain?

13.* D, E, and F hired a pasture on the 20th of May for 5 months, paying \$125 for its use. On that day D put in 200 sheep, E 150, and F 80; June 20, D put in 40, E 200, and F 275; July 20, D took out 100, E 75, and F put in 80; Sept. 20, D put in 25, and E and F took out 200 each. What should each pay?

14.* Weeks, Wyman & Wentworth engaged in business for 1 year. Jan. 1, each put in \$4000; March 1, Weeks and Wyman put in \$1500 each, and Wentworth withdrew \$600; Aug. 1, Weeks put in \$800, Wyman withdrew \$300, and Wentworth put in \$1000; Oct. 1, Weeks withdrew \$400; Nov. 1, Wyman put in \$650, and Wentworth put in \$1500. At the end of the [year t]hey divided \$3500 profits. What was the gain of each?

15. A, B, C, and D put \$5700 in trade. A's money was in 8 months, and his gain was \$160; B's was in 5 months, and his gain was \$200; C's was in 2 months, and his gain was \$180; D's was in 6 months, and his gain was \$240. What stock did each have in? *Ans.* A, \$600; B, \$1200; C, \$2700; D, \$1200

☞ For Dictation Exercises, see Key.

378. Questions for Review.

Ratio. — What is ratio? what is arithmetical ratio? geometrical ratio? What is the first term of a ratio called? the second term? both terms when taken together? What is a ratio of equality? of greater inequality? of less inequality? Give an example. In what respect do ratios resemble fractions? How, then, may ratios, at any time, be written? How do you multiply a ratio? how divide a ratio? Suppose you multiply or divide both terms by the same number? What is a simple ratio? a complex? a compound ratio? How do you reduce a complex ratio to a simple one? a compound ratio? Write a simple ratio; a complex ratio; a compound ratio.

Proportion. — What is proportion? Explain the proportion 2 : 4 = 7 : 14. What are the 1st and 4th terms called? the 2d and 3d? the 1st and 3d? the 2d and 4th? the 1st and 2d? What is inverse proportion? compound proportion? What is a mean proportional between two numbers? Upon what important principle does the solving of examples by proportion depend? Write a proportion, and illustrate that principle. How can you find an extreme, when the other three terms are given? how a mean? how a mean proportional between two given numbers? Give the rule for solving an example by simple proportion, and illustrate it by an example of your own. Perform the same example by analysis. What is meant by analysis? Give your rule for solving an example by compound proportion, and illustrate it. In solving any example by proportion, the two terms of a ratio must be *of the same kind;* why?

Partnership. — What is partnership? Who are the partners? How are profits and losses usually shared? What is simple partnership? (*Ans.* It is partnership where persons enter into business for the same time.) How do you find each person's share of gain or loss in simple partnership? What is compound partnership? How do you find the shares of gain or loss in compound partnership? Why is partnership sometimes called partitive proportion?

INVOLUTION.

379. Involution consists in raising a number to a required power. (Art. 89.)

380. The required power is indicated by a small figure called the *index* or *exponent*, placed at the right, and a little above the number. (Art. 90.)

381. The *first power* of a number is the number itself. The *second power* or *square* of a number is obtained by using the number as a factor twice. The *third power* or the *cube* results from using the number as a factor three times, and so on.

NOTE. — The most important applications of Involution are in the use of the second and third powers.

382. Any power may be obtained by the following

RULE. *Employ the given number as a factor as many times as there are units in the exponent of the required power.*

EXAMPLES.

1. Find the squares of the integers from 1 to 25 inclusive, and commit them to memory.*

Ans.

Numbers,	1,	2,	3,	4,	5,	6,	7,	8,	
Squares,	1,	4,	9,	16,	25,	36,	49,	64,	
Numbers,	9,	10,	11,	12,	13,	14,	15,	16,	
Squares,	81,	100,	121,	144,	169,	196,	225,	256,	
Numbers,	17,	18,	19,	20,	21,	22,	23,	24,	25.
Squares,	289,	324,	361,	400,	441,	484,	529,	576,	625.

2. Find the cubes of the integers from 1 to 10 inclusive, and commit them to memory.*

Ans.

Numbers,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Cubes,	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

*At the option of the teacher.

3. $92^2 = ?$ *Ans.* 8464.
4. $(\frac{7}{8})^2 = ?$ *Ans.* $\frac{49}{64}$.
5. $.3^2 = ?$ *Ans.* .09.
6. $(7\frac{5}{8})^2 = ?$ *Ans.* $58\frac{9}{64}$.
7. $3.08^2 = ?$ *Ans.* 9.4864.
8. $.371^2 = ?$ *Ans.* .137641.
9. $4372^2 = ?$
10. $5.8^2 = ?$
11. $47.6^2 = ?$
12. $(\frac{17}{21})^2 = ?$
13. $(124\frac{3}{7})^2 = ?$
14. $97^3 = ?$
15. $5.75^3 = ?$
16. $(3\frac{5}{8})^3 = ?$
17. $11^7 = ?$
18. $10^{10} = ?$
19. $(\frac{1}{2})^{12} = ?$
20. $.5^9 = ?$
21. Involve 1.3 to the 6th power.
22. Raise $18\frac{3}{4}$ to the 5th power.

23. What is the difference between the square and the cube of 24.

24. What is the compound interest of $1.10 for 4 years, at 10 per cent.?

25. How many paving stones 13 inches square will be required to pave 100 rods of a street 3 rods in width?

26. How many dice measuring $\frac{1}{2}$ an inch each way may be made from a cubical foot of ivory, allowing $\frac{1}{10}$ for waste in the manufacture?

EVOLUTION.

383. Evolution consists in finding the roots of numbers.

384. The root of a number is one of the equal factors which produce that number.

The square root is one of the two equal factors; the cube root, one of the three equal factors; the fourth root, one of the four equal factors, and so on.

385. $\sqrt{\ }$ is the *radical sign*, and by itself signifies the square root, and with a figure above it, signifies the degree of the root indicated by the figure; thus, $\sqrt[3]{27}$ signifies the third root of 27.

The root may also be indicated by a fractional exponent; thus, $16^{\frac{1}{4}}$ (read, 16 to the $\frac{1}{4}$ power) $= \sqrt[4]{16} = 2$.

SQUARE ROOT.

386. Table, showing the places occupied by the square of any number of units, tens or hundreds.

Roots.		Squares.
1 squared	=	$\dot{1}$
9 "	=	$8\dot{1}$
10 "	=	$\dot{1}0\dot{0}$
99 "	=	$9\dot{8}0\dot{1}$
100 "	=	$\dot{1}0\dot{0}0\dot{0}$
999 "	=	$9\dot{9}8\dot{0}0\dot{1}$

387. By the above we perceive that the square root of any whole number expressed by one or two figures, must be units; expressed by three or four figures, must be units and tens; expressed by five or six figures, must be units, tens, and hundreds. Hence, generally, *if a number be separated into periods of two figures each, beginning with the units, the number of figures in the square root will be indicated by the number of periods.*

NOTE I. — The left hand period may contain but one figure.

NOTE II. — The principle above elucidated applies also to decimal fractions; but every period in decimal fractions must contain two figures.

388. That the pupil may comprehend the method of extracting the square root of a number, we will multiply 64 by itself, *i. e.*, square it, and keep the separate products, instead of reducing them and adding as in ordinary multiplication.

$$64 \times 64 = (60 + 4) \times (60 + 4).$$

$$\begin{array}{lll} (1.) & 60 \times 60 = & 60^2 = 3600. \\ (2.) & \left\{ \begin{array}{r} 60 \times 4 \\ 4 \times 60 \end{array} \right\} = 2 \times (60 \times 4) = & 480. \\ (3.) & 4 \times 4 = & 4^2 = \quad 16. \end{array} \left. \vphantom{\begin{array}{l} a \\ b \\ c \\ d \end{array}} \right\} = 4096.$$

By the above it will appear that a square whose root is composed of tens and units, contains

(1.) *The square of the tens ;*

(2.) *Twice the tens multiplied by the units ;* and

square of the units.

. We will now extract the square root of 4096.

OPERATION. (Tens. Units.)

$$\begin{array}{lr} & \dot{4}0\dot{9}6\ (64 \\ 6(0)^2 = & 36 \\ \text{isor, } 2 \times 6(0) = 12(0) & 496 \\ 4 & 496 \\ \text{isor, } \ldots\ 124 & 000 \end{array}$$

Or simply,

$$\begin{array}{r} \dot{4}0\dot{9}6\ (64 \\ 36 \\ 124\ )\ 496 \\ 496 \end{array}$$

Separating the number into periods of two figures each, we find that the root will consist of two figures, and the square of the tens must be contained in the 40 (hundreds); the largest square contained in 40 (hundreds) is 36 (hundreds), the root of which is 6 (tens); this we write as the tens' figure of the root, and subtract its 6 (hundreds) from the 40 (hundreds), and to the remainder 4 ls), bring down the next period, 96.

emainder (496) must contain *two times the tens of the root ;d by the units, plus the square of the units, or the product of the tens, plus the units, multiplied by the units.* If it contained times the tens multiplied by the units, we should obtain the ure by dividing the remainder (496) by two times the tens. e 2×6 (tens) $= 12$ (tens) the trial divisor, which is contained ns) 4 times. We write 4 as the units' figure of the root, and e right of the 12 (tens), and have 124 for the true divisor. This ply by 4, and thus complete the square, — obtaining at once, *product of the tens by the units, and the square of the units.* root consists of more than two figures, having obtained the we can consider them as tens in reference to the next figure, eed with them in all respects as above. Thus, suppose it be required to extract the square root of 4138.94: find the first two places as before; bring down the next period, 94, and form a new trial divisor by doubling 64 (the root already found); find how many times this, considered as tens, is contained in 429 tens, for the third

OPERATION.

$$\begin{array}{r} \dot{4}1\dot{3}8.\dot{9}4\ (64.33+ \\ 36 \\ 124\ )\ 538 \\ 496 \\ 1283\ )\ 4294 \\ 3849 \\ 12863\ )\ 44500 \\ 38589 \\ 5911 \end{array}$$

figure of the root. To obtain a fourth figure in the root, form another period by annexing two zeros, double 643, and so continue.

From the above, we deduce the following

RULE FOR EXTRACTING THE SQUARE ROOT OF A NUMBER. *Point off the given number into periods of two figures each, by placing a dot over the units' figure and every alternate figure to the left in whole numbers, and to the right in decimals.*

Find the greatest square number in the left hand period, and write its root as the first term in the answer. Subtract the square number from the left hand period, and to the remainder bring down the next period for a dividend.

Take twice the root already found for a trial divisor; rejecting the right hand figure of the dividend, divide it by the trial divisor; place the result, as the second term in the root, also at the right of the trial divisor, making a true divisor; multiply the true divisor thus obtained by the last term of the root, and subtract this product from the dividend; to the remainder bring down the next period for a new dividend.

Double the terms of the root already found for a new trial divisor, and proceed as before.

NOTE I. — When a zero occurs in the root, annex a zero to the trial divisor, bring down another period, and proceed as before.

NOTE II. — If a root figure proves too large, substitute a smaller, and repeat the work.

NOTE III. — When a remainder occurs after all the periods are brought down, the root may be more nearly approximated by annexing periods of zeros, and continuing the operation.

NOTE IV. — The square root of a common fraction may be obtained by extracting the root of both terms when they are perfect squares; when they are not, the fraction may first be reduced to a decimal.

NOTE V. — Mixed numbers may be reduced to the decimal form, or to improper fractions when the denominator of the fractional part is a square number.

390. The above rule may be illustrated by diagrams.

Fig 1.

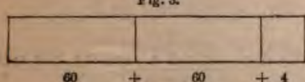
Let A B C D (Fig. 1) represent a square court containing 4096 square feet, the length of whose side we wish to determine. Having found (Art. 389) that the greatest square of tens in 4096 is 3600, the root of which is 6 tens, we deduct 3600 from 4096, and have left 496 square feet, which are to be disposed on two sides of the square already found. The width of these additions we wish to ascertain.

Fig. 2.

By extending the lines a and b, we shall divide the addition into three parts, M, N, and O; M and N having for one side the tens of the root, and O being a square whose side is equal to the width of the side additions.

If the 496 square feet equalled the feet in the side additions, M and N, the width of the additions would be determined by dividing 496 by twice the length of the square already found, 2×60. Using this as the trial divisor, we obtain 4 as the width, which is the units' term of the root; but the *entire* length of the additions is *two times the tens, plus the units*, or 124 (Fig. 3), the product of which by 4, the units' term, is 496. There being no remainder, 4096 is found to be a square of which 64 is the root, and the length of the court is 64 feet.

Fig. 3.



391. Examples in Square Root.

1. What is the square root of 841? *Ans.* 29.
2. What is the square root of 763876? *Ans.* 874.
3. What is the square root of 13616100? *Ans.* 3690.
4. What is the square root of 253009? *Ans.* 503.
5. What is the square root of 1012036? *Ans.* 1006.
6. What is the square root of 447.3225? *Ans.* 21.15.
7. What is the square root of .005625? *Ans.* .075.
8. What is the square root of .169? *Ans.* .41109+.
9. What is the square root of $\frac{49}{144}$? *Ans.* $\frac{7}{12}$.

10. What is the square root of $\frac{12}{27}$?

NOTE.—$\frac{12}{27}=\frac{4}{9}$. *Ans.* $\frac{2}{3}$

11. What is the square root of $10\frac{9}{16}$? *Ans.* $3\frac{1}{4}$

12. What is the square root of $\frac{3}{4}$? *Ans.* .86602+.

13. What is the square root of $8\frac{1}{5}$? *Ans.* 2.8635+.

14. What is the square root of $9\frac{9}{64}$? *Ans.* 3.02334+.

15. What is the square root of $\frac{3}{8}$ of $\frac{27}{32}$? *Ans.* $\frac{9}{16}$.

OPTIONAL EXAMPLES.

NOTE.—Extract the root in the following to five places.

16. $\sqrt{2\frac{1}{4}\times(9\frac{2}{3})^2}=?$
17. $\sqrt{21025}=?$
18. $\sqrt{980100}=?$
19. $\sqrt{502681}=?$
20. $\sqrt{22\frac{9}{16}}=?$
21. $\sqrt{14002564}=?$
22. $\sqrt{.4\times 25}=?$
23. $\sqrt{2213.7025}=?$
24. $\sqrt{152399025}=?$
25. $\sqrt{2.}=?$
26. $\sqrt{4028049}=?$
27. $\sqrt{20\frac{1}{4}}=?$
28. $\sqrt{9569534976}=?$
29. $\sqrt{16\times 75^2}=?$
30. $\sqrt{\frac{29736}{2985984}}=?$
31. $\sqrt{\frac{1}{2}\text{ of }\frac{3}{5}\text{ of }1\frac{1}{5}}=?$
32. $\sqrt{.144}=?$
33. $\sqrt{8}=?$
34. $\sqrt{81.10083136}=?$
35. $\sqrt{.7}=?$
36. $\sqrt{746841.64}=?$
37. $\sqrt{\frac{9}{16}\times(6\frac{1}{4})^2}=?$
38. $\sqrt{769.987}=?$
39. $\sqrt{.42025}=?$
40. $\sqrt{\frac{4}{25}\text{ of }.05^2}=?$
41. $\sqrt{.538}=?$
42. $\sqrt{(.25\div.06\frac{1}{4})\times(\frac{2}{3})^2}=?$
43. $\sqrt{100\frac{25}{49}}=?$

392. PRACTICAL EXAMPLES.

1. There is a field of corn having an equal number of rows and hills in a row, which contains 1020100 hills in all; what is the number of rows in the field? *Ans.* 1010 rows.

2. A body of troops, consisting of 2601 men, has an equal number in rank and file; how many are there in each? *Ans.* 51 men.

3. A company of persons spent \$3.24; each person spending as many cents as there were persons, how many cents did each spend? *Ans.* 18 cents.

4. What is the length of one side of a square farm containing 302 acres, 2 roods of land? *Ans.* 220 rods.

5. What is the length of a square park which contains 2 square miles? *Ans.* 1.4142+ miles.

6. There is a circular lot which contains 3 acres; what is the length of a square lot whose area is the same?

Ans. 21.9089+ rods.

7. What is the size of a square lot whose area is thirty times that of the above? *Ans.* 120 rods.

8. What is the cost of fencing a square lot which contains 1 acre, at $5 per rod? *Ans.* $252.98.

9. The side of a square is 8 ft. 6 in.; what is the side of a square having 25 times the area?

10. A owned a lot of land 51 rods by 80 rods, and another 180 rods by 100 rods, which he bartered with B for a square lot containing 138 acres; how many rods less of fencing are there in the square lot than in the other two? *Ans.* 228 rods nearly.

11. I have two square lots of land, the larger of which contains 270 acres; the ratio of the smaller to the larger is as 5 to 6; what is the length of one side of the smaller?

Ans. 189.73+ rods

12. On a roof there are laid 5000 slates,—the number in the length being twice the number in the breadth; what is the number each way?

NOTE.—It is evident that the slates are laid in two equal squares; hence the square root of $\frac{1}{2}$ of 5000 ($\sqrt{\frac{1}{2} \text{ of } 5000}$) will equal the breadth.

Ans. 50 slates in breadth; 100 slates in length.

13. Suppose the above roof to have had 10000 slates, and the breadth to have been one third of the length, what would have been the number of slates in the length and breadth?

Ans. 173.205+ length; 57.735+ breadth.

14. What is the difference between the fencing of a 34-acre lot, whether it be a square or a rectangular lot, twice as long as it is wide? *Ans.* 17.89 rods.

15. My orchard contains 5400 trees; the number of trees in

width is to the number in length, as 2 to 3; what is the number each way?

NOTE. — $\frac{2}{3}$ of the trees will be a square, whose square root will be the number of trees in the width of the orchard.

16. Suppose, in the above orchard, the outer rows of trees to stand upon the boundary line, and all to stand 30 feet apart, what is the area covered by the orchard? *Ans.* $108\frac{119}{242}$ acres.

17. There is a rectangular court paved with 1728 paving-stones 15 inches square; the length of the court is to the width as 4 to 3; what is the number of stones each way?

18. How many square feet in the superficial contents of the above court?

19. What is the side of a square that will contain as many square feet as a rectangle whose sides are 150 and 70 feet?

20. What is the mean proportional between 6 and 24? (Art. 373.)

APPLICATION OF SQUARE ROOT TO RIGHT-ANGLED TRIANGLES

DEFINITIONS.

393. An **Angle** is the opening between two lines that meet each other.

394. A **Right Angle** is the angle formed by two lines that are perpendicular to each other. (Art. 191.)

395. A **Triangle** is a figure having three angles, and bounded by three straight lines.

396. A **Right-angled Triangle** is a triangle having one of its angles a right angle.

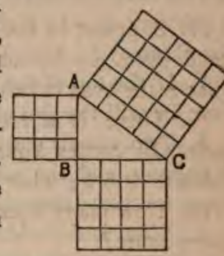
397. The **Hypothenuse** of a **Right-angled Triangle** is the side opposite the right angle.

398. The **Base** of a **Right-angled Triangle** is the side upon which it is supposed to stand.

399. The **Perpendicular** of a **Right-angled Triangle** the side perpendicular to the base.

400. To find either Side of a Right-angled Triangle, the other two Sides being known.

Suppose the figure A B C to be a right-angled triangle, whose sides are 3, 4 and 5 feet respectively. A square formed upon the hypothenuse, A C, will contain 25 square feet; one formed upon the base, B C, will contain 16 square feet, and one formed upon the perpendicular, A B, will contain 9 square feet. Thus, it appears that the square upon the line A C is equal to the two squares upon A B and B C; and generally,



The square upon the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides. Hence,

Rule I. To find the hypothenuse, the base and perpendicular being given: *Square the base and perpendicular, and extract the square root of their sum.*

Rule II. To find the base or perpendicular, the hypothenuse and other side being given: *Square the hypothenuse and the given side, and extract the square root of their difference.*

401. Examples.

1. The base of a right-angled triangle being 30 feet, the perpendicular 40 feet, what is the hypothenuse? *Ans.* 50 feet.

2. The hypothenuse of a right-angled triangle being 32.5 feet, the base 30 feet, what is the perpendicular? *Ans.* 12.5 feet.

3. What must be the height of the eaves of a house that can be reached by a ladder 30 feet long, the foot of the ladder standing 18 feet from the underpinning of the house? *Ans.* 24 feet.

4. How far from the foot of a post 15 feet high can a horse feed that has a rope fastened around his neck and attached to the top of the post, the distance being 37 feet to the neck, and the horse feeding two feet beyond the end of the rope in a direct line with the rope? *Ans.* 36 feet.

5. G. W. Bailey had a tree, which being partially broken off

24 feet from the ground, the top struck the ground 10 feet from the foot of the tree, and on a level with it; what was the height of the tree? *Ans.* 50 feet

6. What must be the length of a ladder to reach to the top of a chimney 48 feet high, the foot of the ladder being 20 feet from the chimney? *Ans.* 52 feet.

7. If the top of the ladder mentioned above be lowered 6 feet, how far will the foot stand from the chimney?

8. Two vessels start at the same point, and sail, one due south 6 degrees, and the other due east 8 degrees; how many miles apart are they, reckoning $69\frac{1}{6}$ miles to a degree?

9. What is the width of a street from a point in which a ladder $32\frac{1}{2}$ feet long will reach a window 26 feet high on one side, and one $24\frac{1}{2}$ feet high on the other side? *Ans.* 40.85+.

10. What is the width of a common, on which stands a flagstaff 195 feet high, from the top of which to one side of the common is 675 feet, and to the other 360 feet?

11. How far from the foot of a flagstaff 24 feet high, must a ladder 23 feet long be placed that a person may ascend to within 5 feet of the top?

12. My house is 40 feet wide, and the ridge-pole is 15 feet above the middle of the beam which connects the eaves; what is the length of the rafters?

13. Provincetown, Erie, and Elmira are in nearly the same latitude; suppose Elmira to be 243 miles directly north of Washington, Erie to be 305 miles north-westerly, and Provincetown 380 miles north-easterly, how far is Provincetown from Erie?

14. Four persons, Messrs. Ames, Barnes, Carnes, and Doane, are residing around Cincinnati, as follows: Ames, 20 miles north; Barnes, 60 miles east; Carnes, 27 miles south; and Doane 36 miles west of the city; what is the shortest distance one of these persons must travel to visit all the rest, and reach his own home?

15. What is the length of the diagonal, that is, the distance from one corner to the opposite corner, of a square lot which contains 16 square rods? *Ans.* 5.6568+ rods

16. The diagonal of the floor of a square room is 110 feet; what is its area? *Ans.* 6050 ft.

17. The diagonal of a square lot is 75 rods; what is one side?

18. What is the diagonal of the floor of a room which is 15 feet square?

19.* Suppose the above room to be 10 feet high; what is its diagonal, that is, its distance from the lower corner to the opposite upper corner?

NOTE. — The diagonal of the floor (Ans. to Ex. 18) becomes the base of the triangle, of which the diagonal of the room is the hypothenuse; but the square of the diagonal of the floor is equal to the sum of the squares of the length and width of the room. Hence, to obtain the diagonal of the room, *Square its three dimensions, and extract the square root of their sum.*

20.* What is the diagonal of a cubical room, each of whose dimensions is 20 feet? *Ans.* 34.64+ feet.

21.* What is the diagonal of a room 36 feet long, 24 feet wide, and 18 feet high?

22.* What is the diagonal of a cubical block whose edge is $2\frac{5}{8}$ inches?

23.* In the centre of a square of land containing one acre is a mound 35 feet high; at the top of this mound, which corresponds with the centre of the square, is a liberty-pole, 120 feet high; what is the distance from the top of the pole to the nearest point in the boundary of the lot?

24.* What is the distance to the farthest point in the boundary?

25.* I have a lot of land 15 feet square, which I design to arrange in five flower-beds as follows: a central square bed, to be bounded by lines connecting the middle points of the sides of the original square, and four equal triangular corner beds, whose sides extend $5\frac{1}{2}$ feet from the right angle at the corner; how many feet of bordering will be required to surround all the beds? *Ans.* 117.54 ft., nearly.

26.* At the summit of a hill, which is 200 feet in height, stands a tower, 20 feet high; from the top of the tower to the foot of the hill is 300 feet; from the top of the tower to the opposite

side of a stream which flows at the foot of the hill is 400 feet; what is the width of the stream?

☞ For Dictation Exercises, see Key.

CUBE ROOT.

402. Table, showing the third power or cubes of units, tens and hundreds.

Roots.			Cubes.
1	cubed	=	1
9	"	=	729
10	"	=	1000
99	"	=	970299
100	"	=	1000000
999	"	=	997002999

By the above it will be seen that the cube root of any whole number, composed of one, two, or three figures, must be units; of four, five, or six figures, must be units and tens; of seven, eight, or nine figures, must be units, tens and hundreds; and hence, generally, that *if we point a number off into periods of three figures each, beginning with the units, the number of figures in the cube root will be indicated by the number of the periods.*

NOTE I. — The left hand period may contain but one or two figures.

NOTE II. — The principle above elucidated applies to decimal fractions; but every period in decimal fractions must contain three figures.

403. Before extracting the cube root, let us involve 64 to the third power, and preserve the separate products.

We have already seen (Art. 388) that the square or second power of 64 is

$$60^2 + 2 \times (60 \times 4) + 4^2.$$

By multiplying this square by 64 $(60 + 4)$, we shall obtain the third power of 64.

$$
\begin{aligned}
64^2 &= 60^2 + 2 \times (60 \times 4) + 4^2 \\
64 &= 60 + 4 \\
\hline
& 60^3 + 2 \times (60 \times 60 \times 4) + (60 \times 4^2) \\
& \qquad\qquad (60^2 \times 4) + 2 \times (60 \times 4^2) + 4^3 \\
\hline
64^3 &= 60^3 + 3 \times (60^2 \times 4) + 3 \times (60 \times 4^2) + 4^3
\end{aligned}
$$

In this product we have four distinct parts, as follows: —

(1), 60^3,	the tens raised to the 3d power,	$= 216000$
(2), $3 \times (60^2 \times 4)$,	$3 \times$ the square of the tens $\times$ the units,	$= 43200$
(3), $3 \times (60 \times 4^2)$,	$3 \times$ the tens $\times$ the square of the units,	$= 2880$
(4), 4^3,	the units raised to the 3d power,	$= 64$
		262144

Thus we see that a cube whose root is composed of tens and units, contains (1) *the cube of the tens*, (2) *three times the square of the tens multiplied by the units*, (3) *three times the tens multiplied by the square of the units, and* (4) *the cube of the units.*

404. Observing, now, that the first of these parts is the *cube of the tens*, and that the *units* is a factor in each of the other parts, we will proceed to extract the cube root in the following

Ill. Ex. What is the cube root of 264609.288?

Operation.

			$26\dot{4}60\dot{9}.28\dot{8}$ (6 4.2 (Tens. Units. Tenths.)
Cube of tens, ...	$60^3 =$		216
$3 \times$ sq. of tens (trial divisor), . .	$3 \times 60^2 =$	10800	48609
$3 \times$ tens $\times$ units, ...	$3 \times 60 \times 4 =$	720	
Square of units, ...	$4^2 =$	16	
True divisor, ...		$11536 \times 4 =$	46144
$3 \times$ sq. of tens (trial divisor), .	$3 \times 640^2 =$	1228800	2465288
$3 \times$ tens $\times$ units, ...	$3 \times 640 \times 2 =$	3840	
Square of units, ...	$2^2 =$	4	
True divisor, ...		$1232644 \times 2 =$	2465288

Solution. Pointing the number off into periods of three figures each, by placing a dot over the units and every third figure to the right and left, we find that the root will consist of three figures, and the cube of the tens must be contained in 264(000). The largest cube

contained in 264(000) is 216(000), the root of which is 6(0); this we write as the tens' term of the root, and subtract its cube, 216(000), from 264(000), and to the remainder, 48(000), bring down the next period, 609.

We know that this remainder, 48609, contains three times the square of the tens (the term already found), multiplied by the units; and though it contains other terms, since this is much the largest, we take 3 times 60^2 (three times the square of the tens) for a trial divisor, and, dividing 48609 by it, obtain 4 for the units' figure.

We multiply 3 $\times$ the tens (60) by the new term of the root (4), and place the product under the trial divisor, and under this, place the square of the units' figure; and thus form our true divisor, consisting of the last three parts of a perfect cube (Art. 403), wanting the units as a factor in each. Multiplying their sum by 4, we have 46144.

This we subtract, and to the remainder bring down the next period.

Considering 64 as the tens in the root, we multiply its square (640^2) by 3 for a new trial divisor, and, proceeding as before, obtain 64.2 for the cube root of 264609.288.

From the above we deduce the following

405. Rule for extracting the Cube Root. *Point off the given number into periods of three figures each, by placing a dot over the units, and every third figure to the left in whole numbers, and to the right in decimals.*

Find the greatest cube in the left hand period, and write its root as the first term in the answer. Subtract the cube from the left hand period, and to the remainder bring down the next period for a dividend.

Multiply the square of the root already found, considered as tens, by three for a trial divisor. Divide the dividend by the trial divisor, and place the result as the next term in the root.

To the trial divisor add three times the former term in the root (considered as tens), multiplied by the last term, also the square of the last. Multiply this sum by the last term, and subtract the product from the dividend.

To the remainder bring down the next period for a new dividend.

Multiply the square of the terms of the root already found (consid-

ered as tens), by three for a trial divisor, with which divide and proceed as before.

Note I. — When a zero occurs in the root, annex two ciphers to the trial divisor, and, bringing down another period, proceed as before.

Note II. — If a root figure proves too large, substitute a lower, and repeat the work.

Note III. — When a remainder occurs after all the periods are brought down, the root may be more nearly approximated by annexing periods of zeros, and continuing the operation.

Note IV. — The cube root of a common fraction may be obtained by extracting the root of each of the terms when they are perfect cubes; when they are not, the fraction may be reduced to a decimal.

Note V. — Mixed numbers may be reduced to decimal fractions, or to improper fractions when the denominator of the fractional part is a cube number.

406. The above rule may be illustrated by means of blocks.

A cube number represents a cube, the edge of which is the cube root of the number.

Let there be a cube of 262144 solid inches, whose edge we wish to determine.

Having found by pointing and trial that the greatest cube of tens in 262144 is 216(000), the root of which is 6 tens, we will let 216000 inches be represented by the following diagram (Fig. 1), having for its edge 6 tens of inches, or 60 inches.

Fig. 1.

Fig. 2.

Subtracting the cube, 216(000) in., from 262144 inches, there will remain 46144 inches, which may be disposed on three sides of the cube already found, so as to retain the cubical form. The *square* contents of the addition upon one side of this cube will be $60^2 = 3600$ inches, and upon three sides 10800 inches. Using this as a trial divisor, we find the thickness of the additions to be 4 inches. The additions are represented by Fig. 2. These additions being made, the solid will be represented by Fig. 3.

Fig. 3.

Fig. 4.

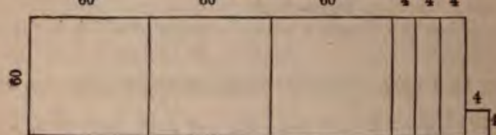
Fig. 5.

Fig. 6

To complete the cube, it also requires three oblong rectangular blocks, whose length is 60 inches, and whose end is 4 inches square (Fig. 4); also a cube, whose edge is 4 inches (Fig. 6). The side of one of the oblong blocks being 60×4, one side of the three will be 3 times $60 \times 4 = 720$ square inches, and one side of the small cube will be $4^2 = 16$ square inches.

If, now, we multiply the sum of these surfaces, $10800 + 720 + 16$, $= 11536$ (Fig. 7), by their thickness, 4, and increase the cube 216000

Fig. 7.



$$3 \times 60^2 + 3 \times 60 \times 4 + 4^2 = 11536.$$

Fig. 8.

by the product, we form a perfect cube (Fig. 8), whose edge is 64 inches. And since there is no remainder, 262144 is a perfect cube, of which 64 is the root.

407. Examples.

1. What is the cube root of 2744? *Ans.* 14.
2. What is the cube root of 24389? *Ans.* 29.
3. What is the cube root of 704969? *Ans.* 89.
4. What is the cube root of 12977875? *Ans.* 235.
5. What is the cube root of 224755712? *Ans.* 608.

6. What is the cube root of 122097755681? *Ans.* 4961.
7. What is the cube root of 729486108008? *Ans.* 9002.
8. What is the cube root of 19683000? *Ans.* 270.
9. What is the cube root of 195.112? *Ans.* 5.8.
10. What is the cube root of .000729? *Ans.* .09.
11. What is the cube root of 329778750? *Ans.* 690.8+.
12. What is the cube root of .57? *Ans.* .8291+.
13. What is the cube root of $32\frac{1}{3}$? *Ans.* 3.185+.
14. What is the cube root of 4? *Ans.* 1.587+.
15. What is the cube root of $\frac{125}{1728}$? *Ans.* $\frac{5}{12}$.
16. What is the cube root of $\frac{8}{64}$? *Ans.* $\frac{1}{2}$.
17. What is the cube root of $\frac{54}{250}$? ($\frac{54}{250} = \frac{27}{125}$) *Ans.* $\frac{3}{5}$.
18. What is the cube root of $\frac{3}{400}$? *Ans.* .1957+.
19. What is the cube root of $1\frac{1}{8}$? *Ans.* 1.04004+.

Optional Examples.

Note. — In the following, the pupil need extract the root to but four places, if decimal fractions be reached.

20. $\sqrt[3]{157464} = ?$
21. $\sqrt[3]{36926037} = ?$
22. $\sqrt[3]{350402625} = ?$
23. $\sqrt[3]{69224023016} = ?$
24. $\sqrt[3]{614255059180216} = ?$
25. $\sqrt[3]{59823325.7} = ?$
26. $\sqrt[3]{525.3425872} = ?$
27. $\sqrt[3]{.0009874} = ?$
28. $\sqrt[3]{9575248.5} = ?$
29. $\sqrt[3]{16} = ?$
30. $\sqrt[3]{42\frac{1}{3}} = ?$
31. $\sqrt[3]{\frac{8000}{15625}} = ?$
32. $\sqrt[3]{\frac{729}{3375}} = ?$
33. $\sqrt[3]{\frac{16}{54}} = ?$
34. $\sqrt[3]{\frac{8}{11}} = ?$
35. $\sqrt[3]{27\frac{1}{8}} = ?$
36. $\sqrt[3]{.27} = ?$
37. $\sqrt[3]{\dfrac{\frac{1}{3}}{1\frac{1}{9}} + \dfrac{1\frac{3}{4}}{25}} = ?$

38. $(\sqrt[3]{125} + \sqrt{819}) - \sqrt[3]{125 + 819} = ?$
39. $\sqrt[3]{28\frac{1}{24}} = ?$
40. $5.4^3 \times 19^4 + \sqrt[3]{27054036008} = ?$
41. Find the difference between the sum of the cube roots of 13824 and .000729, and the cube root of their sum.

408. Practical Examples.

1. What is the length of one side of a cubical block of granite which contains 7077888 solid inches?

2. What will be the edge of a cubical pile of wood, composed of 1000 loads, each 8 feet long, 4 wide, and $2\frac{1}{2}$ feet high?

3. What will be the length of a cubical pile of wood that will contain one cord?

4. What will be the length of a cube which will contain $\frac{1}{8}$ as much as another whose edge is 15 feet? *Ans.* 7.5 feet.

5. What is the depth of a cubical cistern which will contain 9 times as much as one whose depth is 5 feet?
Ans. 10.4004+ feet.

6. What must be the dimensions of a cubical vessel that shall contain 300 gallons of water, reckoning 231 cubic inches to a gallon? *Ans.* 41.075+.

7.* What will be the cost of boards, at $11.25 per thousand feet, to construct the bottom and sides of a cubical bin which shall contain 75 bushels of grain?

Note. — 2150.4 cubic inches = 1 bushel. *Ans.* $1.191+.

8. What will be the cost of lead, at $.12$\frac{1}{2}$ per lb., there being $1\frac{1}{2}$ lbs. to the square foot, to line a cubical box containing $15\frac{5}{8}$ cubic feet?

9. How many yards of paper, $\frac{1}{2}$ yard wide, will be required to line 98 cubical boxes, each containing $5\frac{13}{343}$ cubic feet?
Ans. 384 yards.

10.* The walls of the ancient city of Babylon are said to have been 350 feet high, and built of brick; the city, 15 miles square inside the walls. Suppose the average thickness of the walls to have been 60 feet, what would be the length of a cubical pile composed of the brick in the walls? *Ans.* 1881.2+ feet

☞ For Dictation Exercises, see Key.

MENSURATION.

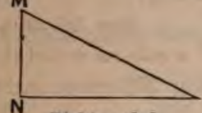
409. The definitions of various surfaces and solids are found on pages 109, 112, 114. Such as are in general use, and not there found, are given in this section.

PLANE SURFACES, RECTILINEAR FIGURES.

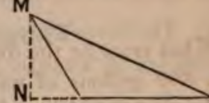
TRIANGLES.

410. The **Right-angled Triangle** contains one right angle.

411. The **Obtuse-angled Triangle** contains one obtuse angle.



Right-angled.



Obtuse-angled.

412. The **Equilateral Triangle** contains three equal sides.

413. The **Isosceles Triangle** contains two equal sides.

414. The **Scalene Triangle** has no sides equal.

Equilateral.

Isosceles.

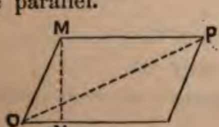

Scalene.

QUADRILATERALS.

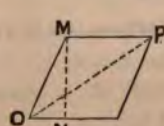
415. A **Parallelogram** is a quadrilateral whose opposite sides are parallel.

416. A **Rhombus** is a parallelogram whose sides are all equal, and whose angles are not right angles.

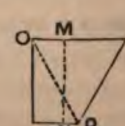
417. A **Trapezoid** is a quadrilateral only two of whose sides are parallel.



Parallelogram.



Rhombus.

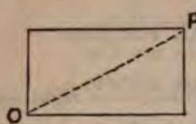


Trapezoid.

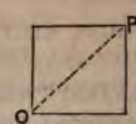
418. A **Rectangle** is a parallelogram whose angles are right angles.

419. A **Square** is a rectangle whose sides are all equal.

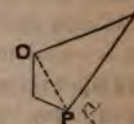
420. A **Trapezium** is a quadrilateral of which no two sides are parallel.



Rectangle.



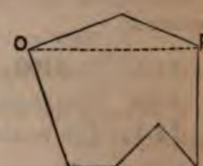
Square.



Trapezium.

421. The term **Polygon** is a general name applied to any figure bounded by straight lines.

422. The **Base** of a figure is the line upon which it is supposed to stand.



Polygon.

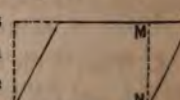
423. The **Altitude** of a figure is its height. The lines M N in the preceding figures indicate altitudes.

424. The **Diagonal** of a figure is a line joining any two angles not adjacent. The lines O P are diagonals.

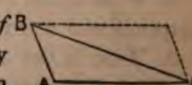
AREAS.

425. The area of a square or rectangle equals *the product of its length and its breadth or height.* (Art. 173.)

426. The area of any parallelogram equals the *product of its base and its height;* for it can be proved to be equal to a rectangle of the same base and height.

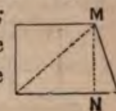


427. The area of a triangle equals *half the product of its base and height;* for every triangle equals one half of a parallelogram of the same base and height.



When the three sides of a triangle are given, the area may be found *by subtracting each side separately from half the sum of the three sides, then multiplying the continued product of these remainders by half the sum of the sides, and extracting the square root.*

428. The area of a trapezoid equals *half of the sum of its parallel sides multiplied by the distance between them;* for it is equal to two triangles whose bases are the two parallel sides of the trapezoid, and whose altitude is the distance between them.



429. The area of any polygon may be found *by dividing it into triangles and obtaining the sum of their areas.*

NOTE. — The student should draw figures for each of the following problems.

430. Examples for Practice.

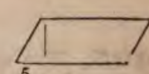
1. What is the area of a rectangle whose length is 20 feet and breadth $6\frac{1}{2}$ feet? *Ans.* 130 sq. ft.

2. How many square feet of canvas in a picture 6 ft. 9 in. long and 4 ft. 2 in. broad?

3. How many square yards in a garden 20 yards square?

4. Required the area of a parallelogram whose base measures 3 ft. 4 in. and altitude 1 ft. 3 in.

5. Required the area of a parallelogram whose base measures 23 feet and the adjacent side 13 feet, from the extremity of which a perpendicular drawn to the base cuts from the base 5 feet.



Ans. 276 sq. ft.

6. What is the area of a triangle, the length of the base being 20 feet, and the height 10 ft. 4 in.? *Ans.* $103\frac{1}{3}$ sq. ft.

7. What is the area of a right-angled triangle whose base and perpendicular are 20 and 18 feet respectively?

8. What is the area of a right-angled triangle whose perpendicular and hypothenuse are 42 and $45\frac{1}{2}$ feet respectively?

Ans. $367\frac{1}{2}$ sq. feet.

9. Required the height and area of an equilateral triangle whose sides are 10 feet long.

Ans. height 8.66+ ft.; area 43.3+ sq. ft.

10. Required the area of a triangle whose sides are 3, 8, and 10 feet long respectively. *Ans.* 9.921+ sq. ft.

11. How many square rods in a triangular lot of land whose sides measure 14 rods, 32 rods, and 20 rods, respectively?

12. What is the area of a trapezoid whose parallel sides are 14 and 32 feet long, and the distance between them 16 feet?

Ans. 368 sq. ft.

13. What is the area of a trapezoid whose parallel sides are twice those of the above, and the distance between them 5 ft. 7 in.?

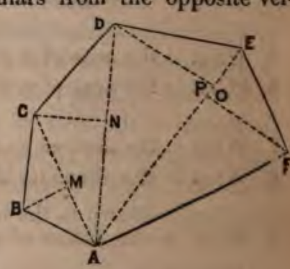
14. How many sq. ft. in the surface of a board which is 18 ft. long, 18 in. wide at one end, and 14 in. wide at the other?

Ans. 24 sq. ft.

15. How many acres in a quadrangular field having 2 parallel sides measuring 10 ch. 5 l. and 16 ch. 8 l. respectively, and the distance between them being 15 ch.? *Ans.* 19.5975 acres.

16. What is the area of a trapezium, the length of a diagonal being 50 feet, and of the perpendiculars from the opposite vertices to the diagonal 10 feet and 35 feet? *Ans.* 1125 sq. ft.

17. Find the area of the accompanying polygon, the dimensions being as follows: A C, 5 ft.; A D, 8 ft.; A E, 10 ft.; B M, 3 ft.; C N, 4 ft.; D O, 5 ft. 6 in.; P F, 4 ft. 6 in.



CIRCLES.

431. The area of a circle equals *one half of the product of the circumference and radius*, or *one fourth of the product of the circumference and diameter;* for it may be considered as made up of triangles, whose bases compose the circumference of the circle, and whose vertices (Art. 190), are at the centre.

432. Geometricians have proved that the circumference of every circle is nearly 3.1416 times its diameter. Hence,

When the Diameter is given,

433. The **Circumference** $=$ Diameter $\times$ 3.1416.

434. The **Area** $=$ (Diameter $\times$ 3.1416) $\times$ $\frac{\text{Diameter}}{4}$ $=$ Diameter^2 $\times$ .7854.

When the Circumference is given,

435. The **Diameter** $= \frac{\text{Circumference}}{3.1416}$.

When the Area is given,

436. The **Diameter** $= \sqrt{\frac{\text{Area}}{.7854}}$.

437. EXAMPLES.

1. Required the circumference of a circle whose diameter is 8 feet. *Ans.* 25.1328 ft.

2. If a radius is 12 feet, what is the circumference? *Ans.* 75.398+ ft.

3. If the circumference is 100 feet, what is the length of the diameter? *Ans.* 31.8309+ ft.

4. What is the area of a circle whose diameter is 21 feet? *Ans.* 346.3614 sq. ft.

5. What is the area of a circle whose diameter is 5 ft. 6 in.?

6. What is the area of a circle whose radius is 2 ft. 1 in.? *Ans.* 13 sq. ft. 91$\frac{1}{2}$ sq. in.

7. What is the area of a circle whose radius is 5 ft. 2 in.?

8. What is the diameter of a circle whose area is 4 sq. rods? *Ans.* 2.256+ rds.

9. What is the radius of a circle whose area is 19 sq. miles? *Ans.* 2.459+ miles.

10. What is the space occupied by a cart-wheel whose spokes are 2 feet long, and the diameter of whose hub is 10 inches? *Ans.* 18.3478+ sq. ft.

11. How many square yards in a circular piece of cloth that will cover a haycock measuring from the ground over the top to the opposite side 10 feet? *Ans.* 8.72$\frac{2}{3}$ sq. yds.

12. How many sq. inches in the bottom of a square box that will contain a ring 20 inches in diameter? *Ans.* 400 sq. in.

13. How many sq. inches in the bottom of a square box that will be contained in a circular box 20 inches in circumference?

Ans. 20.263 sq. in.

14. How many rods square is a plat of ground which contains as much as a circular plat that is 20 rods across?

Ans. 17.724+ rds.

15. How many planks 2 inches thick can be sawed from a log 10 feet in circumference, allowing 2 slabs, each at least 3 inches thick, to be cast aside? *Ans.* 16 planks.

SOLIDS.

Cube.

Parallelopiped.

Prism.

Pyramid.

Cone.

Cylinder.

Frustum of a Pyramid.

Frustum of a Cone.


Sphere.

438. A **Cube** is a solid bounded by six equal squares.

439. A **Parallelopiped** or **Parallelopipedon** is a solid bounded by parallelograms.

440. A **Prism** is a solid whose upper and lower bases are equal and parallel polygons, and whose convex surface is composed of parallelograms.

441. A **Cylinder** is a round body whose bases are equal and parallel circles.

442. A **Pyramid** is a solid whose base is a polygon, and whose convex surface is composed of triangles which terminate in a common point called the vertex.

443. A **Cone** is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the vertex.

444. The **Frustum** of a **Pyramid** or **Cone** is that which remains after cutting off the upper part by a plane parallel to the base.

445. The **Height** of any of the solids here defined is the perpendicular distance from the highest point to the base. (See lines A B in the preceding figures.)

446. The **Slant Height** of a **Regular Pyramid** or of a cone is the shortest distance from the vertex to the perimeter (boundary) of the base. (See lines A m in the preceding figures.)

447. The **Slant Height** of a **Frustum** of a **Pyramid** or **Cone** is the shortest distance between the perimeters of the two bases. (See lines $o\,p$ in the figures.)

448. A **Globe** or **Sphere** is a solid bounded by a curved surface, every part of which is equally distant from a point within called the centre.

SOLIDITIES AND CONVEX SURFACES.

449. The **Solidity** of a **Parallelopiped** equals *the product of its three dimensions.* (Art. 178.)

450. The **Solidity** of a **Cube** equals *the cube of one of its edges.*

451. The **Solidity** of a **Prism** or of a **Cylinder** equals *the area of its base multiplied by its height;* for it is evident that a prism or cylinder 1 inch high must contain as many cubic inches as there are square inches in the base; and if it is 2, 3, or any number of inches high, it must contain 2, 3, or that number of times as many solid inches.

452. The **Convex Surface** of an **Upright Prism** or **Cylinder** equals *the perimeter of one of its bases multiplied by its height;* for it is evident that, if the prism or cylinder is 1 inch high, its convex surface contains as many sq. inches as there are inches in the perimeter; and if the prism or cylinder is any number of times 1 inch in height, its convex surface must contain that number of times as many square inches.

453. The **Solidity** of a **Pyramid** or **Cone** equals *the area of its base multiplied by $\frac{1}{3}$ of its height;* for it can be proved that these solids are each $\frac{1}{3}$ of a prism or cylinder of the same base and height.

454. The **Convex Surface** of a **Pyramid** or **Cone** equals *the perimeter of its base multiplied by $\frac{1}{2}$ of the slant height;* for the convex surface of each may be regarded as composed of triangles whose bases form the perimeter of the base of the solid, and whose height is the slant height of the solid.

455. The **Solidity** of the **Frustum** of a **Pyramid** or **Cone** equals *that of three pyramids or cones whose bases are the* upper and lower bases of the frustum and a mean proportional (Art. 373) between the two, and whose height is the height of the frustum. Hence, the solidity equals *the sum of the two bases plus the square root of their product, multiplied by $\frac{1}{3}$ of the height of the frustum.*

456. The **Convex Surface** of the **Frustum** of a **Pyramid** or **Cone** equals $\frac{1}{2}$ *the sum of the perimeters of the two bases multiplied by the slant height;* for the convex surface of each may be regarded as made up of trapezoids whose parallel sides form the perimeters of the bases, and whose height is the slant height of the frustum.

457. Geometricians have proved that the **Convex Surface** of a **Sphere** equals *the circumference multiplied by the diameter*, or equals *the area of four great circles* of the sphere.*

458. The **Solidity** of a **Sphere** is equal to *its surface multiplied by $\frac{1}{3}$ of the radius, or $\frac{1}{6}$ of the diameter*, for the sphere may be regarded as made up of pyramids whose bases comprise the surface of the sphere, and whose vertices are at the centre.

From the preceding explanations, and by the use of the well established fact that the circumference of every circle is 3.1416 times the diameter, the following formulas for finding the solid contents and convex surfaces of cylinders, cones, frustums of cones, and spheres, are obtained.

To save space, D will be used for diameter of lower base, D for diameter of upper base, h. for height, and s. h. for slant height.

459. The **Solid Contents** of a **Cylinder** $= D^2 \times .7854 \times h.$

460. The **Solid Contents** of a **Cone** $= D^2 \times .7854 \times \frac{h}{3}.$

461. The **Solid Contents** of a **Frustum** of a **Cone** $= (D^2 \times .7854 + D'^2 \times .7854 + D \times D' \times .7854) \times \frac{h}{3} = (D^2 + D'^2 + D \times D') \times .7854 \times \frac{h}{3}.$

462. The **Convex Surface** of a **Cylinder** $= D \times 3.1416 \times h.$

463. The **Convex Surface** of a **Cone** $= D \times 3.1416 \times \frac{s.h.}{2}$

464. The **Convex Surface** of a **Frustum** of a **Cone** $= (D \times 3.1416 + D' \times 3.1416) \times \frac{s.\,h.}{2}.$

465. The **Convex Surface** of a **Sphere** $= D \times 3.1416 \times D = D^2 \times 3.1416.$

466. The **Solid Contents** of a **Sphere** $= D^2 \times \overset{.5236}{\cancel{3.1416}} \times \frac{D}{6} = D^3 \times .5236.$

* A great circle of a sphere is a circle which divides the sphere into two equal parts.

467. Examples.

1. How many cubic feet does a block of granite contain, that is 12 feet long, 4 feet wide, and $1\frac{1}{2}$ feet thick? *Ans.* 72 cu. feet.

2. What number of cubic feet are there in a cube whose edge is 1 foot, 11 inches? *Ans.* 7.041+ cu. feet.

3. How many cubic feet in a prism whose base is a parallelogram 15 feet long and 4 feet wide, and whose height is 9 inches? *Ans.* 45 feet.

4. Required the contents of a prism whose base contains $8\frac{1}{3}$ square yards, and the square of whose height equals 3 times the number of square feet in the base. *Ans.* $41\frac{2}{3}$ cu. yards.

5. Required the contents of a pyramid whose base is the same as the above, and whose height is 5 feet. *Ans.* 4 cu. yards, 17 cu. feet.

6. Required the contents of a pyramid whose base is 7 feet square, and whose height equals the diagonal of the base. *Ans.* 161.69+ cu. feet.

7. Required the contents of the frustum of a pyramid whose bases are 12 and 108 square feet, and whose height is 18 feet. *Ans.* 936 cu. feet.

8. What is the convex surface of a prism, the perimeter of whose base is 7 yards, 2 feet, and whose height is 5 yards, 1 foot? *Ans.* $40\frac{8}{9}$ sq. yards.

9. Required the number of square feet in the surface of a four-sided pyramidal roof, the length of each side being 20 feet, and the slant height 18 feet. *Ans.* 720 sq. feet.

10. What would be the square contents of a four-sided pyramidal roof, the length of each side being 48 feet, and the highest point 10 feet above the eaves? *Ans.* 2496 sq. feet.

11. Required the number of square feet in the sides of an octangular (eight-sided) tower, the length of each side of the base being 2 feet, 9 inches, that of each side of the top 1 foot, 10 inches, and the height of the tower to the roof, measured on the side 12 feet. *Ans.* 220 sq. feet.

12. Required the capacity of a cylindrical cistern, measuring feet across and 8 feet deep. *Ans.* 226.195+ cu. feet.

13. Required the capacity of a conical pit, measuring 8 feet across and 5 feet from the edge to the deepest part.

Ans. 50.2656 cu. feet.

14.* How many quarts of water will a circular tin pan contain, that measures across the bottom $11\frac{1}{2}$ inches, across the top 14 inches, the slant height being $3\frac{1}{4}$ inches? *Ans.* 6.65+ quarts.

15. How many cubic feet in a ball 5 feet in diameter?

Ans. 65.45 cu. feet.

16. How many square feet in the surface of the ball?

Ans. 78.54 sq. feet.

17. How many square inches of leather will cover a ball 4 inches in diameter?

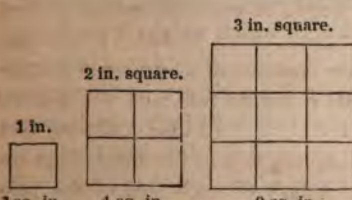
18. What proportion do the cubic contents of a cone bear to the contents of a cylinder which will just contain it? *Ans.* $\frac{1}{3}$.

19. What proportion do the cubical contents of a sphere bear to the contents of a cylinder which will just contain it? *Ans.* $\frac{2}{3}$.

20.* Suppose, when the moon is 238600 miles from the earth,† that its shadow just reaches the earth's surface, how many cubic miles in the shadow, allowing the diameter of the moon to be 2160 miles, and that of the earth to be 8000 miles?

Ans. 283,914,786,355.2 cu. miles.

RELATIONS OF CIRCLES, SIMILAR TRIANGLES, AND POLYGONS.



468. It will be apparent, by the annexed diagrams, that a figure 1 inch square will contain 1 square inch, one 2 inches square will contain 4 square inches, one 3 inches square will contain 9 square inches, and thus, generally, that *the areas of squares are to each other as the squares of their edges.*

† The distance is measured from the centre of the earth to the centre of the moon.

The same principle applies to circles, triangles, and all figures that are similar to each other;* hence,

469. I. *The* **Areas** *of* **Similar Triangles** *and* **Polygons** *are to each other as the squares of their corresponding dimensions.*

Ill. Ex. A triangle whose base is 10 feet has an area of 15 feet; what is the area of a similar triangle whose base is 12 feet?

By Proportion, $10^2 : 12^2 = 15 : 21.6$ square feet, *Ans.*

470. II. *The* **Areas** *of* **Circles** *are to each other as the squares of their diameters, semi-diameters, and circumferences.*

Ill. Ex. If a pipe of 2 inches diameter will empty a cistern in 3 hours, what must be the diameter of a pipe to empty the same cistern in $1\frac{1}{2}$ hours?

By Proportion, $1\frac{1}{2} : 3 = 2^2 : 8$, the square of the diameter of the required pipe. $\sqrt{8} = 2.828 +$ inches, *Ans.*

471. Examples.

1. If the pot to a furnace which consumes 60 lbs. of coal a day is 24 inches in diameter, what amount of coal will be consumed in the same time by a furnace whose pot is 15 inches, all other conditions being the same? *Ans.* 23.437+ lbs.

2. If a rope 3 inches in diameter weighs 20 lbs., what is the diameter of a rope of the same length which weighs 9 lbs.? *Ans.* 2.012+ in.

3. If a pipe 4 inches in diameter fills a cistern in 20 minutes, 15 seconds, in what time will a pipe that is $2\frac{1}{2}$ inches in diameter fill the same cistern? *Ans.* 51.84 minutes.

4. If it costs $10.50 to cover a roof whose length is 7 feet, what will it cost to cover a similar roof whose length is 21 feet? *Ans.* $94.50.

* Angular figures are similar when their angles are equal, and their corresponding sides proportional; and, conversely, similar figures have their corresponding sides proportional.

5 The hypothenuse of a right-angled triangle is 40 feet; what must be the hypothenuse of a similar triangle that it may contain twice the area? *Ans.* 56.568+.

6. If a circular lot of land which is 10 rods in diameter contains 78.5398 square rods, what number of rods will a lot contain which is 5 rods in diameter? *Ans.* 19.63495.

7. The area of a triangle whose base is 24 feet is 120 feet; what is the area of a similar triangle whose base is 96 feet?

Ans. 1920 feet.

8. The Winchester bushel is $18\frac{1}{2}$ inches in diameter and 8 inches deep; what must be the diameter of a circular measure 6 inches deep, that it may hold a bushel? *Ans.* 21.36+ inches.

9. I have a circular flower-garden, the circumference of which is bordered with 75 yards in length of sodding; how many yards will be required to border a circular garden of $\frac{2}{3}$ the area?

Ans. 61.237+ yards.

10. Having a triangular board $7\frac{1}{2}$ feet long, what distance from the base end shall I cut it to divide it into two equal parts?

Ans. 2.197— ft.

SIMILAR SOLIDS.

NOTE. — Angular solids are similar when their angles are equal each to each, and arranged in the same way, and their corresponding edges proportional.

The following proposition may be easily proved by geometry: —

472. *The* **Solidities** *of* **Cubes, Spheres,** *and all* **Similar Solids** *are to each other as the cubes of their corresponding dimensions.*

ILL. EX., I. How many globes of 6 inches diameter can be made from a globe of 48 inches diameter?

By Proportion, $6^3 : 48^3 = 1$ (globe) : 512 globes, *Ans.*

ILL. EX., II. If a conical stack of hay which contains $\frac{3}{7}$ of a ton is 6 feet high, what is the height of a similar stack which contains $3\frac{3}{7}$ tons?

By Proportion,

$\frac{3}{7} : 3\frac{3}{7} = 6^3 : 1728$, the cube of the height of the larger stack.

$\sqrt[3]{1728} = 12$ ft., height of larger stack, *Ans.*

Examples.

1. If an ounce ball is $\frac{5}{8}$ inch in diameter, how many ounce balls can be made from a globe of lead 6 inches in diameter?

Ans. $884\frac{92}{125}$ balls.

2. A pyramid which is 9 feet in height contains 48 cubic feet; what is the height of a similar pyramid that contains 100 cubic feet? *Ans.* 11.494+ feet.

3. If a cube of granite whose edge is 2 feet weighs 1336.32 pounds, what will be the weight of a cube whose edge is 4 feet, 9 inches? *Ans.* 17901.99.

4. If an egg of $2\frac{1}{4}$ inches in circumference weigh 1 ounce, what would another of the same form and consistency weigh whose circumference is 6 inches?

5. What must be the height of a cone to contain 125 times as many solid inches as a similar cone 3 inches in height?

Ans. 15 inches.

6. If a bushel measure is $18\frac{1}{2}$ inches in diameter and 8 inches deep, what must be the diameter and depth of a half-bushel measure similar in form?

Ans. Diam. 14.683+ in.; depth 6.349+ in.

7. If an elephant's tusk $9\frac{1}{2}$ feet long and 8 inches in diameter at base weigh 214 pounds, what would be the dimensions of a similar tusk weighing 75 pounds?

8. Estimating the mean diameter of the earth at 7912 miles, and that of the moon at 2160 miles, how many bodies of the size of the moon could be made from the bulk of the earth?

9. If the bulk of Saturn be 1000 times as great as that of the earth, what is the diameter of Saturn?

10. At what distance from the top, parallel with the base, must a conical sugar-loaf 12 inches high be cut that it may be divided into two equal parts?

11. Mr. Root has three stacks of hay of similar shape, the

diameters of their bases being, respectively, 10, 12, and 14 feet; if the one whose diameter is 10 feet contains $2\frac{1}{2}$ tons, what will each of the others contain?

☞ For Dictation Exercises, see Key.

473. Questions for Review.

What is **Involution**? a power? an exponent? What is the first power? second power? third power? fourth power?

What are the second and third powers generally called?

Rule for Involution? How may a mixed number be raised to a given power? Repeat the squares of the integers from 1 to 25;* the cubes of the integers from 1 to 10.*

How does **Evolution** differ from Involution?

What is a root? What is the square root of a number? the cube root? How is the square root indicated? the cube root? How otherwise may the root of a number be indicated?

If a power contain one or two figures only, of how many figures will its square root consist? If a power contain three or four figures? If five or six?

What three terms does every square number contain whose root consists of tens and units?

Give the rule for extracting the square root. Of how many figures *may* the left-hand period in whole numbers consist? of how many *must* every period, except this, consist? of how many, every period in decimals?

How do you proceed when a zero occurs in the root? how when a root figure proves too large? how when there is a remainder? How do you extract the square root of a common fraction whose terms are square numbers? whose terms are not squares? How extract the root of a mixed number? Explain the extraction of the square root by an example. Illustrate by diagrams.

What is an angle? a right angle? a triangle? a right-angled triangle? its hypothenuse? its perpendicular? its base?

To what is the square on the hypothenuse of a right-angled triangle equal? Rule to find the hypothenuse; to find base or perpendicular.

If a cube number contain one, two, or three figures only, of how many figures will its cube root consist? if it contain four, five, or six figures only?

To what four terms is every cube number equal whose root consists of tens and units?

* At the option of the teacher.

Give the rule for extracting the cube root. Of how many figures *may* the left-hand period consist? of how many *must* every other period consist in whole numbers? in decimals?

How do you proceed when a zero occurs in the root? how when a root figure proves too large? how when there is a remainder? How do you extract the cube root of a common fraction when the terms are cubes? how when the terms are not? How extract the cube root of a mixed number?

Explain the extraction of the cube root by an example. Illustrate by blocks.

What is **Mensuration**? Name and describe the different kinds of triangles. Draw a right-angled triangle; an obtuse-angled triangle; an equilateral triangle; an isosceles triangle; a scalene triangle. Name and describe the different kinds of quadrilaterals. Draw a square; a rectangle; a rhombus; a trapezoid; a trapezium; a circle; a polygon of 5 sides with 2 diagonals.

How do you find the area of a square? a rectangle? any parallelogram? a triangle? a trapezoid? a trapezium? any polygon?

How do you find the circumference of a circle when the diameter is given? when the radius is given? How do you find the area of a circle when the diameter is given? when the radius is given? How do you find the diameter when the circumference is given? How find diameter when the area is given? How do you find the radius when the area is given?

Define a cube; parallelopiped; prism; cylinder; pyramid; cone; frustum of a pyramid or cone; a sphere. Draw or mention something in the form of each of these solids. What is the height of any solid? the slant height of a pyramid or cone? the slant height of a frustum of a pyramid or cone?

How do you obtain the solid contents of a cube? a parallelopiped? a prism or cylinder? a pyramid or cone? a frustum of a pyramid or cone? How do you find the convex surface of each of these solids?

When the diameters and altitude are given, how do you find the solid contents of a cylinder? of a cone? of a frustum of a cone? of a sphere? How do you find the convex surface of a cylinder? of a cone? the frustum of a cone? a sphere?

What proportion exists between the areas of squares? of circles? of all similar triangles and polygons? When are angular figures similar?

What proportion exists between spheres? between all similar solids? When are angular solids similar?

474. General Review, No. 8.

1. Supply the 2d term in the proportion $3\frac{4}{5} : ? = 8 : 25 \times 6$.

2. What is the mean proportional between .8 and .72?

3. Divide \$1900 between two men, in the proportion of 3 to 5.

4. Divide \$45 between three boys, so that one shall have as much as the other two, whose shares are as 2 to 7.

5. If 15 gallons of oil cost 7 £, 10 s., what will $25\frac{1}{2}$ gallons cost?

6. How many pounds can 5 horses draw, if 6 horses can draw as much as 10 oxen, and 2 oxen can draw 2400 pounds?

7. Smith and Lee formed a partnership. Smith put in \$1000 for 6 months, and Lee \$600 and his services for 8 months, his services being equal to \$100 a month. They gained \$1506; what was each one's share?

8. What is the 5th power of 23? the cube of 96?

9. What is the largest number of men in a regiment of 1000 that can be arranged in a square; and how many men will remain? How many men will there be on each side of the square?

10. How many feet of fencing around a square farm containing 15 acres?

11. A ladder $27\frac{6}{7}$ feet long reaches a window $25\frac{5}{7}$ feet from the ground; how far does the foot of the ladder stand from the house?

12. Required the diameter of a circular grass plat which contains $314\frac{4}{25}$ square feet.

13. How many rods of fencing on both sides of a road which surrounds a circular park containing $\frac{3}{4}$ of a square mile, the road being 3 rods wide?

14. What must be the depth of a pail, that is 10 inches across, to contain 5 gallons (the sides being upright)?

15. How many feet of canvas are required to construct a conical tent 14 feet across the bottom and $9\frac{1}{3}$ feet from the highest point to the ground?

16.* How many gallons will a circular vat contain, which measures across the top 8 ft., across the bottom 7 ft., the sides sloping uniformly and measuring on the slope $6\frac{1}{2}$ ft. in depth?

☞ For changes, see Key.

ALLIGATION.

475. **Alligation,** or **Average,** treats of the mixing of different ingredients.

476. **Alligation Medial** is the process of determining the average or mean value of given quantities of different values.

477. **Alligation Alternate** is the process of determining what quantities of different values may be so combined that the mixture shall be of a given value.

NOTE. — The word alligation means a *tying together*, and is applied to these processes because, in the solutions of many examples, the amounts or prices of articles are linked or tied together. Average is perhaps the better name to use, as it applies to all the examples.

ALLIGATION MEDIAL.

478. ILL. EX. Let it be required to mix 10 lbs. of sugar at 7 cents per lb. with 7 lbs. at 9 cents, and 8 lbs. at 11 cents; what will be the value of the mixture?

OPERATION.

$$\begin{array}{ll} 10 \times 7 = 70 \\ 7 \times 9 = 63 \\ 8 \times 11 = 88 \\ \hline 25 \quad) 221 \\ \quad 8\tfrac{21}{100} \text{ cents, } \textit{Ans.} \end{array}$$

The price of 10 lbs. at 7 cents per lb. = 70 cents; of 7 lbs. at 9 cents = 63 cents; of 8 lbs. at 11 cents = 88 cents. Adding, we find the value of the mixture to be 221 cents, and the number of pounds to be 25. As 25 lbs. are worth 221 cents, 1 lb. is worth $\frac{1}{25}$ of 221 cents $= 8\frac{21}{100}$ cents, *Ans.* Hence we deduce the following

RULE. To find the mean value of given quantities of different values: *Divide the sum of the values of the several quantities by the sum of the quantities.*

EXAMPLES.

1. If 10 lbs. of raisins worth 10 cents per lb. be mixed with 4 lbs. worth 15 cents per lb., what is the value of the mixture per pound? *Ans.* $11\frac{3}{7}$ cents.

2. There are in a certain school, 10 pupils 14 years old; 9 pupils 12 years old; 5, 11 years; 8, 9 years, and 17, 10 years old; what is their average age?

3. A family spent, during the year, as follows: in January \$89.75, in February \$70.16, in March \$85.32, in April \$90.21, in May \$87.00, in June \$66.14, in July \$69.42, in August \$72.68, in September \$80.65, in October \$90.45, in November \$98.54, in December \$109.63; what was their average expense per month?

4. In Philadelphia, during the year 1861, rain or snow fell as follows: in January on 13 days, in February on 9 days, in March on 9, in April on 9, in May on 13, in June on 15, in July on 14, in August on 12, in September on 6, in October on 10, in November on 11, in December on 4; what was the average number of days per month when rain or snow fell?

5. In Massachusetts, during the year 1850, the value of home manufactures was \$205,333. During the year 1860, it was \$245,886. What was the average rate of increase per year during the 10 years?

6. A flour merchant sold 50 bbls. flour at \$7.50 per bbl., 60 bbls. at \$9.00 per bbl., 25 bbls. at \$8.50, 40 at \$8.75, and 100 at \$9.50; what did his sales average per barrel?

7. A baker made wedding-cake of the following ingredients: 5 lbs. flour worth 5 cents per lb., 5 lbs. sugar at 11 cents per lb., 5 lbs. of butter at 22 cents per lb., 6 lbs. raisins at 17 cents per lb., 12 lbs. currants at 20 cents per lb., 2 lbs. citron at 50 cents per lb., 50 eggs, $1\frac{1}{2}$ lbs. to the dozen, 18 cents per dozen, $\frac{1}{2}$ pint wine at $37\frac{1}{2}$ cents per pint, 3 oz. cinnamon at 56 cents per lb., 3 oz. nutmegs at \$1.00 per lb., $1\frac{1}{2}$ oz. mace at \$1.00 per lb. Allowing \$2.00 for labor and fuel, $\frac{1}{2}$ lb. for the weight of the wine, and 1 oz. in every lb. for loss of weight in baking, what was the cost of the cake per lb.? *Ans.* \$.24$\frac{1480}{4053}$.

2. How shall oil at 80, 95, and \$1.50 per gallon, be proportioned that the mixture may be worth \$1.00 per gallon?

3. How shall tea at 62, 75, 68, 90, and 98 cents, be proportioned that the mixture may be worth 80 cents per lb.?

4. A grocer makes a mixture of syrup, worth 62 cents per gall., from syrups worth 45, 60, 75, and 80 cents per gall.; how many gallons of each may he use?

5. A grocer has cider at 28 and 30 cents per gall., which he wishes to mix with vinegar at 27 cents per gall., and water, so that the mixture may be worth 25 cents per gall.; what proportions may he use?

Ans. 1 gal. of each of the other ingredients to $\frac{2}{5}$ gal. water, etc.

483. When one of the quantities is limited, *find the entire gain or loss on that quantity, and take such quantities of the other ingredients that their gains and losses shall balance each other and the gain or loss on the limited quantity.*

When more than one quantity is limited, *find the resulting loss or gain from taking the limited quantities, and balance as before.*

Ill. Ex. How much tea at 60, 75, and 87 cents per lb., may be mixed with 30 lbs. of tea at 95 cents per lb., that the mixture shall be worth 85 cents per lb.?

Operation.

$$85\begin{cases} 60+25\times 12=+300 \\ 75+10\ \rceil \quad 1 \\ 87-\ 2\ \rfloor \quad 5 \\ 95-10\times 30=-300 \end{cases}$$

Ans. 12 lbs. at 60 cents, 1 at 75 cents, and 5 at 87 cents.

Examples.

6. How many lbs. split peas at 5 cents per lb., must be put with 40 lbs. coffee at 21 cents per lb., that the mixture shall be worth 14 cents per lb.? *Ans.* $31\frac{1}{9}$ lbs.

7. A goldsmith has gold 16 carats fine, which he wishes to mix with 4 oz. gold 17 carats fine, 5 oz. 20 carats fine, 2 oz. 22 carats fine, and 3 oz. 24 carats fine, that the mixture may be 18 carats fine; how many oz. of it shall he use?

NOTE. — The term *carat* is a word used in indicating the proportion of pure gold in any given quantity of the metal; thus, if the metal be pure gold, it is said to be 24 carats fine; if two thirds gold, 16 carats fine; if 17 parts gold and 7 parts alloy, 17 carats fine, etc.

8. How much wool, of equal quantities, at 35 and 40 cents per lb., must be mixed with 100 lbs. at 60 cents per lb., that the mixture may be worth 45 cents per lb.?

484. When the entire quantity is limited, *find the proportion of the ingredients as before, and then divide the given quantity among the ingredients in the proportion found.*

EXAMPLES.

9. J. Blake has an order from New York for 1000 bushels of wheat, at $1.25 per bushel. How shall he mix his wheat, which he values at $1.20, $1.22, and $1.30, to fill the order?

Ans. 100 bu. at $1.20, 500 bu. at $1.22, 400 bu. at $1.30.

10. J. Smith wishes to purchase a farm of 200 acres, at $100 an acre. How much woodland at $125 per acre, mowing upland at $90 per acre, pasture land at $70 per acre, and tillage ground at $128 per acre, may he purchase?

11. How many lbs. of cotton at 60, 73, and 98 cents per lb., must be mixed with 750 lbs. at 90 cents, that the mixture may contain 2000 lbs. at 80 cents per lb.?

NOTE. — First balance the loss on the 750 lbs. with gain on one of the other ingredients taken; then proceed to make a mixture of the other ingredients equal to the entire quantity given, minus the quantities balanced.

ARITHMETICAL PROGRESSION.

485. Arithmetical Progression is progression by equal differences.

486. An **Arithmetical Series** is a succession of numbers which increase or decrease by a common difference.

If the numbers increase from the first term, the series is an **Increasing Series**: *e. g.*, 2, 4, 6, 8, 10, 12, &c.

If the numbers decrease from the first term, the series is a **Decreasing Series**; *e. g.*, 13, 11, 9, 7, 5, &c.

487. In every series, five things are to be considered; viz., *the First Term, the Last Term, the Number of Terms, the Common Difference,* and *the Sum of the Terms;* any three of which being given, the other two may be found. This gives rise to twenty distinct cases, a few of the more important of which will be here presented.

NOTE I. — For the remaining cases, also for full discussions of Geometrical Progression and Annuities, the student is referred to works on Algebra.

NOTE II. — Increasing series only will be considered in this book, as rules that apply to increasing series apply to decreasing series also, provided that, wherever the common difference is introduced, it is used with the contrary sign.

488. TO FIND ANY TERM IN A SERIES, WHEN THE FIRST TERM, COMMON DIFFERENCE, AND NUMBER OF TERMS ARE GIVEN.

Let $5 =$ first term, $2 =$ common difference, and $6 =$ the number of terms. The series will be constructed as follows: —

(1.)

1st term.	2d term.	3d term.	4th term.	5th term.	6th term.
5.	$5+2$.	$5+2\times 2$.	$5+3\times 2$.	$5+4\times 2$.	$5+5\times 2$.

We find that the second term equals the first term, plus the common difference; the third term equals the first term, plus two times the common difference; the fourth term equals the first term, plus three times the common difference, &c.; and that the last or sixth term equals the first term, plus five times the common difference. Hence,

I. To find any term of the series: *Add the first term to the product of the common difference multiplied by the number of terms which precede it.*

II. To find the last term: *Add the first term to the product of the common difference multiplied by the number of terms less one.*

EXAMPLES.

1. In an increasing series the first term is 4, and the common difference is 8; what is the seventh term? *Ans.* 52.

2. The first term is 7, the common difference $\frac{1}{3}$, and the number of terms 20; what is the last term?

3. If 5 lbs. of power is imparted to a fly-wheel at each revolution, what is its power at the end of the tenth revolution from a state of rest, provided its average loss of power from friction and other causes is 1 lb. during each revolution? *Ans.* 40 lbs.

4. If a stone, in falling to the earth, descends $16\frac{1}{12}$ feet during the first second, $3 \times 16\frac{1}{12}$ feet during the next, $5 \times 16\frac{1}{12}$ feet during the third, and so on; how far will it fall during the eleventh second?

5. What is the amount of $200 at simple interest for 8 years, at 6 per cent.?

NOTE.—The amount will be the ninth term of the series, of which the first term is $200.

489. TO FIND THE COMMON DIFFERENCE IN A SERIES, ALSO THE NUMBER OF TERMS.

If, in series (1.) we subtract the first term from the last, we have remaining 5×2, that is, the common difference multiplied by the number of terms less one. Hence,

I. To find the common difference: *Divide the difference between the first and last term by the number of terms less one.*

II. To find the number of terms: *Divide the difference between the first and last term by the common difference, and add one to the quotient.*

EXAMPLES.

6. The first term of a series is 7, the last term 19, and the number of terms 13; what is the common difference? *Ans.* 1.

7. The first term is 30, the last term is 3, and the number of terms 10; what is the common difference?

8. The first term is 8, the last term 23, and the common difference $1\frac{1}{2}$; required the number of terms.

9 A boy, in picking up stones 2 feet apart, and carrying them, one at a time, to a deposit 2 feet from the first, found that to carry the last one, he had walked 60 feet; how many stones did he carry in all? *Ans.* 15 stones.

490. To find the Sum of the Series.

Let 2, 4, 6, 8, 10, 12, 14, 16, be a series, of which we wish to find the sum. We write under it the same series in an inverted order, and add the terms as follows:—

2	4	6	8	10	12	14	16
16	14	12	10	8	6	4	2
18	18	18	18	18	18	18	18

We then have the sum of both series $= 8 \times 18$,

or the sum of one series $= \frac{8 \times 18}{2}$.

But 8 equals the number of terms, and 18 the sum of the extremes. Hence,

To find the sum of a series: *Multiply one half the sum of the extremes by the number of terms.*

Examples.

10. The first term of a series is 4, the last 40, and the number of terms 11; what is the sum of the series? *Ans.* 242.

11. What is the sum of the odd numbers from 1 to 99 inclusive?

12. What is the sum of the multiples of 3 from 6 to 45 inclusive?

13. How many notes must a person sing in ascending two octaves, if he goes back to the first note each time he strikes a new one, and sounds all the intermediate notes each time he ascends? *Ans.* 120 notes.

14. Two of Dio Lewis's pupils tried their skill in running for pegs. Each set up 5 pegs 6 feet apart, and commenced running 6 feet from the first peg. How far did each run to place the pegs at his starting-point?

15. How far would the first boy of a row of 21 scholars travel, in gathering writing-books from the row, if the scholars were $2\frac{1}{2}$ feet apart, and he brought one book at a time to his own desk?

GEOMETRICAL PROGRESSION.

491. **Geometrical Progression** is progression by equal multipliers.

492. A **Geometrical Series** is a succession of numbers which increase or decrease by a common multiplier. Thus,

2, 4, 8, 16, 32, 64, is an increasing geometrical series, in which the multiplier is 2.

2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, is a decreasing geometrical series, in which the multiplier is $\frac{1}{2}$.

493. The common multiplier is called the **Ratio.**

494. In every geometrical progression, five things are to be considered; viz., *the First Term, the Last Term, the Number of Terms, the Common Ratio*, and *the Sum of the Terms;* any three of which being given, the other two may be found.

495. TO FIND THE LAST TERM OF A SERIES, THE FIRST TERM, THE RATIO, AND NUMBER OF TERMS BEING GIVEN.

Let 3 be the first term, 2 the ratio, and 5 the number of terms. The series will then become,

(1.)

1st term.	2d term.	3d term.	4th term.	5th term.
3,	3×2,	3×2^2,	3×2^3,	3×2^4,

in which the second term equals the first term multiplied by the ratio, the third term equals the first term multiplied by the second power of the ratio, the fourth term equals the first term multiplied by the third power of the ratio, and the fifth term equals the first term multiplied by the fourth power of the ratio. Hence,

I. To find any term of the series: *Multiply the first term by the ratio raised to a power equal to the number of terms which precede the required term.*

II. To find the last term of the series: *Multiply the first term by the ratio raised to a power equal to the number of terms less one.*

EXAMPLES.

1. What is the seventh term of the series 2, 6, 18, 54, &c.?

2. What is the fifteenth term of the series 5, $2\frac{1}{2}$, $1\frac{1}{4}$, $\frac{5}{8}$, $\frac{5}{16}$, &c.?

3. What is the amount of $500 for 7 years, at 6 per cent., compound interest?

NOTE. — 1.06 is the ratio, and the amount the eighth term of the series.

4. Naturalists have found that the ratio of increase of some kinds of animalculæ (microscopic animals) is often four in a single day. At that rate, what would be the increase of one animalcula and its descendants in ten days? *Ans.* 1,048,576

496. To find the Ratio, the First Term, the Last Term, and Number of Terms being given.

In series (1), if the last term, 3×2^4, be divided by the first term, 3, the quotient will be 2^4, or the fourth power of the ratio, the fourth root of which will equal the ratio. Hence,

To find the ratio: *Divide the last term by the first term, and extract that root of the quotient whose index equals the number of terms less one.*

EXAMPLES.

5. The first term of a series is 2, the last term 128, the number of terms 3; what is the ratio? *Ans.* 8.

6. The first term is 4, the last term $\frac{1}{2}$, and the number of terms 4; what is the ratio? *Ans.* $\frac{1}{2}$.

7. The extremes are 5 and 625, and the number of terms 4; what is the ratio?

497. To find the Sum of a Series.

Let 3, 9, 27, 81, 243, be a series, of which we wish to find the sum.

OPERATION.

3 times the first series =		9	27	81	243	729;	subtracting
the first series =	3	9	27	81	243,		we have
twice the first series =	—3	0	0	0	0	729,	or 729 — 3.

$\therefore$ the first series $= \frac{729-3}{2}$.

By multiplying each term of the series by the ratio, 3, we have a second series, whose sum is 3 times that of the first series, from which we subtract the first series; the remainder equals twice the sum of the first series, which we find by dividing by 2. Hence,

To find the sum of the series: *Subtract the first term from the product of the last term multiplied by the ratio; divide the remainder by the ratio less one.*

NOTE. — If the series is descending, the last term multiplied by the ratio should be taken from the first term, and the remainder be divided by one less the ratio.

EXAMPLES.

8. What is the sum of the series 3, 12, 48, 192, 768, 3072? *Ans.* 4095.

9. The first term is 5, the last term 3125, and the number of terms 5; what is the sum of the series? *Ans.* 3905.

10. What is the sum of 7 terms of the series 4, 8, 16, 32, &c.?

11. If $\frac{1}{2}$ of the air in a receiver be taken from it by an air-pump at the first stroke of the piston, and $\frac{1}{2}$ of the remainder at the second stroke, and so on, what will be the amount taken from the receiver by 8 strokes? *Ans.* $\frac{255}{256}$.

ANNUITIES.

498. **Annuities** are periodical payments of fixed sums of money, in consideration of money paid or services rendered.

499. When an annuity is made for a definite number of years, it is called a *certain annuity;* when it is made forever, a *perpetuity;* when it depends upon the life of one or more persons, a *life annuity;* when it does not commence till a given time has elapsed, it is said to be *in reversion.*

500. When annuities are granted by government, they are called **Pensions.**

501. The **Amount** of an annuity is the sum of all the payments, plus their interest, from the time they become due.

502. The **Present Worth** of an annuity is such a sum of money as, put at interest, will exactly pay the annuity.

503. Annuities are said to be in **Arrears,** or **Foreborne,** when they remain unpaid after they become due.

504. Annuities are generally computed at compound interest.

ANNUITIES AT SIMPLE INTEREST.

505. ILL. EX. What is the amount of an annuity of $200 a year, at 6 per cent. simple interest, 5 years in arrears?

The payment due at the end of the fifth year is $200; that which was due at the end of the fourth year amounts, at the end of the fifth year, to $200 plus the interest on the same for 1 year; that which was due at the end of the third year, to $200 plus its interest for 2 years; that due at the end of the second year, to $200 plus its interest for 3 years; that due at the end of the first year, to $200 plus its interest for 4 years. Hence, the sums due at the end of the fifth year would form an arithmetical series, 200, $200 + 12$, $200 + 24$, $200 + 36$, $200 + 48$, of which the first term is $200, the last term the amount of $200 for the number of years less 1, and the number of terms the number of years. Hence the sum may be found by Art. 490; and, generally,

To find the amount of an annuity at simple interest: *Find the sum of an arithmetical series, of which the first term is the last payment, the last term the amount of the first payment, and the number of terms the number of payments.*

EXAMPLES.

1. What is the amount of an annuity of $300 for 6 years, at 6 per cent., simple interest? *Ans.* $2070.

2. What is the amount of an annuity of $600 for 7 years, at 7 per cent., simple interest?

3. A gentleman's salary of $1200 a year, payable quarterly, remained unpaid for 4 years; what was then his due?

ANNUITIES AT COMPOUND INTEREST.

506. ILL. EX. What is the amount of an annuity of $36 for 4 years, at 6 per cent., compound interest?

We will first find the amount of an annuity of $1 for the same time. The last payment, due at the end of 4 years, will be $1. The sum due on the third payment, at the end of the fourth year, will be the amount of $1 for 1 year; that due on the second payment will be the amount of $1 for 2 years; that due on the first payment will be the amount of $1 for 3 years. Hence the four sums due will form the geometrical series,

$$1, \quad 1.06, \quad 1.1236, \quad 1.19101, \text{ or}$$
$$1, \quad 1.06, \quad 1 \times (1.06)^2 \quad 1 \times (1.06)^3,$$

of which the first term is the last payment, the last term the amount of the first payment, and the number of terms the number of payments. Finding the sum of this series (Art. 297), and multiplying by 36, we obtain the required amount. Hence, the

RULE. To find the amount of an annuity at compound interest: *Find the amount of an annuity of* $1 *for the given time by geometrical progression* (Art. 497), *and multiply the sum thus obtained by the annuity.*

EXAMPLES.

1. What is the amount of an annuity of $1 for 2 years, at 6 per cent.? for 3 years? for 5 years? for 10 years?

2. What is the amount of an annuity of $20 for 8 years, at 5 per cent.? *Ans.* $190.98.

507. TABLE I.,

Showing the amount of $1 *or* £1 *annuity from* 1 *year to* 20.

Years.	5 Per Cent.	6 Per Cent.	Years.	5 Per Cent.	6 Per Cent.
1	1.000000	1.000000	11	14.206787	14.971643
2	2.050000	2.060000	12	15.917127	16.869941
3	3.152500	3.183600	13	17.712983	18.882138
4	4.310125	4.374616	14	19.598632	21.015066
5	5.525631	5.637093	15	21.578564	23.275970
6	6.801913	6.975319	16	23.657492	25.672528
7	8.142008	8.393838	17	25.840366	28.212880
8	9.549109	9.897468	18	28.132385	30.905653
9	11.026564	11.491316	19	30.539004	33.759992
10	12.577893	13.180795	20	33.065954	36.785591

NOTE.—The following examples may be performed by the use of Table I.

What are the amounts of the following annuities?

3. $100 for 7 years, at 5 per cent.

4. $200 for 10 years, at 6 per cent.

5. £150 for 18 years, at 6 per cent.

6. A gentleman, on his daughter's first birthday, and on each succeeding birthday, deposited $10 in a savings-bank, which yielded 5 per cent. compound interest, and presented her with the amount on her eighteenth birthday. What was the value of the present?

508. To find the present worth of an annuity: *Divide the amount of the annuity by the amount of $1 compound interest for the time given.* It may also be obtained by the use of the following table: —

TABLE II.,

Showing the present value of an annuity of $1 or £1 from 1 year to 20.

Years.	5 Per Cent.	6 Per Cent.	Years.	5 Per Cent.	6 Per Cent.
1	0.952381	0.943396	11	8.306414	7.886875
2	1.859410	1.833393	12	8.863252	8.383844
3	2.723248	2.673012	13	9.393573	8.852683
4	3.545950	3.465106	14	9.898641	9.294984
5	4.329477	4.212364	15	10.379658	9.712249
6	5.075692	4.917324	16	10.837770	10.105895
7	5.786373	5.582381	17	11.274066	10.477260
8	6.463213	6.209794	18	11.689587	10.827603
9	7.107822	6.801692	19	12.085321	11.158116
10	7.721735	7.360087	20	12.462210	11.469421

7. What is the present worth of an annuity of $200 for 4 years, at 5 per cent.? *Ans.* $709.19.

8. What must I pay for an annuity of $300 for 10 years, at 6 per cent.?

509. QUESTIONS FOR REVIEW.

Of what does ALLIGATION treat? What is Alligation Medial? Alligation Alternate? What other name might be used for Alligation? Make an example in Alligation Medial; perform and explain it, and

give the rule. Give the proof. Make an example in Alligation Alternate; perform and explain it, and give the rule. Give the proof. How many answers may you have to examples in Alligation Alternate? How do you proceed when one quantity is limited? when several quantities are limited? when the entire quantity is limited?

What is ARITHMETICAL PROGRESSION? What is an Arithmetical Series? When is a series increasing? when decreasing? Give examples of each. How many things are to be considered in a series? How many must be known, that the rest may be found? To what is the last term of a series equal? Show why? How do you find the common difference? Explain. How the number of terms? Explain. How the sum of the series? Explain.

What is GEOMETRICAL PROGRESSION? a Geometrical Series? What is the constant multiplier called? When the ratio is greater than unity will the series be increasing or decreasing? What will it be when the ratio is less than one? How many things are to be considered in a Geometrical Series? How many must be known, that the rest may be found? What is your rule for finding the last term? explain it. Your rule for finding the ratio? explain it. Your rule for finding the sum? explain it.

What are ANNUITIES? What is a Certain Annuity? a Perpetuity? a Life Annuity? a Pension? What is the amount of an annuity? the present worth? When are annuities in arrears? How are they generally computed? Give your rule for finding the amount of an annuity at simple interest. Illustrate it by an example. Give your rule for finding the amount of an annuity at compound interest. Illustrate it by an example. How do you use the table? How do you find the present worth of an annuity?

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510. MISCELLANEOUS EXAMPLES.

1. $\frac{1}{8}$ of a number exceeds $\frac{1}{9}$ by 20; what is that number?

2. The sum of two numbers being $4\frac{2\frac{1}{3}}{5\frac{1}{2}}$, and one of them being the difference between $\frac{45\frac{1}{10}}{11}$ and $\frac{24\frac{1}{2}}{7}$, what is the other?

3. From the product of the sum and difference of 3.6 and 2.24, take the difference between the squares of 3.6 and 2.24.

4. $\frac{3}{7}$ l. yr. $-\ \frac{3}{5}$ d. $+\ \dfrac{3.2}{1\frac{1}{2}}$ h. $-\ \dfrac{2.13\frac{1}{3}}{2\frac{2}{3}}$ min. $=$?

5. A can mow 2 acres in a day, B $2\frac{1}{2}$ acres, and C 3 acres; what is the smallest number of acres that will give a number of whole days' work for either?

6. What is the longest rail that will exactly fence either side of a lot of ground, the sides being 56 ft., 42 ft., 63 ft., 77 ft., and 49 ft.?

7. $\dfrac{27.5}{\frac{9}{8}} \times \dfrac{21}{.87\frac{1}{2}} = ?$ 8. $\dfrac{66\frac{2}{3}}{100} \times 4.75 \times 83\frac{1}{3} \times 27 \div \dfrac{3\frac{4}{9}}{2} = ?$

9. A quotient being $95\frac{5}{8}$, and the divisor $.33\frac{1}{3}$ of $3024\frac{9}{11}$, what is the dividend?

10. Bought a piece of cloth for \$75.30, sold $\frac{4}{7}$ of it to one person, and $\frac{4}{5}$ of the remainder to another; what is the value of the unsold part at 10% advance upon the cost?

11. An aeronaut ascends at the rate of $4\frac{1}{2}$ miles an hour for 40 minutes, after which he maintains the same elevation; if his balloon is driven east 7 miles during the first hour from the time of his starting, and in an opposite direction at the rate of 10 miles an hour for the remaining time, how far from his starting-point in a straight line is he at the end of 5 hours?

12. What is the weight of a bale of cotton cloth, containing 13 pieces, 42 yards to the piece, every 3 yards weighing $1\frac{1}{4}$ lbs.?

13. A trader bought apples at \$1.62$\frac{1}{2}$ per barrel, and immediately sold them at \$2.25, making \$234.37$\frac{1}{2}$; how many barrels were bought?

14. Divide $380 + 20 + 5 + (81 - 69) \times 5$ by $7.5 \div .5 + \dfrac{9.9 \div 3 - 1.8}{.1}$.

15. The sum of three numbers is $55\frac{1}{2}$; two of them are $14\frac{1}{4}$ and $24\frac{1}{12}$; what is the third?

16. Suppose a dividend to be 241.3, and the quotient .127; what was the divisor?

17. When the ice upon a pond is 10 inches thick, what will be the value of the ice taken from one acre of the pond at $\frac{1}{4}$ ct. per lb., 1 cu. ft. containing $58\frac{1}{8}$ lbs.?

18. A can do $\frac{1}{3}$ of a piece of work in 4 days, B $\frac{1}{2}$ of it in 5

days, and C $\frac{1}{4}$ of it in 2 days; in what time will all together do the whole?

19. $\frac{1}{2}+\frac{1}{3}+\frac{1}{5}+\frac{1}{6}$ of a certain number increased by $3\frac{1}{2}+10\frac{5}{7}=50\frac{5}{7}$; what is the number?

20. A man left $\frac{1}{2}$ of his property, wanting \$2000, to his daughter, $\frac{1}{5}$ of it to his son, and the balance to his widow, whose share was \$500 more than that of his daughter; what was the share of each? *Ans.* Widow's, \$7250; daughter's, \$6750; son's \$3500.

21. Of what number is $\frac{3}{4}$ the $\frac{7}{8}$ part?

22. A can do a piece of work in $1\frac{1}{5}$ hours, A and B in 48 min. in what time can B do it alone?

23. A cubic inch of infusorial earth in Germany, weighing 220 grains, was found to consist of 41 billions of infusoria; what was the weight of one skeleton?

24. CAMBRIDGE, *Sept.* 1, 1864.

Mr. JAMES H. EATON,

To FRANK CALDWELL, *Dr.*

To 40000 ft. pine boards,	@	\$12.00	per	M.
" 15625 " walnut,	@	15.00	"	"
" 2875 " scantling,	@	3.75	"	"
" $23\frac{1}{2}$ thousand shingles,	@	4.50	"	"
" 23 planks, 12 ft. long, 11 in. by 3 in.,*	@	12.50	"	"
" 12 sticks timber, 29 ft. long, 10 in. by 12 in.,	@	13.62	"	"

Required the cost of the above? *Ans.* \$887.79

25. A merchant buys gloves at 3s., and sells them at 4s. 6d.; what does he gain in laying out 50 £?

26. Divide 52 into two such parts that $\frac{1}{5}$ of one part shall equal $\frac{2}{3}$ of the other. *Ans.* 12 and 40.

27. A person being asked the time of day, answered that it was between two and three o'clock, and that $\frac{3}{4}$ of the time past two equalled $\frac{5}{6}$ of what it wanted of three. Required the time.

28. There is a pole 18 ft. long, standing in such a position that $\frac{1}{4}$ of the part in the mud is equal to $\frac{1}{6}$ of the part in the water, and to $\frac{1}{14}$ of the part in the air; how many feet are there in each element?

Ans. 3 ft. in the mud, $4\frac{1}{2}$ ft. in the water, and $10\frac{1}{2}$ ft. in the air.

* See Note, page 141.

29. Suppose the water to rise so that $\frac{1}{4}$ of the part in the mud is equal to $\frac{1}{8}$ of the part in the water, how many feet are there in each element?

Ans. 3 ft. in the mud, 6 ft. in the water, and 9 ft. in the air.

30. Butter being worth 25 cents per lb., if $\frac{3}{5}$ of a lb. will pay for $\frac{5}{7}$ of a dozen eggs, how many eggs will be required to pay for 6 lbs. of raisins, 7 lbs. of which cost 98 cents?

31. A, B, and C shared \$102 between them, so that A had \$17 more than B, and B had \$20 more than C; what had each?

32. The captain of a ship at sea finds by his chronometer, at 12 o'clock at noon, that it is 45 m. past 8 o'clock in the evening at London; what is his longitude?

33. When it is 10 A. M., in X, which is 44° 15′ 2″ W. long., what is the time in Y, which is 8° 4′ 40″ E. long.?

34. When it is half past 10 o'clock, P. M., of Dec. 31, 1865, in Albany, N.Y., what is the time in Constantinople? (See Art. 206.)

35. What is the exact time from Nov. 19, 1858, 18 min. of 4, P. M., to Apr. 9, 1863, 10 min. past 2, P. M.?

Ans. 4 yrs. 140 d. 22 h. 28 min.

36. Paid $6\frac{1}{4}$ cents a pound for 3200 lbs. of pork; on the sale of $\frac{2}{3}$ of it I gained $\$8\frac{1}{3}$; with the money received I purchased $7\frac{1}{4}$ tons of plaster; how many cords of wood, of which 3 cords cost $\$22\frac{1}{2}$, would be worth 1 ton of the plaster?

37. If \$1239 were paid for harvesting the wheat on a lot of land 400 rods long, 350 rods wide, what should be paid for harvesting the oats upon a lot 500 rods long, 450 rods wide, the cost of harvesting oats being $\frac{4}{5}$ as much as for harvesting wheat?

38. A coal dealer purchased 500 tons of coal at \$7.50 per long ton, paid \$1 per ton for freighting, and sold it for \$11 by the short ton; what per cent. did he gain? *Ans.* $\$44\frac{16}{17}\%$.

39. When gold sells at 59% advance, how much can be bought with \$100 of good bank bills?

40. A city collector has .8% for collecting taxes; he pays into the treasury \$94625.64 after deducting his commission; what was the whole sum collected?

41. An army of 50000 men besieged a city for three months,

during the first month, 5% were lost by sickness and desertion; at the beginning of the second month an accession was made to their force equal to 20% of what remained; during the second month they lost 25% of the men, and during the last month 30% of what then remained were lost and detailed for service elsewhere; how many were left?

42. A grocer imported 75 galls. of oil, which cost him $2 a gall. and a duty of 10%. Suppose 5 galls. to leak out, for what must he sell the remainder per gall. to gain 10% on the money spent?

43. If you buy figs at the rate of 9 lbs. for $1.50, and sell them at the rate of 10 lbs. for $2, what do you gain per cent.?

44. Pedrick & Closson sold at auction,

2 mattresses,	at $16.00,	which	cost	$13.50.
8 chairs,	at 4.62,	"	"	3.75.
1 rocker,	at 17.50,	"	"	17.00.
1 set furniture,	at 38.00,	"	"	42.00.
1 " "	at 83.50,	"	"	62.00.

They also sold on commission, at 10%,

5 chairs, at $8.00	1 table, at $8.00.
12 " at 1.70.	1 lounge, at 12.00.
1 bureau, at 18.00.	1 stove, at 17.00.

What were their net proceeds from the above sales?

45 I found, on going to Rand & Tyler's dry goods store, that they had that morning marked up their goods 15%; what did I save by purchasing the following goods the day before?

18 yds. blk silk, at	$1.12.
13 " de laine, at	.27.
9 " cambric, at	.15.
3 " silesia, at	.25
1 waterproof, at	8.00.

46. Paid 84 cents a gallon for a cask containing 27 galls. of kerosene, 10% of which leaked out; if the remainder was sold 25% on the gallon higher than it cost, what was the gain or loss on the money invested?

47. Sold 6 sewing machines at $72 each. On two of them I

gained 20%, on two others $33\frac{1}{3}$%, and on the others I lost 25%; what was the balance of gain or loss?

48. I sell a lot of carrots at \$13.26 per ton, and take in payment a note for 2 months without interest, gaining $8\frac{1}{3}$%; what did I pay per ton?

49. What is the interest on £73 from Oct. 21, 1858, to May 11, 1860?

50. What is the amount, at compound interest, of \$100, from Apr. 1, 1860, to Jan. 1, 1863, at 5% per annum, interest payable semi-annually?

51. The interest on a note for 2 y. 10 d., at 7%, is \$141.94$\frac{4}{5}$; what is the face of the note?

52. In what time will \$98, on interest at 7%, amount to \$123.48?

53. What principal will amount to \$185.50 in 3 y. 9 m. 20 d., at 7%?

54. What would be due May 1, 1865, on a note for \$1000, dated March 26, 1860, at 8% interest, on which \$200 were paid at the end of each year from the date of the note?

55. N. T. Allen bought, June 8, 1861, 10 bales of cotton cloth, 14 pieces in a bale, 43 yds. in a piece, at 8 cts. per yd., for which he gave his note on interest at 6%. On the 4th of Nov., 1863, he sold 1 bale at 30 cts. a yd., and with the proceeds made part payment of his note. On the 3d of May, 1864, he sold 1 bale at 40 cts., and paid on his note the amount he received. On the 17th of Sept., 1864, he sold the remainder at 60 cts., and settled the note. What did he gain by his speculation?

56. George Rivers bought \$1000 worth of government stock at par, bearing $7\frac{3}{10}$% interest in U. S. currency. At the end of 3 years, he converted it into five-twenty 6% bonds, interest payable in gold. Gold being at a premium of 100%, does he gain or lose by the exchange, and what %? *Ans.* Gains annually $4\frac{7}{10}$%.

57. I buy United States ten-forty bonds at par, interest being 5% per annum, payable semi-annually in gold, and gold being at a premium of 150%. What rate per cent. in currency, payable annually, do my proceeds equal? *Ans.* $12\frac{1}{8}\frac{1}{9}$%

58. A pays $1075 for United States five-twenty 6% bonds, at a premium of 7½%, the interest on the bonds being paid semi-annually in gold. If the average premium on gold be 112%, does he make more or less, and how much, than B, who invested an equal sum in railroad stocks, at 14% below par, which paid a semi-annual dividend of 4%? *Ans.* $13.60 more, semi-annually.

59. The city tax of Lowell being ¾%, and the state and county tax .15%; for what sum is Samuel Lowe taxed, who pays $56.22, including $1.50 poll-tax?

60. An agent, who purchased a lot of wheat, forwarded his bill for $568.87½. If this included his commission of 2½% on the purchase, what sum was paid for the wheat?

61. What is the difference between the true and bank discount of $700, due in 90 days, where the legal rate is 7%?

62. Write a note for 60 days, for which you could get $300 at a bank, discount being 6%.

63. How much would you receive from a bank, June 12, 1860, for a note of $820, dated April 12, 1860, payable 6 months after date?

64. A bookbinder holds a bill against a publisher for work to the amount of $600, payable in 6 months without interest. He offers to discount 5% of the bill for present payment. If the publisher pays $300 of the debt, what will still be due?

Ans. $284.21+.

65. The government tax on all bank dividends for 1864 was 3%. A certain bank, having declared a dividend of $15000, paid to the government $450, and subsequently paid the full sum of $15000 to the stockholders. To this the government objected. What was the error at the bank? What sum should have been paid to the government?

66. Blake Brothers & Co. purchased to order oil stocks to the amount of $5714.25, including their commission of ¼%; the stock, the par value of which was $50 per share, was purchased at 95%; how many shares were purchased? *Ans.* 120 shares.

67. Received from India 75 tierces of rice, invoiced at 220 lbs. each, for which I paid 3 cents a pound. A duty of ¾ cents per

pound was paid at the custom house, after 5% of the weight had been deducted for tare. If the rice should net what it was invoiced at, for what should it be sold to gain 20%?

68. A, B, and C formed a partnership. A furnished $\frac{4}{9}$ of the capital, B $\frac{1}{3}$, and C the remainder. Their gains were equal to $12\frac{1}{2}$% of their capital. Of these C took $25\frac{1}{4}$ eagles as his share. Allowing a premium for the gold of $24\frac{1}{2}$%, what was the whole capital, and what the gain of A and B in bank notes?

69. Breck and Loring traded in hides for one year. Breck put in \$2000 at first; at the end of 3 months he withdrew \$700, and at the end of 7 months put in \$1000. Loring put in \$1200 at first, and \$500 more in 4 months. At the end of 6 months he withdrew \$200. The gain for the year was \$2355.75, of which Loring received \$1000 for conducting the business. What was the share of each?

70. What should be the date of a note given in payment of the balance of the following account?

Dr. MILES STANDISH in a/c with P. SINON. *Cr.*

1859.				1859.			
May 14	To balance old acc't,	\$960	30	Jan. 11	By Mdse.,	\$800	00
July 8	" Mdse.,	519	00	Sept. 8	" Cash,	475	60

Ans. May 5, 1860.

71. Required the cost of bricks to build the walls of a storehouse 25 ft. long, 20 ft. wide, and 30 ft. high, containing 2 windows 8 ft. by 4 ft. each, and 1 door 7 ft. by 6 ft., the walls being 2 ft. thick, and bricks \$5.50 per thousand, measuring 8 in. by 4 in. by 2 in.

72. Required the cost of boards, at \$20 per thousand feet, to make a box 7 ft. 10 in. long, 3 ft. 8 in. wide, and 2 ft. 6 in. high; boards to be 1 in. thick. *Ans.* \$2.20$\frac{2}{3}$.

73. A wine merchant used the following receipt for port wine: 35 gallons prepared cider, worth \$1.00 per gallon; 5 gallons red wine, at \$2.00 per gallon; 5 gallons port wine, at \$5.00 per gallon; 3 gallons spirits, at \$1.00 a gallon; 3 pounds sugar, at 16 cents per pound; 2 ounces tincture of kino, at 6 cents an ounce;

and 1 ounce tartaric acid, at 13 cents. Suppose the sugar, kino, and acid do not add to the bulk of the mixture, and that the merchant sells it at \$4.50 per gallon, what per cent. does he gain?

74. If a pipe $2\frac{1}{2}$ inches in diameter will fill a cistern in two hours, in what time will a pipe 5 inches in diameter fill the same?

75. What is the length of the edge of the largest cube that can be sawed from a globe 9 inches in diameter?

76. The ridge-pole of a house is 46 ft. from the ground, the eaves 38 ft., the rafters on each side of the roof being 18 ft. long, what is the width of the house?

77. Required the edge of a cubical box that will contain 125 times as much as a box measuring 1 foot each way.

78. The pyramid of Cheops, in Egypt, is said to contain 82111000 cubic feet of masonry, and to have been 480 feet high. Allowing 7000000 cubic feet, which are required to perfect its pyramidal form and to fill its chambers, what is the length of one side of its base, which is a square? *Ans.* 746.2 ft.+.

79. How many yards of cloth $\frac{5}{8}$ yd. wide will be required to cover the sides and top of a cubical box containing 6751.269 cubic inches?

80. What will be the cost of digging a ditch outside a square garden containing 12.75 rods, the ditch to be 7 ft. wide and 5 ft. deep, at 1 cent per cubic foot?

81. How much would the earth taken out of the ditch raise the surface of the garden?

82. How many gallons in a cylindrical jar, the radius of whose base is 1 foot, and whose altitude is 4 ft.?

83. What will be the thickness of a square stick of timber which contains 4.542098 tons (50 cu. ft. = 1 ton), the stick being 100 ft. long?

84. Supposing a cubic foot of snow to weigh 31 lbs., what will be the pressure of a body of snow 9 inches deep upon a flat roof 100 ft. by 25 ft.?

85. Required the number of square feet in the surface of a ditch surrounding a circular garden which is 25 yards across, the ditch being $2\frac{1}{2}$ ft. across.

86. How many gallons will the above ditch contain, it being 5 ft. deep?

87. The circular outlet to a cistern being 4 inches in diameter, what must be the width of a rectangular receiving-pipe, whose depth is 2 inches, that its capacity may be the same as the discharging-pipe?

88. What will a pine log weigh whose length is 18 ft., measuring 3 ft. across the larger end, and $2\frac{1}{2}$ ft. across the smaller, pine being .6 as heavy as water, which weighs $62\frac{1}{2}$ lbs. to a cubic foot? (See Art. 455.)

89. How many cubic yards in a cellar whose side walls measure, on the outside, 70 ft., and whose end walls measure 48 ft., the cellar being 10 ft. deep, and the walls 3 ft. thick?

90. Suppose there is a globe of ice in the region of the Alps weighing 243474 lbs.; there being 930 oz. to a solid foot, what is its diameter? *Ans.* 20 ft.

91. Suppose $\frac{1}{4}$ of the above globe of ice to melt away each year, what will be the length of its diameter each succeeding year?

92. An engineer planted a battery near the bank of a river to shell a fort upon the opposite side. To ascertain the distance of the fort, he noted the direction of the fort from the mortar; then, placing himself at a point eight rods higher up the river, he caused a line to be drawn from a point six feet distant from himself, in range with the mortar, to be extended parallel with the line first noted till it ranged between himself and the fort. This line he found to be 480 feet. What was the distance of the fort from the mortar? (Page 294, Note.) *Ans.* 2 miles.

93. Wishing to know the height of a flagstaff which was 60 feet distant, I held my cane perpendicularly so that its lower end was 2 ft. 4 in. in a horizontal line from my eye, and found the range of the top of the staff was 35 inches from the bottom of the cane. Required the height of the flagstaff, allowing my eye to have been 5 ft. from the ground, which was a horizontal plane. *Ans.* 80 ft. high.

APPENDIX.

SOME OF THE PROPERTIES OF 9.

511. *Any number may be separated into two parts, one of which is divisible by* 9, *and the other of which is equal to the sum of its digits.*

ILLUSTRATION. Let 5864 be the number considered.

$$5864 = \begin{cases} 5000 = 5 \times (999+1) = 5 \times 999 + 5 \\ +800 = 8 \times (99+1) = 8 \times 99 + 8 \\ +60 = 6 \times (9+1) = 6 \times 9 + 6 \\ +4 = 4 \end{cases}$$

$$5864 = (5 \times 999 + 8 \times 99 + 6 \times 9) + (5 + 8 + 6 + 4).$$

$\therefore$ 5864 is separated into two parts, the first $(5 \times 999 + 8 \times 99 + 6 \times 9)$ being divisible by 9, and the second $(5+8+6+4)$ being the sum of its digits. The same can be shown of any number.

The following principles are derived directly from Art. 511 :—

512. *Any number is divisible by* 9, *if the sum of its digits is divisible by* 9.

513. *If any number is divided by* 9, *the remainder is equal to the remainder when the sum of its digits is divided by* 9.

514. PROOF OF MULTIPLICATION BY CASTING OUT THE 9's. (See Art. 50.)

MULTIPLICATION.	PROOF.
326	$3+2+6=\not 9+2$
42	$4+2=6$
652	$\overline{12}.\ 1+2=3$, remainder.
1304	$1+3+6+2=12=\not 9+3$, remainder.
13692	These remainders being equal, the work is probably correct.

DEMONSTRATION OF PROOF.

$326 \div 9$ leaves an excess of 2.
$42 \div 9$ leaves an excess of 6.

$$\begin{array}{r} 326 = 324 + 2 \\ 42 = 36 + 6 \\ \hline 324 \times 6 + 2 \times 6 \\ 324 \times 36 + 2 \times 36 \\ \hline 324 \times 36 + 324 \times 6 + 2 \times 36 + 2 \times 6 \end{array}$$

We separate the multiplicand and multiplier each into two parts, the first part being divisible by 9, and the second part being the excess of 9's. Multiplying these numbers thus separated, we obtain four terms for a product, the first three of which are divisible by 9, and the last is the product of the two excesses. The entire product divided by 9 must, therefore, leave the same remainder as the product of the excesses in the multiplicand and multiplier divided by 9.

515. PROOF OF DIVISION BY CASTING OUT THE 9'S. (See Art. 62.)

DIVISION.

$$\begin{array}{r} 75\,)\,3929\,(\,52 \\ 375 \\ \hline 179 \\ 150 \\ \hline 29 \end{array}$$

PROOF.

$$\begin{array}{r} 7 + 5 = 12 = \cancel{9} + 3 \\ 5 + 2 = 7 \\ \hline 21 \end{array} \quad 2 + 1 = 3, \text{ remainder.}$$

$3929 - 29 = 3\cancel{9}00, 3$, remainder.

These remainders being equal, the work is probably correct.

DEMONSTRATION OF PROOF.

The dividend, minus the remainder, equals the product of the divisor and integral part of the quotient; therefore, if we divide the dividend, minus the remainder, by 9, the remainder thus obtained must be the same as that which results from dividing the product of the excess of 9's in the divisor and the integral part of the quotient by 9.

CONTRACTIONS IN MULTIPLICATION AND DIVISION.

Arithmetical operations may sometimes be shortened materially by the use of contractions in Multiplication and Division. A few have been suggested in Articles 52, 53, and 64. Some additional contractions are here given, which pupils are cautioned *against* using until they are so familiar with the common methods *as to make no mistakes.*

516. To Multiply by 9, 99, 999, &c.

9 being one less than 10, 99 one less than 100, and 999 one less than 1000, &c.,

To multiply by any number whose terms are all 9's: *Annex as many zeros to the multiplicand as there are 9's in the multiplier, and from that product subtract the multiplicand;* thus, $27 \times 99 = 2700 - 27 = 2673$.

EXAMPLES.

1. $36 \times 99 = ?$	4. 36841×9999?
2. $264 \times 999 = ?$	5. $7 \times 9999999 = ?$
3. $58 \times 9999 = ?$	6. $245 \times 999999 = ?$

7. $241 \times 998 = ?$ $(241 \times 998 = 241 \times 1000 - 241 \times 2.)$
Ans. 240518.

8. $356 \times 9995 = ?$	9. $54932 \times 999997 = ?$

517. To Multiply by a Composite Number, *i. e.*, by a Number that is itself the Product of two or more Numbers.

Separate the multiplier into convenient factors, multiply the multiplicand by one of the factors, and that product by another factor, and so on, till all the factors are employed; the last product is the true answer; thus, $41 \times 25 = 41 \times 5 \times 5$.

EXAMPLES.

1. Multiply 368 by 72; by 36.
2. Multiply 4079 by 81; by 48.
3. Multiply 2145 by 108; by 144.
4. Multiply 50411 by 55; by 150.

518. To Multiply by Aliquot Parts of 10, 100, 1000, &c.

Multiply by 10, 100, 1000, *&c., as the case may require, and then find the required part;* thus, to multiply by 5, multiply by 10, and divide the product by 2; to multiply by 25, multiply by

100, and divide by 4; by 125, multiply by 1000, and divide by 8; by $33\frac{1}{3}$, multiply by 100, and divide by 3; by $16\frac{2}{3}$, multiply by 100, and divide by 6; by $12\frac{1}{2}$, multiply by 100, and divide by 8.

EXAMPLES.

1. $8743008 \times 5 =$ what?	4. $875402 \times 3\frac{1}{3} =$ what?
2. $8003478 \times 25 =$ what?	5. $1090806 \times 16\frac{2}{3} =$ what?
3. $786342 \times 12\frac{1}{2} =$ what?	6. $543297 \times 125 =$ what?

519. TO DIVIDE BY A COMPOSITE NUMBER.

ILL. EX. Divide 390 by 15.

OPERATION.

3) 390
5) 130
26, *Ans.*

To divide by a composite number: *Separate the divisor into convenient factors, divide by one factor, and the quotient thus obtained by another factor, and so on till all the factors are employed. The last quotient is the answer required.*

EXAMPLES.

1. Divide 243872 by 32.	3. $8954 \div 121 =$ what?
2. Divide 39726 by 18.	4. $49176 \div 72 =$ what?

520. TO FIND THE TRUE REMAINDER.

ILL. EX. Divide 83248 by 84.

OPERATION.

	Rem.	
3) 83242		
4) 27747	1.	1
7) 6936	3. $3 \times 3 =$	9
990	6. $6 \times 12 =$	72
	True remainder,	82

In this example we have remainders after the several divisions. The first remainder is of the same denomination as the first dividend, or units. The second remainder is of the same denomination as the second dividend, or 3's. 3 threes $=9$ units. The third remainder is of the same denomination as the third dividend or 12's. 6 twelves $=72$ units. The entire remainder equals the sum of these several remainders, or $1+9+72=82$. Hence,

To find the true remainder: *Commence with the remainder resulting from the second division, and multiply each partial remainder by all the preceding divisors except the one which gave that remainder,* and add the sum of the products to *the remainder resulting from* the first division.

EXAMPLES.

1. $86543 \div 117 =$ what?
2. $234567 \div 324 =$ what?
3. $359762 \div 187 =$ what?
4. $32947 \div 132 =$ what?
5. $927638 \div 2800 =$ what?
6. $7362851 \div 693 =$ what?

521. TO DIVIDE BY ALIQUOT PARTS OF 10, 100, 1000, &c.

To divide by 5, divide by 10, and multiply the quotient by 2; to divide by 25, divide by 100, and multiply by 4; by 125, divide by 1000, and multiply by 8; by $33\frac{1}{3}$, divide by 100, and multiply by 3; by $16\frac{2}{3}$, divide by 100, and multiply by 6; by $166\frac{2}{3}$, divide by 1000, and multiply by 6, &c.

EXAMPLES.

1. $9876 \div 25 = ?$
2. $34543 \div 125 = ?$
3. $87096 \div 16\frac{2}{3} = ?$
4. $432872 \div 12\frac{1}{2} = ?$
5. $687904 \div 250 = ?$
6. $110748 \div 166\frac{2}{3} = ?$

MODES OF ESTIMATING THE TIME BETWEEN TWO DATES.

522. The mode adopted in this book for estimating the time between two dates, when interest is computed in months and days, is in common use among business men, and is the one most consistent with the ordinary method of computing interest at 30 days to the month.

523. Another mode of estimating the time between two dates, is to find the number of ENTIRE calendar months, and count the remaining days.

524. A third mode of finding the time between two dates, consists in counting the exact days from one date to the other.

ILLUSTRATION.

By the first of these modes, if a note for months, which matures* Feb. 10th, 1865, were discounted at a bank the 11th of December previous, the time would be estimated at 1 day less than two months; viz., 1 month and 29 days.

By the second mode, it would be estimated at 1 month and 30 days, *i. e.*, 2 months.

* A note is said to mature when it becomes due.

By the third mode, it would be estimated at 61 days, or 2 months and 1 day.

In the above, and in all similar cases, the second of these methods will give more time than the first, and the third will give more time than the first or second.

If, however, the above note matured March 10, and were discounted Jan. 11, whilst the time by the first mode would be 1 day less than 2 months, — viz., 1 month, 29 days, — by the second mode it would be 1 month, 27 days, and by the third mode, 58 days, or 1 month, 28 days. In this case, the third mode gives less time than the first, and the second, less time than the third or first. These differences arise from the difference in the length of the calendar months; and, since a majority of the months have 31 days, the second and third modes will, on the whole, give more time than the first, and the third more time than the first or second.

To avoid these irregularities, it is customary to make notes, running for short times, payable in 30, 60, or 90 days, instead of 1, 2, or 3 months.

Note 1. — By custom, at the banks, a note which is given for months matures on the day corresponding with its date: if the month in which it matures has no corresponding day, it matures on the last day of the month. Thus, four notes dated severally the 28th, 29th, 30th, and 31st of Dec., 1864, and given for two months, all mature Feb. 28, with grace, March 3; while one dated Feb. 28 would mature April 28 and May 1.

Note 2. — A note falling due on the Sabbath, or on a legal holiday, must be paid on the business day next preceding. Thus, when a holiday occurs on Monday, notes maturing that day must be paid on the previous Saturday.

To Compute Interest for Exact Days.

525. The interest for days is calculated, in some of the United States, in Great Britain, and by the United States government, at 365 days to the year; 1 day's interest being considered $\frac{1}{365}$ of a year's interest. By the ordinary method it is calculated at 360 days to the year; this gives a year's interest for $\frac{365}{360}$ of a year, which is $\frac{5}{365}$, or $\frac{1}{73}$ too much. Hence,

To obtain the true interest for days: *Subtract from the interest found by the ordinary method, $\frac{1}{73}$ of itself.*

A TABLE

Showing the Number of Days

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January,	365	31	59	90	120	151	181	212	243	273	304	334
February,	334	365	28	59	89	120	150	181	212	242	273	306
March,	306	337	365	31	61	92	122	153	184	214	245	275
April,	275	306	334	365	30	61	91	122	153	183	214	244
May,	245	276	304	335	365	31	61	92	123	153	184	214
June,	214	245	273	304	334	365	30	61	92	122	153	183
July,	184	215	243	274	304	335	365	31	62	92	123	153
August,	153	184	212	243	273	304	334	365	31	61	92	122
September,	122	153	181	212	242	273	303	334	365	30	61	91
October,	92	123	151	182	212	243	273	304	335	365	31	61
November,	61	92	120	151	181	212	242	273	304	334	365	30
December,	31	62	90	121	151	182	212	243	274	304	335	365

Note. — In leap years, if the last day of February is included in the time, a day must be added to the number obtained from the table.

MEASUREMENT OF LUMBER.

526. The contents of boards, and of hewn and round timber, whether they are of uniform dimensions throughout or taper regularly, may be found by rules explained in Mensuration. The following additional directions for finding their contents may be serviceable: —

Board Measure.

527. If a board be 1 inch or less in thickness, its contents are found *by multiplying its length by its mean breadth.* The mean breadth of a board tapering regularly is *half the sum of the breadth of its two ends.* If it is irregular in shape, *the average of a number of measurements at equal distances must be used as the mean breadth.* If the board is more than 1 inch thick, *the*

square contents in feet must be multiplied by the thickness in inches. Thus, a board whose length is 7 feet, mean breadth 2 feet, and thickness $\frac{1}{2}$ inch, contains 14 feet, board measure; but a board whose length and breadth are the same as given above, and whose thickness is 2 inches, contains 28 feet, board measure.

EXAMPLES.

1. Required the contents of a board 10 ft. long and 1 in. thick, which is 2 ft. wide at one end, and 1 ft. 6 in. wide at the other.

2. Required the contents of a board 14 ft. long, $\frac{3}{4}$ in. thick, and measuring at the ends 2 ft. 3 in. and 1 ft. 9 in. respectively.

3. Required the contents of a board 16 ft. long and $1\frac{1}{2}$ in. thick, whose mean breadth is 2 ft. 5 in.

MEASUREMENT OF TIMBER.

528. The contents of timber are often estimated in board measure, and are found *by multiplying the product of the length and mean breadth expressed in feet, by the mean depth expressed in inches.* If estimated in cubic measure, *the product of the length, mean breadth, and mean depth, expressed in the same dimensions, must be found.*

EXAMPLES.

1. Required the contents of a piece of joist 20 ft. long, whose mean breadth is 4 inches, and mean depth 3 inches, board measure.

2. Give the contents in cubic measure.

3. How many feet, board measure, in a stick of timber 15 ft. long, the breadth at the ends being 6 in. and 4 in., and the depth at the ends 4 in. and $2\frac{1}{2}$ in. respectively?

529. The contents of round timber may be obtained *by ascertaining its mean diameter, and from that, as a basis, estimating its contents as if it were a cylinder,* or *by squaring $\frac{1}{5}$ of the mean girt, and multiplying the square by twice the length.*

1. How many cubic feet in a stick of round timber, whose mean girt is 8 ft., and whose length is 12 ft.?

MISCELLANEOUS.

530. **Shingling,** and other plain work, as flooring and partitioning, are generally estimated by the square of 100 feet. 1000 shingles are allowed for a square.

531. **Painting** is measured by the square yard.

532. **Plastering** is measured by the square foot, square yard, or square of 100 ft.

533. **Glazing** is measured by the square foot, including the sash.

534. **Paving** is measured by the square foot, or square yard.

535. **Bricklaying** is generally estimated by the thousand bricks; sometimes it is estimated by the square yard, square rod, or square (of 100 ft.), allowing $1\frac{1}{2}$ bricks, or 12 in. in thickness.

A great variety of methods for measuring prevail. Some workmen make no allowance for doors and windows, others make allowance of half the space occupied by doors and windows, and others still estimate the exact amount of material and labor employed. Measurements are taken on the outside of walls, no allowance being made for corners. In estimating the number of bricks used, an allowance of one tenth of the solid contents is made for mortar.

GAUGING.

536. **Gauging** is the process of finding the capacity of casks, or other vessels, in gallons or bushels.

537. To find the capacity of a cask or barrel:

Add the squares of the head diameter, of the bung diameter, and of twice the middle diameter. Multiply the sum thus obtained by .0005667 *times the length, and the result will be the contents in gallons.*

NOTE. — The middle diameter is the diameter of the section midway between the bung and the head. The dimensions used in the above rule should be expressed in inches.

1. Find the capacity, in gallons, of a cask whose length is 40 inches, the head diameter 25 inches, the bung diameter 32 inches, and the middle diameter 29 inches. Ans. 113.634 gal.+

New Hampshire Rule for Annual Interest with Partial Payments.

538. When notes are given upon "annual interest" (Art. 277), and partial payments are made during the year, instead of following the United States rule (Art. 274), which makes a new principal at the time of each payment, —

1. *Compute annual interest upon the principal to the end of the first year in which any payments are made; also compute interest upon the payment or payments from the time they are made to the end of the year.*

2. *Apply the amount of such payment or payments, first to cancel any interest that may have accrued upon the yearly interests, then to cancel the yearly interests themselves, and then towards the payment of the principal.*

3. *Proceed in the same way with succeeding payments.*

4. *If, at the time of any payment, no interest is due except what is accruing during the year, and the payment or payments are less than the interest due at the end of the year, deduct such payment or payments at the end of the year, without interest added.*

Note. No interest should be computed beyond the time of settlement.

539. Examples.

1. A note for $2500, dated Oct. 1, 1860, was given on demand, with interest payable annually at 6 %.

Endorsements,	June 1, 1862,	$500.00.
	March 17, 1863,	87.94.
	Dec. 1, 1865,	1000.00.

What was due April 1, 1867?

Operation.

Principal,	$2500.00		Oct. 1, 1860.
Yearly interest,	150.00		to Oct. 1, 1861.
Yearly interest,	150.00		to Oct. 1, 1862.
Simple interest on $150.00,	9.00		from Oct. 1, 1861, to Oct. 1, 1862.
Amount,		2809.00	Oct. 1, 1862.
Payment,	500.00		June 1, 1862.
Interest,	10.00		to Oct. 1, 1862.
Amount of payment,		510.00	
Principal,		2299.00	Oct. 1, 1862.

PRINCIPAL bro't forw'd		$2299.00	
Interest,	137.94		to Oct. 1, 1863.
Payment,	87.94		March 17, 1863.
Balance yearly int.,	50.00		due Oct. 1, 1863.
Simple interest,	9.00		to Oct. 1, 1866.
Yearly interest,	137.94		to Oct. 1, 1864.
Yearly interest,	137.94		to Oct. 1, 1865.
Yearly interest,	137.94		to Oct. 1, 1866.
$137.94 at simple int.,	24.83		for 2 yrs. + 1 yr. = 3 yrs.
		497.65	
Amount,		2796.65	
Payment,	1000.00		Dec. 1, 1865.
Interest,	50.00		to Oct. 1, 1866.
Amount of payment,		1050.00	
PRINCIPAL,		1746.65	Oct. 1, 1866.
Interest,		52.40	to April 1, 1867.
Balance due,		1799.05	" "

Ans. $1799.05.

2. A note for $4200, given May 27, 1862, was payable on demand, with interest at 6 % annually; what was due on the note May 27, 1867, the following payments having been made:

Payments, { Aug. 9, 1865, $50.72.
Jan. 27, 1867, 3000.00. *Ans.* $2500.48.

3. A note given for $5000, on demand, at 6 % annual interest, was dated January 1, 1860, and endorsed as follows:

July 1, 1860, $50. July 1, 1865, $500.
" 1, 1861, 500. " 1, 1866, 2500.

What remained due after the last payment was made? *Ans.* $3530.65.

540. To Compute Interest at $7\frac{3}{10}$ %.

Multiply the principal by twice the number of days, and point off four places if the principal contains only dollars, six places if it contains cents.

Examples.

1. What is the simple interest of $100 for 12 days, at $7\frac{3}{10}$ %? *Ans.* $.24.

2. What is the simple interest of $300 for 9 days, at $7\frac{3}{10}$ %? *Ans.* $.54.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

NOTE.—The Metric System of weights and measures was first adopted in France in 1795. A length supposed to be one ten millionth of a quadrant, or one forty millionth of a circumference of the earth measured over the poles, was taken as a provisional measure for the base of the system; this length was called a METER.

In order to ascertain more accurately the length of a quadrant, new measurements of the earth were subsequently instituted under the direction of eminent mathematicians, who measured the arc of a meridian between the parallels of Dunkirk and Barcelona. From their measurements, the length of the meter now in use was determined. This length was adopted as the base of the system, in 1799. The use of the metric system was not, however, legally enforced, to the exclusion of any other system, until January 1, 1840.

In Spain, Portugal, and Belgium, this system is also used exclusively, while in many other countries it is adopted wholly or in part. Among these are Holland, Italy, Greece, Austria, Switzerland, and Poland, in Europe, and Mexico, Chili, Venezuela, Brazil, Ecuador, Guatemala, San Salvador, and the Argentine Republic, on this continent.

Movements are also being made to adopt it in England, Germany, Sweden, and Norway. Its use in the United States was legalized by an act of Congress, passed in July, 1866.

Notwithstanding so much has been done to make the meter exactly one ten millionth of a quadrant, it is now thought to be too short by a small fraction, which is, however, less than one eight thousandth of itself. The length of the meter is nearly 39.37079 English inches, or 39.3685 United States inches; but for ordinary purposes, may be considered 39.37 inches.

541. The Metric System is so called from the METER, which is the base of all the weights and measures which it employs.

The **Meter** is the primary unit of length, and equals about 39.37 inches, or nearly 3 ft. $3\frac{3}{8}$ in.

Upon the Meter are based the following primary units: the **Square Meter**, the unit of measure for small surfaces; the **Are**, the unit of land measure; the **Cubic Meter**, or **Stere**, the unit of volume; the **Liter**, the unit of capacity; and the **Gram**, the unit of weight.

From these primary units the higher and lower orders of [illegible] are derived decimally.

542. The *names* of the higher orders of units are formed by prefixing to the name of the primary unit the following, from the Greek numerals:—

Deka (10), **Hecto** (100), **Kilo** (1000), **Myria** (10000).

The names of the lower orders of units are formed by prefixing to the name of the primary unit the following, from the Latin numerals:—

Deci (10th), **Centi** (100th), **Milli** (1000th).

Consequently, the word *dekameter* signifies ten meters; *dekaliter*, ten liters; *hectometer*, one hundred meters; *hectogram*, one hundred grams; *kilometer*, one thousand meters; *myriameter*, ten thousand meters, etc.

So, also, the word *decimeter* signifies the tenth part of a meter; *centigram*, the hundredth part of a gram; *milliliter*, the thousandth part of a liter, etc.

MEASURES OF LENGTH.

NOTE.—In this table, and in those which follow, the name of the primary unit is designated by capitals, and the names of other important units by italics.

543. TABLE.

10 *mil′limeters* (mm)	= 1 centimeter,	marked	(cm).
10 cen′timeters	= 1 decimeter,	"	(decim).
10 dec′imeters	= 1 METER,	"	(m).
10 METERS	= 1 dekameter,	"	(dekam).
10 dek′ameters	= 1 hectometer,	"	(hm).
10 hec′tometers	= 1 *kilometer*,	"	(km).
10 kil′ometers	= 1 myriameter,	"	(myriam).

EXERCISES.

1. How many meters equal 1 dekameter? 1 hectometer? 1 kilometer? 1 myriameter?

2. How many dekameters equal 1 hectometer? 1 kilometer?

3. 1 meter equals how many decimeters? how many centimeters? how many millimeters?

544. The outer diagram in the margin represents a measure 4 inches in length; the inner diagram represents a measure 1 decimeter or 10 centimeters in length.

These diagrams will enable the pupil to compare the units of length of the metric system with those in common use.

NOTE 1.—The new five-cent piece (of 1866) is 2 centimeters in diameter.

NOTE 2.—25 millimeters, or $2\frac{1}{2}$ centimeters, nearly equal 1 inch.

NOTE 3.—5 meters are nearly equal to 1 rod.

NOTE 4.—1 kilometer is a little less than $\frac{5}{8}$ of a mile.

NOTE 5.—1 myriameter is a little more than $6\frac{1}{5}$ miles.

Although the meter is generally considered the unit of length, yet in estimating great distances, as the length of a road, of a river, the distance between two cities, etc., the kilometer is regarded as the unit; thus, the length of the Ohio River is 1528 kilometers, the distance from Troy to New York is 267 kilometers.

545. The manner of writing the different orders of units of length is illustrated by the following

TABLE.

Myriameter.	Kilometer.	Hectometer.	Dekameter.	METER.	Decimeter.	Centimeter.	Millimeter.
1	1	1	1	1.	1	1	1

In writing numbers by the metric system, the decimal point is usually placed at the right of the figure denoting the primary unit; thus, the number 5 meters, 9 decimeters, is written 5.9^{m}.

If in writing a number any intermediate orders of units are wanting, their places should be supplied by zeros; thus, 1 dekameter, 2 millimeters, is written 10.002^{m}.

Exercises.

Write the following in figures:—

1. Three dekameters, four meters. *Ans.* 34^{m}.
2. Seven hectometers, three meters. *Ans.* 703^{m}.
3. Three hectometers, one dekameter, five meters.
4. Three kilometers, two hectometers, seven meters.
5. Nine myriameters, five hectometers.
6. Two meters, four decimeters. *Ans.* 2.4^{m}.
7. Two meters, two centimeters, four millimeters.
8. Five dekameters, two decimeters, eight centimeters.

546. When numbers are expressed by figures, the part of the expression at the left of the decimal point is usually read in the denomination of the primary unit; the part at the right of the decimal point may be read either as a decimal part of the unit, or in the denomination indicated by the place of the last figure. Thus, in reading the expression 34.62^{m}, we may say either 34 and 62 hundredths meters, or 34 meters and 62 centimeters.

Exercises.

Read the following, giving the name of each order of units:—

1. 23^{m}. 2. 25.1^{m}. 3. 321.05^{m}. 4. 7137.008^{m}.

5. Read the above in the denomination of the units as indicated by the abbreviation ^{m}.

6. Read the same, giving each decimal part the denomination indicated by the place of the last figure.

Since the metric system is a decimal system, a number expressed in units of one order may be reduced to units of another order by multiplying or dividing by ten, or some power

of ten. If the number is written in figures, it is only necessary to remove the decimal point to the right of the figure indicating the required order, and give the expression its proper abbreviation; thus, 59.36^{m} may be reduced to 5.936^{dekam}, $.5936^{hm}$, $.05936^{km}$, $.005936^{myriam}$, 593.6^{decim}, $5936.^{cm}$, $59360.^{mm}$.

547. Examples.

1. Express 5.24 meters as centimeters. *Ans.* 524^{cm}.
2. Express 37.2 meters as kilometers. *Ans.* $.0372^{km}$.
3. Express 12 hectometers as meters. *Ans.* 1200^{m}.
4. Express 25 millimeters as meters. *Ans.* $.025^{m}$.
5. In 518 meters how many decimeters? how many centimeters? how many millimeters?
6. In 3687 metres how many dekameters? how many hectometers? how many kilometers?
7. Express in meters the following, and add them: 4075 centimeters, 27 dekameters, .075 kilometers. *Ans.* 385.75^{m}.
8. Express in kilometers the following, and add them: 2400 meters, 500 dekameters, .79 myriameters. *Ans.* 15.3^{km}.
9. If 7.08 kilometers are taken from 42 kilometers, how many meters will remain? *Ans.* $34{,}920^{m}$.
10. The distance round a certain park is 2.58 kilometers; how many meters will a man go who rides around it six times? *Ans.* $15{,}480^{m}$.
11. A schoolboy walked one third around the above park in 12 minutes; how many meters did he walk in 1 minute? *Ans.* $71.66^{m}+$.
12. The latitude of Chicago is 42° N.; how many kilometers is it from Chicago to the equator? *Ans.* $4666.66^{km}+$.

MEASURES OF SURFACE.

548. In the measurement of small surfaces the **Square Meter** is the primary unit.

Each side of a square meter is 10 decimeters in length, and ·e a square meter contains $10 \times 10, = 100$, square deci- ·s. Each side of a square decimeter is 10 centimeters in

length, and hence a square decimeter contains $10 \times 10, = 100$, square centimeters, etc. Thus it may be seen that while measures of length increase and decrease by a scale of tens, measures of surface increase and decrease by a scale of hundreds.

Since the values of units of surface increase and decrease by a scale of hundreds, it is necessary, in writing numbers denoting surfaces, to allow two places for sq. decimeters, two for sq. centimeters, and two for sq. millimeters: thus,

4 sq. meters, 8 sq. decimeters, are written $4.08^{\text{sq m}}$.

In what places at the right of the decimal point are sq. decimeters written? sq. centimeters? sq. millimeters?

Examples.

1. Express the following numbers in sq. meters and add them: 5 sq. decimeters, 87 sq. meters, 26 sq. centimeters, 5.9 sq. meters. *Ans.* $92.9526^{\text{sq m}}$.

2. How many sq. meters are there in a rectangular court 5 meters long and 22 meters wide? *Ans.* $110^{\text{sq m}}$.

3. What is the cost of polishing the surface of a rectangular piece of marble 2 meters 8 decimeters long, and 1 meter 2 decimeters wide, at $2.50 per sq. meter? *Ans.* $8.40.

549. We have seen that the meter and its subdivisions are used to measure small surfaces; but to measure surfaces of great extent, as a field, a township, etc., the **Are** is the primary unit.

The Are is a square whose side is 10 meters and whose surface contains 100 square meters.

In land measure, centares, ares, and hectares only are used.

Note.—The are equals 119.6 square yards, nearly 4 square rods, or about $\frac{1}{40}$ of an acre. The hectare equals about $2\frac{1}{2}$ acres.

Table.

100 cen′tares ($^{\text{ca}}$)	= 1 ARE,	marked	($^{\text{ar}}$).
100 ĀRES	= 1 *hectare*	"	($^{\text{ha}}$).

550. The following table shows the method of writing numbers in land measure, also the relation of the units to the square meter and its subdivisions.

TABLE.

Hectare.	Are.	Centare, or Sq. Meter.	Sq. Decimeter.	Sq. Centimeter.	Sq. Millimeter.
1	0 1.	0 1	0 1	0 1	0 1

EXAMPLES.

1. Express the following in ares and add them: 1.3 hectares, 155.5 ares, 43 hectares, 26 centares. *Ans.* 4585.76$^{ar.}$

2. Mr. Jenks owned 25 hectares, 32 ares, 16 centares of land, and afterwards bought 36 hectares, 5 ares, 8 centares; how many ares did he then have? *Ans.* 6137.24$^{ar.}$

3. A had 6 hectares, 7 ares, 9 centares of land, and sold $\frac{2}{11}$ of it at $54 an are; how many dollars did he receive for what he sold? *Ans.* $5960.52.

MEASURES OF VOLUME.

551. In the measurement of solids, the **Cubic Meter** is the primary unit.

Each edge of a cubic meter is 10 decimeters in length, and hence a cubic meter contains $10 \times 10 \times 10, = 1000$, cubic decimeters. Each edge of a cubic decimeter is 10 centimeters in length, and hence a cubic decimeter contains $10 \times 10 \times 10, = 1000$, cubic centimeters, etc.

Thus it may be seen that while measures of length increase and decrease by a scale of tens, measures of volume increase and decrease by a scale of thousands. Hence, in writing numbers denoting volume, three places must be allowed for cu. decimeters, three for cu. centimeters, and three for cu. millimeters.

In what places at the right of the decimal point are cu. decimeters written? cu. centimeters? cu. millimeters?

EXAMPLES.

1. Express the following numbers in cu. meters and add them: 58.5 cubic meters, 1.7 decimeters. *Ans.* 58.5017$^{cu\ m}$.

2. How many cu. meters in a cube whose edge is 2.7 meters?
Ans. 19.683$^{cu\ m}$.

3. How many cubic meters of air will a room contain whose length is 5.2 meters, whose breadth is 4 meters, and whose height is 35 decimeters? *Ans.* 72.8$^{cu\ m}$.

552. For measuring firewood, stone, etc., the **Stere** is the primary unit. Dekasteres and decisteres are also used.

The stere is a cubic meter, or 1.308 cubic yards, and is a little more than ¼ of a cord.

TABLE.

10 dec′isteres (^{ds})	= 1 STERE,	marked (^{s}).
10 STERES	= 1 dekastere	" (^{dekas}).

The method of writing numbers in wood measure is the same as that of writing numbers in measures of length. (Art. 545.)

EXAMPLES.

1. How many steres will a pile of wood contain that is 1 meter long, 1 meter wide, and 1 meter high? 2 meters high?

2. What part of a stere will a pile of wood contain that is 1 meter long, 1 meter wide, and 1 decimeter high?

3. How many steres in a pile of stone that is 1 meter wide, 8.24 meters long, and 4 decimeters high? *Ans.* 3.296^{s}.

4. What must be the height of a pile of wood 2.5 meters long and 1 meter wide, to contain a stere? *Ans.* 4^{decim}.

MEASURES OF CAPACITY.

553. In measuring liquids, as milk, and dry articles, as beans, barley, and salt, the **Liter** is the primary unit.

The Liter is 1 cubic decimeter, and contains .908 of a quart dry measure, or 1.0567 quarts liquid measure.

TABLE.

10 mil′liliters (^{ml})	= 1 *centiliter*,	marked (^{cl}).
10 cen′tiliters	= 1 deciliter,	" (^{decil}).
10 dec′iliters	= 1 LITER,	" (^{l}).
10 LĬTERS	= 1 dekaliter,	" (^{dekal}).
10 dek′aliters	= 1 *hectoliter*,	" (^{hl}).
10 hec′toliters	= 1 kiloliter,	" (^{kl}).

NOTE 1. — A kiloliter has the same capacity as a stere, or cu. meter.

NOTE 2. — A hectoliter equals about $2\frac{5}{6}$ bushels.

Numbers denoting capacity are written in the same manner as numbers denoting length. (Art. 545.)

EXAMPLES.

1. Express the following numbers as liters and add them: 458 centiliters, 82 dekaliters, 765 milliliters. *Ans.* 825.345^{l}.

2. From a vessel containing 1 hectoliter of oil were drawn 25 liters, 6 centiliters; how many liters remained? *Ans.* 74.94^{l}.

3. How many liters of wheat can be put into a bin that is 2 meters long, 1.3 meters wide, and 1.5 meters high? *Ans.* 3900^{l}.

4. What must be the length of a bin that is $1\frac{3}{5}$ meters wide and 1 meter high, to contain 4000 liters of grain? *Ans.* 2.5^{m}.

MEASURES OF WEIGHT.

554. The **Gram** is the primary unit of weight.

The Gram is the weight, in a vacuum, of a cubic centimeter of distilled water at the temperature when it is most dense, which is at $39\frac{1}{5}°$ Fahrenheit.

NOTE 1. — The gram equals 15.432 grains.

NOTE 2. — The new five-cent piece (of 1866) weighs 5 grams.

TABLE.

10 mil′ligrams (^{mg})	=	1 centigram,	marked	(^{centig}).
10 cen′tigrams	=	1 decigram,	"	(^{decig}).
10 dec′igrams	=	1 GRAM,	"	(^{g}).
10 GRĂMS	=	1 dekagram,	"	(^{dekag}).
10 dek′agrams	=	1 hectogram,	"	(^{hg}).
10 hec′tograms	=	1 KILOGRAM,	"	(^{kg}) or (^{k}).
10 KIL′OGRAMS	=	1 myriagram,	"	(^{myriag}).
10 myr′iagrams	=	1 quintal,	"	(^{q}).
10 quintals	=	1 *tonneau*,	"	(^{t}).

The kilogram, sometimes called the kilo, is considered the unit in weighing gross, heavy articles. The kilogram equals about $2\frac{1}{5}$ pounds avoirdupois, or, more nearly, 2.2046 pounds. The *tonneau* equals a little more than 2204 pounds.

Numbers denoting weight are written in the same manner as numbers denoting length. (See Art. 545.)

EXAMPLES.

1. Express the following numbers as grams and add them: 8.5 dekagrams, 1000 centigrams, 225 decigrams. *Ans.* 117.5^{g}.

2. Express the following numbers as kilograms and add them: 7.2 hectograms, 8294 grams, 4 quintals. *Ans.* 409.014^{kg}.

3. How many papers, each containing $\frac{1}{2}$ a kilogram, may be filled from 32 myriagrams of coffee? *Ans.* 640 papers.

4. Bought 1 tonneau of coal for $12; what is the cost of 1 kilogram of coal at the same rate? *Ans.* 12 mills.

5. What weight of mercury will a vessel contain whose capacity is 10 cubic centimeters, mercury being 13.5 times as heavy as water. *Ans.* 135 grams.

6. In 77.2 grams of gold how many cubic centimeters, gold being 19.3 times as heavy as water? *Ans.* 4$^{\text{cu centim}}$.

555.

METRIC MEASURES LEGALIZED BY THE UNITED STATES WITH THEIR EQUIVALENTS NOW IN USE.

NOTE.—Although the equivalents here given are not entirely accurate, they are those which are established by Congress for use in legal proceedings, and in the interpretation of contracts, and are sufficiently exact for all practical purposes.

Measures of Length.

METRIC DENOMINATIONS AND VALUES.		EQUIVALENTS IN DENOMINATIONS IN USE.
Myriameter	10,000 m. . .	6.2137 miles.
Kilometer	1,000 m. . .	0.62137 mile, or 3280 ft. 10 in.
Hectometer	100 m. . .	328 ft. 1 in.
Dekameter	10 m. . .	393.7 in.
Meter.	1 m. . .	39.37 in.
Decimeter	.1 m. . .	3.937 in.
Centimeter	.01 m. . .	0.3937 in.
Millimeter	.001 m. . .	0.0394 in.

Measures of Surface.

METRIC DENOMINATIONS AND VALUES.		EQUIVALENTS IN DENOMINATIONS IN USE.
Hectare	10,000 sq. m.	2.471 acres.
Are	100 sq. m.	119.6 sq. yards.
Centare	1 sq. m.	1550. sq. inches.

Measures of Capacity.

METRIC DENOMINATIONS AND VALUES.			EQUIVALENTS IN DENOMINATIONS IN USE.	
Names.	No. of Liters.	Cubic Measure.	Dry Measure.	Liquid or Wine Measure.
Kiloliter or Stere	1000	1 cu. m.	1.308 cu. yds.	264.17 gal.
Hectoliter . . .	100	.1 cu. m.	2 bu. 3.35 pks.	26.417 gal.
Dekaliter	10	10 cu. dm.	9.08 qts. . . .	2.6417 gal.
Liter	1	1 cu. dm.	0.908 qt. . . .	1.0567 qts.
Deciliter	.1	.1 cu. dm.	6.1022 cu. in.	0.845 gill.
Centiliter	.01	10 cu. cm.	0.6102 cu. in.	0.338 fld oz.
Milliliter	.001	.1 cu. cm.	0.061 cu. in. .	0.27 fld dr.

Weights.

METRIC DENOMINATIONS AND VALUES.			EQUIVALENTS IN DENOMINATIONS IN USE.
Names.	No. of Grams.	Weight of what quantity of water at maximum density.	Avoirdupois Weight.
Millier or Tonneau .	1,000,000	1 cu. meter . .	2204.6 pounds.
Quintal	100,000	1 hectoliter . .	220.46 "
Myriagram	10,000	10 liters	22.046 "
Kilogram or Kilo . .	1,000	1 liter	2.2046 "
Hectogram	100	1 deciliter . . .	3.5274 ounces.
Dekagram	10	10 cu. centim. .	0.3527 "
Gram	1	1 cu. centim. . .	15.432 grains.
Decigram	.1	.1 cu. centim. .	1.5432 "
Centigram	.01	10 cu. millim. .	0.1543 "
Milligram	.001	1 cu. millim. . .	0.0154 "

REDUCTION OF NUMBERS IN THE METRIC SYSTEM TO EQUIVALENTS NOW IN USE.

556. ILL. EX. In 5 meters how many inches? how many feet?

OPERATION.

$$\frac{39.37 \times 5}{12} = 16.40\tfrac{5}{12} \text{ ft., } Ans.$$

EXPLANATION. Since 1 meter equals 39.37 inches, 5 meters must equal 5 times 39.37 inches, and since 12 inches equal 1 foot, there must be as many feet as there are times 12 in 5 times 39.37, which is $16.40\frac{5}{12}$ times.

Ans. $16.40\frac{5}{12}$ ft

EXAMPLES.

1. In 1 meter, 2 decimeters, how many feet? *Ans.* 3.937 ft.
2. In 25 millimeters how many inches? *Ans.* .985 in.
3. How many inches long is a silkworm that measures 5.2 centimeters in length? *Ans.* 2.047 in.+.
4. What is the height of a person in feet and inches whose height is 1 meter, 728 millimeters? *Ans.* 5 ft. 8 in.+.
5. In 21 ares how many square yards? *Ans.* 2511.6 sq. yds.
6. In a field of 7 hectares, 2 ares, how many acres? *Ans.* 17.346 acres+.
7. In 32 centares how many square yards? *Ans.* 38.272 sq. yds.
8. In 12 steres of wood how many cubic yards? how many cords? *Ans.* 15.696 yds.; 3.31 cords+.
9. In 24 kiloliters how many gallons? *Ans.* 6340.08 gal
10. How many gallons of vinegar may be put into a cask containing 8 dekaliters, 2 liters? *Ans.* 21.66 gals.+.
11. In 2 hectoliters how many bushels and pecks? *Ans.* 5 bu. 2.7 pks.
12. In 23 kilograms how many pounds avoirdupois? *Ans.* 50.7 lbs.+.
13. In 28 kilograms, 7 hectograms, how many pounds avoirdupois? *Ans.* 41.22 lbs.+.
14. In 27 tonneaux how many tons? *Ans.* 29.762 tons+.

557. The following are some of the measures in common use, with their equivalents in measures of the metric system :—

An inch	= 2.54 centimeters.	A cu. yard	= .7646 cu. meter.
A foot	= .3048 meter.	A cord	= 3.624 steres.
A yard	= .9144 meter.	A liquid quart	= .9464 liter.
A rod	= 5.029 meters.	A gallon	= 3.786 liters.
A mile	= 1.6093 kilometers.	A dry quart	= 1.101 liters.
A sq. inch	= 6.452 sq. centimet's.	A peck	= 8.811 liters.
A sq. foot	= .0929 sq. meter.	A bushel	= 35.24 liters.
A sq. yard	= .8361 sq. meter.	An ounce av.	= 28.35 grams.
A sq. rod	= 25.29 sq. meters.	A pound av.	= .4536 kilogram.
An acre	= .4047 hectare.	A ton	= .9072 tonneau.
A sq. mile	= 259 hectares.	A grain Troy	= .0648 gram.
A cu. inch	= 16.39 cu. centimet's.	An ounce Troy	= 31.104 grams.
A cu. foot	= .02832 cu. meter.	A pound Troy	= .3732 kilogram.

REDUCTION OF MEASURES NOW IN USE TO EQUIVALENTS IN THE METRIC SYSTEM.

558. ILL. EX. In 5 feet how many meters?

OPERATION.

$.3048 \times 5 = 1.524^{m}$, *Ans.*

EXPLANATION. Since 1 foot equals .3048 meters, 5 feet must equal 5 times .3048 meters, which is 1.524 meters.

Ans. 1.524^{m}.

EXAMPLES.

1. In 2 rods how many meters? *Ans.* 10.058^{m}.
2. In 25 sq. yards how many sq. meters? *Ans.* $20.9^{sq\,m}$.
3. In 23 acres how many hectares? *Ans.* $9.308^{ha}+$.
4. In 8.2 cords how many steres? *Ans.* $29.725^{s}+$.
5. In 9.2 liquid quarts how many liters? *Ans.* $8.706^{l}+$.
6. In 28 grains how many grams? *Ans.* $1.814^{g}+$.
7. In 3 yards 1 foot how many meters? *Ans.* 3.048^{m}.
8. In 2 bushels 3 pecks how many liters? *Ans.* 96.921^{l}.
9. In 8 pounds 7 ounces how many kilos? *Ans.* $3.627^{k}+$.

☞ For a fuller treatment of this subject see Walton's pamphlet edition of "The Metric System of Weights and Measures."

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W. G. Whipple